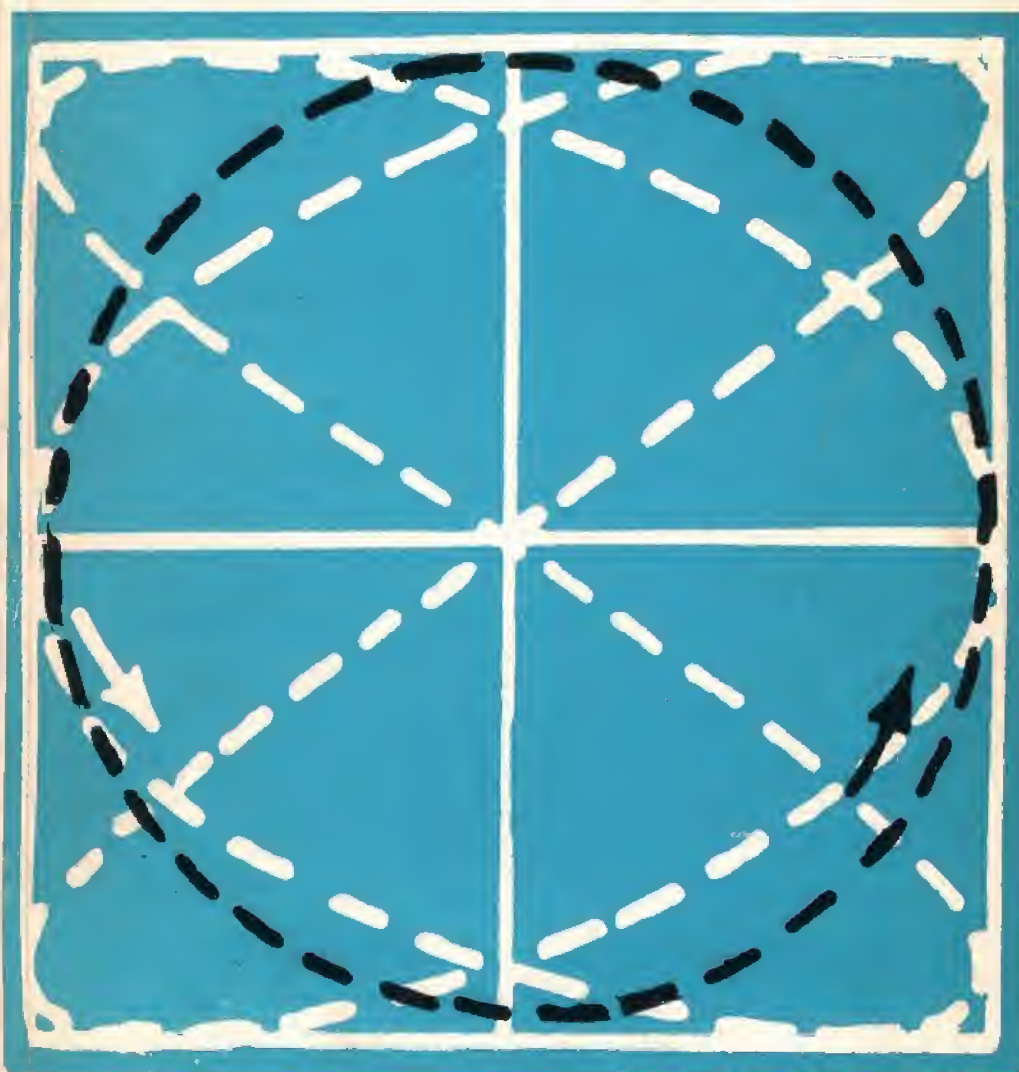F. A. BENSON D.Eng., Ph.D., M.I.E.E., Sen.M.I.E.E.E., F.I.E.S.

Problems in Electronics with Solutions

Fourth Edition
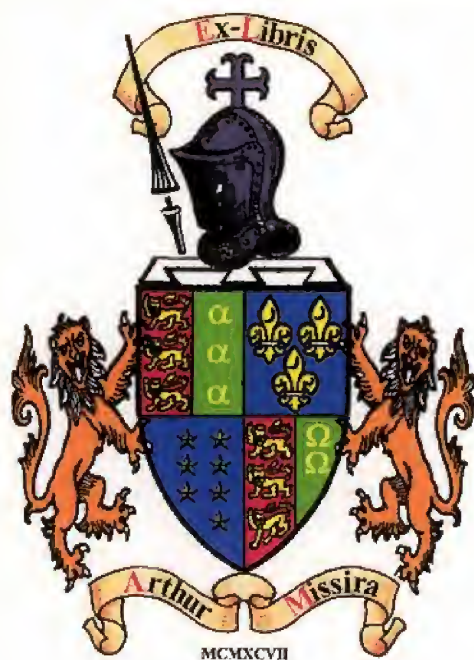
PROBLEMS IN ELECTRONICS WITH SOLUTIONS

SOME OTHER SPON BOOKS

Problems in Electronics with Solutions

FOURTH EDITION REVISED AND ENLARGED

F. A. BENSON

D.Eng., Ph.D., M.I.E.E., Sen. M.I.E.E.E., F.I.E.S.

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of Sheffield*

E. & F. N. SPON LIMITED

1965

CONTENTS

PREFACE TO THE FOURTH EDITION

THE book now contains 349 problems. Extra pages have again been made available to enable a limited number of additional problems to be given. Short chapters have been included on semiconductor fundamentals and kinetic theory of gases. Other new problems deal with transistor characteristics and circuits, noise, gas discharges, amplifiers, oscillators, dielectrics, Fourier transforms, transmission lines and circuit analysis. A few of the previous problems have been extended and some further minor modifications and corrections have been made.

A number of the questions on kinetic theory of gases, noise and dielectrics have been taken from problem papers produced for second-year students in the Department of Electronic and Electrical Engineering at the University of Sheffield. The author wishes to thank Dr. P. N. Robson, B.A., A.M.I.E.E., for giving permission for these problems to be incorporated.

<div align="right">F. A. BENSON</div>

Electronic and Electrical Engineering Department,
The University of Sheffield,
1964.

PREFACE TO THE FIRST EDITION

THIS book is based largely on problems which the author has collected over the last ten years, many of which have been given to undergraduate engineers. The purpose of the book is to present the problems, together with their solutions, in the hope that they will prove of value to other teachers and students. It is thought that the book covers almost the complete undergraduate electronics courses in engineering at Universities, but it has not been written to match any particular syllabus, and it should also be found useful by postgraduate students and research workers as a reference source. In fact, a few questions of postgraduate standard are included.

Within the author's knowledge, there is no other problem book on electronics with solutions which covers such a wide field as the present one. The few problem books which are available, while being excellent in some ways, suffer from certain disadvantages. In some, answers are given but not solutions, in others there are no questions at all on electronics, or if there are they form only a small part of the whole. In other instances, where solutions are given, the questions and solutions are not separated. Descriptive questions are given in some problem books; and, while it may be argued that these should be included to assist readers engaged in private study, the answers to these can easily be found in standard textbooks. The purpose of a problem book should surely be the application of theory rather than the teaching of it.

Textbooks form other useful sources of problems; and in fact, most textbooks give some worked examples too, but these are generally used merely to illustrate points which have just previously been made in the text and do not encourage students to think for themselves.

The author is very much in favour of problem papers and tutorials as a method of education, because it is well known that young students encounter many difficulties when they first try to apply their theoretical knowledge to practical problems.

The 282 problems are divided up into 23 sections and the solutions are separated from the problems so that the students shall not see

solutions by accident. The answer is also given, however, at the end of each problem for convenience. A thorough grasp of the principles involved in any particular problem cannot be obtained by merely reading through the solution. Students should not therefore consult the solutions until they have either repeatedly tried hard and failed to obtain the stated answer, or successfully solved the problem and wish to compare the method of solution with that given. Wherever possible the problems are based on practical data, so as to familiarize the student with practical orders of magnitude.

At first it was thought that, because of the enormous range of subjects to be included, two books might be published, one featuring the elementary topics and the other the more advanced ones. It was finally decided that one volume rather than two was much more desirable; but, to keep the price of the book at a figure reasonable for students, it was necessary to limit the number of examples to about the same as already given in the author's existing book *Electrical Engineering Problems with Solutions*. It was also obvious at the outset that, because of length limitations, it would not be possible to include step-by-step mathematics, but only the electrical steps in the solutions. It is therefore assumed that the reader knows the necessary mathematics. It has been felt desirable to include a few problems of importance which are just standard textbook material but, in such cases, the solutions simply give references to suitable textbooks. Some topics which readers may expect to find included, e.g. kinetic theory of gases, sound equipment, polyphase rectifiers, vacuum techniques, have had to be omitted, and others have had less space devoted to them than one would have liked.

The author cannot possibly claim that all the problems in the collection are original, but it is impossible to acknowledge the sources of those which are not. Most of the problems are new, however, and in many cases they have been carefully formulated to try to encourage thought and understanding; but some, which require only numerical substitutions in formulae are included, in the hope that they will develop the student's sense of magnitudes.

To avoid repetition, all the general data required have been collected together and are given at the beginning of the book.

While great care has been taken to try to eliminate errors some will inevitably have crept in, and the author will be glad to have any such brought to his notice.

The author is indebted to Dr. J. Allison, B.Sc. (Eng.) for providing some experimental figures for use in several questions on transmission lines. He also expresses appreciation to a number of his former students who have confirmed the answers to many of the problems. The encouragement given and many helpful suggestions made by Professor A. L. Cullen, B.Sc. (Eng.), Ph.D., M.I.E.E., M.I.R.E., throughout the preparation of the manuscript are also gratefully acknowledged.

F. A. BENSON

Electrical Engineering Department,

The University of Sheffield,

1957.

GENERAL DATA

Charge on an electron (e) $= 1 \cdot 602 \times 10^{-19}$ coulomb

Mass of an electron (m) $= 9 \cdot 107 \times 10^{-31}$ kg

Planck's constant (h) $= 6 \cdot 624 \times 10^{-34}$ joule-sec

Boltzmann constant (k) $= 1 \cdot 38 \times 10^{-23}$ joule/$^\circ$K

Permittivity of free space (ε_0) $= 8 \cdot 855 \times 10^{-12}$ farad metre^{-1}

Excitation potential of argon $= 11 \cdot 6$ V

Ionization potential of mercury $= 10 \cdot 4$ V

Ionization potential of neon $= 21 \cdot 5$ V

Resistivity of copper $= 1 \cdot 7 \times 10^{-6}$ ohm-cm

Resistivity of nickel $= 9 \cdot 39 \times 10^{-6}$ ohm-cm

Section 1

PROBLEMS

CIRCUIT ANALYSIS

1. A series L, C, R circuit, with $R = 4\,\Omega$, $L = 100\,\mu\text{H}$ and $C = 200\,\mu\mu\text{F}$ is connected to a constant-voltage generator of variable frequency. Calculate the resonant frequency, the value of 'Q' and the frequencies at which half the maximum power is delivered.

[*Ans.* 1,126 kc/s; 177; 1,129 kc/s; 1,122 kc/s]

2. The graph shows the variation of current through a series L, C, R circuit when connected to a 5-V constant-voltage generator of variable frequency. Find the values of 'Q', R, L and C.
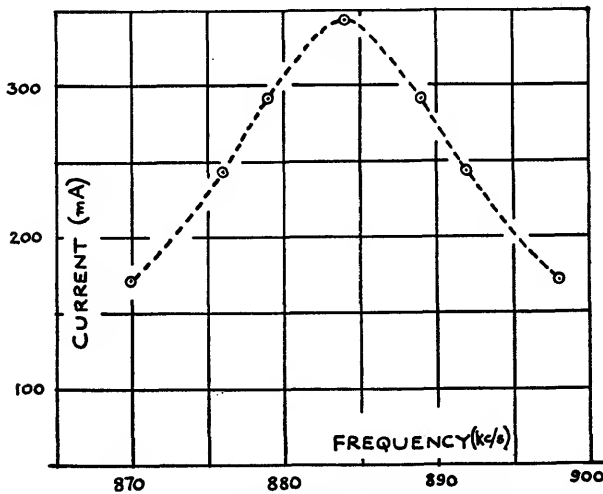
[*Ans.* 55·25; 14·53 Ω; 144·5 μH; 224 $\mu\mu\text{F}$]

3. A coil of inductance $88\,\mu\text{H}$ is placed in series with a 4·8-Ω resistor. The combination is connected in parallel with a 375-$\mu\mu\text{F}$ capacitor. Calculate the frequency of the circuit for which the effective impedance is a pure resistance.

[*Ans.* 876·4 kc/s]

4. Determine the 'Q' factor of the parallel damped circuit shown below.
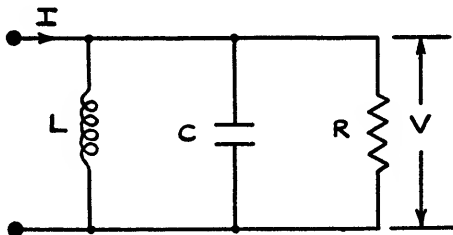
[*Ans. $R/\omega_r L$*]

5. A parallel resonant circuit is tuned to a frequency of 1 Mc/s and contains a 200-$\mu\mu$F capacitor. When a source of constant voltage is injected in series with the circuit the current falls to 0·707 of its resonant value, for a frequency deviation of 5 kc/s from the resonant frequency. Calculate the circuit 'Q' and the parallel resonant impedance.
[*Ans.* 100; 79·6 kΩ]

(a)

6. A parallel resonant circuit employs a 50-$\mu\mu$F capacitor and has a bandwidth of 250 kc/s. Calculate the maximum impedance of the circuit.
[*Ans.* 12,740 Ω]

(b)

7. Reduce the two circuits shown at (a) and (b) to the simple coupled circuit of (c) by assigning suitable values to Z_p, Z_s and M.

What are the coefficients of coupling for the circuits (a) and (b)?

[*Ans.* (a) $Z_p = j\omega(L_1 + L_m)$,
$\quad Z_s = j\omega(L_2 + L_m)$, $M = L_m$;
(b) $Z_p = (C_1 + C_m)/\omega C_1 C_m$,
$\quad Z_s = (C_2 + C_m)/\omega C_2 C_m$,
$\quad M = 1/\omega^2 C_m$; coefficients
of coupling are
$L_m/\sqrt{(L_1 + L_m)(L_2 + L_m)}$ for (a) and
$\sqrt{C_1 C_2/(C_1 + C_m)(C_2 + C_m)}$ for (b)]
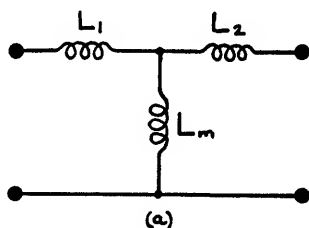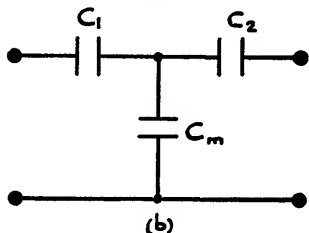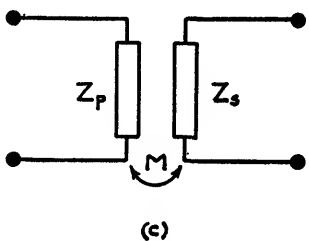
(c)

8. The two resonant circuits shown are tuned to the same frequency $\omega_r/2\pi$ and coupled together. Obtain an expression for the secondary current I_s in terms of the voltage E, the circuit Q's, Q_p and Q_s, the coefficient of coupling k and the ratio of the actual frequency to the resonant frequency, α.

Show that I_s reaches its maximum value when the circuits are in resonance and when $\omega_r M = \sqrt{R_p R_s}$ and that the value of k for critical coupling is $1/\sqrt{Q_p Q_s}$.
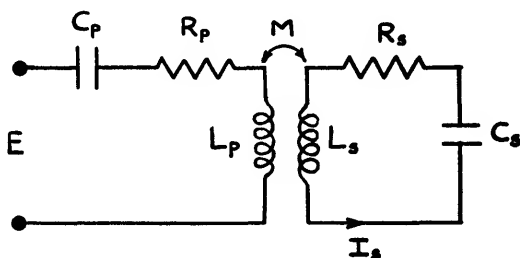
$$[Ans.\ I_s = -jEk/\alpha\omega_r\sqrt{L_pL_s}\{k^2 +$$

$$1/Q_pQ_s - (1 - 1/\alpha^2)^2 + j(1 - 1/\alpha^2)(1/Q_p + 1/Q_s)\}]$$

9. Two series circuits, each consisting of a 300-μH inductor and a 1,000-$\mu\mu$F capacitor, are magnetically coupled so as to have a mutual inductance of 60 μH. An e.m.f. of 10 V having a frequency of $1/\pi$ Mc/s is injected into one circuit. Determine the current in the other circuit and the coefficient of coupling (k).

[Ans. $-j0\cdot273$ A; $0\cdot2$]

10. Evaluate the input impedance of the circuit shown, at a frequency of 1 Mc/s. The coefficient of coupling is $0\cdot1$.

[Ans. $(6\cdot1 + j1{,}249\cdot1)\ \Omega$]

2
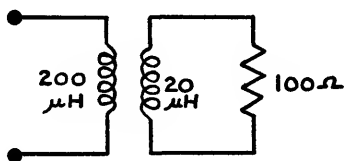
11. A voltage of 100 V at a frequency of $[10^6/2\pi]$ is applied to the primary of the coupled circuit illustrated. Calculate the total effective resistance and reactance referred to the primary.

M = 75μH
R₁ = 15 Ω   R₂ = 8 Ω
L₁ = 200μH   L₂ = 100μH   C₂ = 10,000μμF
C₁ = 5,000μμF

Determine also the primary and secondary currents.

[*Ans.* 718 Ω; 0; 0·139 A; 1·306 A]

12. A transformer has a tuned primary winding and an untuned secondary. The inductance of each winding is 1 mH and the mutual inductance between them is 0·5 mH. The primary winding is tuned with the secondary open-circuited, and resonates at a frequency of 500 kc/s. If the secondary circuit is now short-circuited find the change of tuning capacitance required to keep the same resonant frequency. Neglect the resistances of the windings.

[*Ans.* 34 μμF]

13. In the circuit illustrated $\omega L_2 = 1/\omega C_2$. Determine the value of the input impedance, if C_1 is chosen to make it purely resistive. The frequency is 1 Mc/s.
[*Ans.* 202·4 Ω]

C₁
R₂ = 20 Ω
L₁   L₂   C₂
R₁ = 5 Ω
M = 10μH
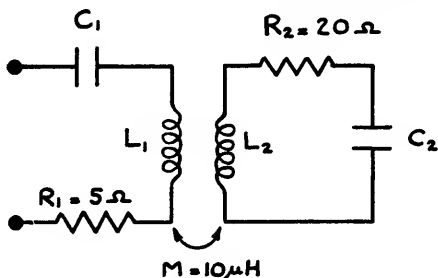
14. In the circuit illustrated $e_1 = 169\cdot7 \sin 1{,}885t$ volts and $e_2 = 141\cdot4 \sin(1{,}885t + 45°)$ volts. Calculate the primary and secondary currents and draw a complete vector diagram for the circuit.

R₁ = 60 Ω   R₂ = 80 Ω
I₁   I₂
e₁   L₁ = 50 mH   L₂ = 70 mH   e₂
M = 17·75 mH
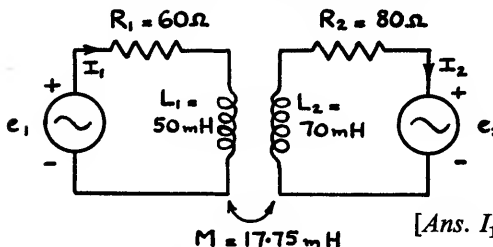
[*Ans.* $I_1 = 1\cdot168\underline{/-45\cdot6°}$ A; $I_2 = 0\cdot903\underline{/-13\cdot6°}$ A]

15. Determine the equivalent impedance of the two magnetically-coupled parallel circuits illustrated.
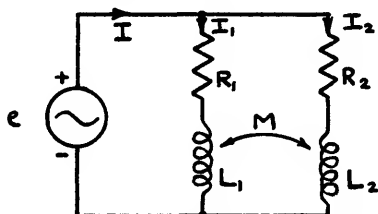
[*Ans.* $(Z_1Z_2 - Z_m{}^2)/(Z_1 + Z_2 - 2Z_m)$
where $Z_1 = R_1 + j\omega L_1$
$Z_2 = R_2 + j\omega L_2$
and $Z_m = \pm j\omega M$]

16. Experiments carried out on a variometer, whose two coils were series-connected, showed that the inductance values obtainable varied from 40 mH to 360 mH. Assuming that the self-inductances of the two coils are equal, determine the range of inductance values obtainable if the coils are reconnected in parallel. Neglect resistances.

[*Ans.* 10 to 90 mH]

17. Calculate the current flowing in resistor R_6 of the network, involving several coupled circuits, as shown. The angular frequency $\omega = 2 \times 10^6$ radians/sec.
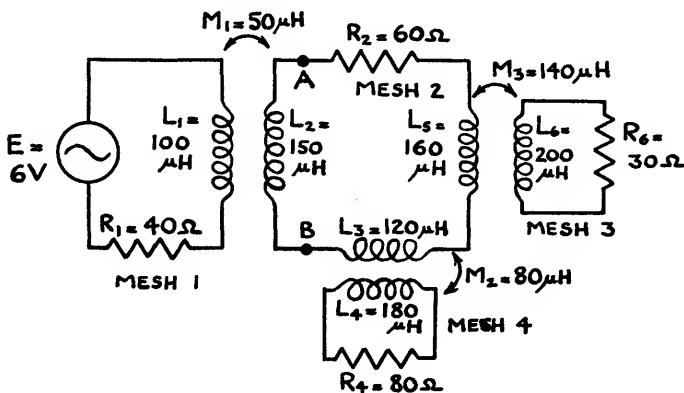
[*Ans.* $3 \cdot 69 / - 64° \ 6'$ mA]

18. For the circuit shown, calculate the total loop impedances, the mutual impedance, the apparent impedance of the primary loop and the currents in the two loops.
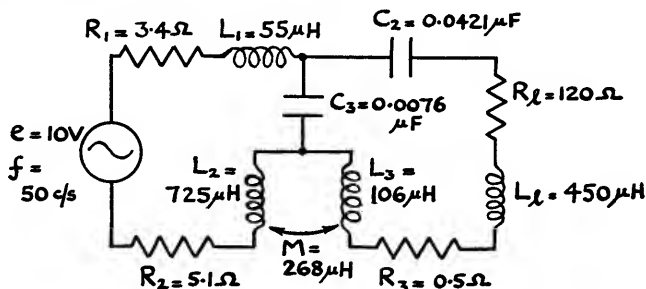
[*Ans.* $(8\cdot5 - j174)\,\Omega$; $(120\cdot5 - j320)\,\Omega$;
$j503\,\Omega$; $(271 + j522)\,\Omega$;
$0\cdot017\underline{/- 62\cdot6°}$ A; $0\cdot0251\underline{/- 83\cdot2°}$ A]

19. A wavemeter consists of a variable capacitor, having a range of 50 to 1,000 $\mu\mu$F, and two coils of inductances 300 and 100 μH respectively. If the coils are fixed so that their mutual inductance is 25 μH, what range will the wavemeter have when the coils are used (*a*) in series aiding (*b*) in series opposing, (*c*) in parallel aiding and (*d*) in parallel opposing?

[*Ans.* (*a*) 283 to 1,265 m; (*b*) 249 to 1,115 m;
(*c*) 122 to 546 m; (*d*) 107 to 481 m]

20. A coil has an inductance of 5 mH, a self-capacitance of 5 $\mu\mu$F and a high-frequency resistance of 100 Ω. Determine the effective resistance and inductance of the coil at a frequency of 500 kc/s.

[*Ans.* 177 Ω; 6·67 mH]

21. A coil is tuned to a certain frequency by a 250-$\mu\mu$F capacitor. To tune the coil to the second harmonic of this frequency a capacitance of 55 $\mu\mu$F is required. Determine the self-capacitance of the coil.

[*Ans.* 10 $\mu\mu$F]

22. Two coils, of inductances 50 μH and 200 μH respectively, are magnetically coupled. Find the effective value of the mutual inductance between them, at a frequency of 2 Mc/s, their self-capacitances being 5 and 7 $\mu\mu$F respectively, and the coefficient of coupling being 0·05.

[*Ans.* 6·3 μH]

23. Derive the conditions which must be satisfied for the two circuits illustrated to present identical impedances at all frequencies.

(a)   (b)
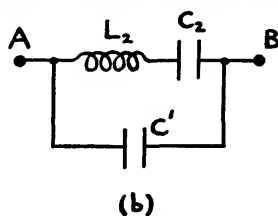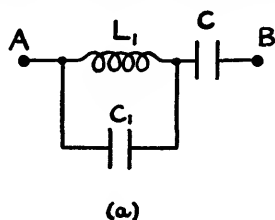
[*Ans.* $C' = CC_1/(C + C_1)$; $L_2 = L_1(C + C_1)^2/C^2$;
$$C_2 = C^2/(C + C_1)]$$

24. Prove that if $R = \sqrt{L/C}$ the impedance of the circuit shown is independent of frequency, and determine the value of this impedance.
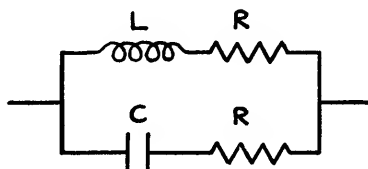
[*Ans. R*]

25. A non-inductive resistor of resistance R ohms is connected in parallel with a coil of inductance L henrys and negligible resistance. Calculate the values of R and L so that the impedance of the parallel combination, at a given frequency, is the same as that of a single coil of resistance r ohms and inductance l henrys.

[*Ans.* $R = \{r + (\omega^2 l^2/r)\}$; $L = \{l + (r^2/\omega^2 l)\}$]

26. Prove that the load impedance which absorbs the maximum power from a source is the conjugate of the impedance of the source.

A loudspeaker is connected across terminals A and B of the network illustrated. What should its impedance be to obtain maximum power dissipation in it?
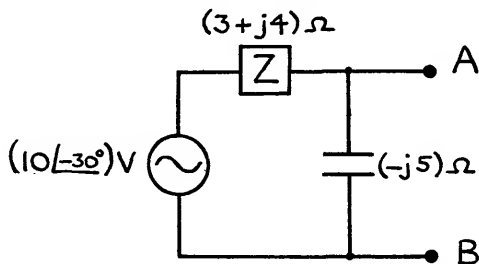
[*Ans.* $(7{\cdot}5 + j\,2{\cdot}5)\ \Omega$]

27. Derive an expression for the relationship between the series resistance ρ and the shunt resistance r which may be used alternatively to represent the losses in a capacitor of capacitance C.

For a particular capacitor at a certain frequency the product ρC was found to be 25×10^{-10} and the power factor was known to be 0.001. Determine the frequency at which the measurement was carried out.

[*Ans.* $\omega^2 C^2 \rho r = 1$, where $\omega = 2\pi \times$ frequency; 63.7 kc/s]

28. The diagram shows a phase-shifting network. R and C are always adjusted so that the magnitude of their total impedance in series is $5,000 \, \Omega$. The supply frequency is $1,000$ c/s. Determine the values of R and C which produce a phase shift of $30°$ between V_i and V_o. Does V_o lag or lead with respect to V_i?
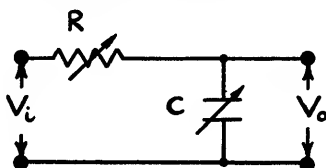
[*Ans.* $2,500 \, \Omega$; $0.037 \, \mu$F; lags]

29. Find the voltages V_1, V_2 and V_3 in the circuit illustrated using (*a*) mesh analysis, (*b*) nodal analysis.
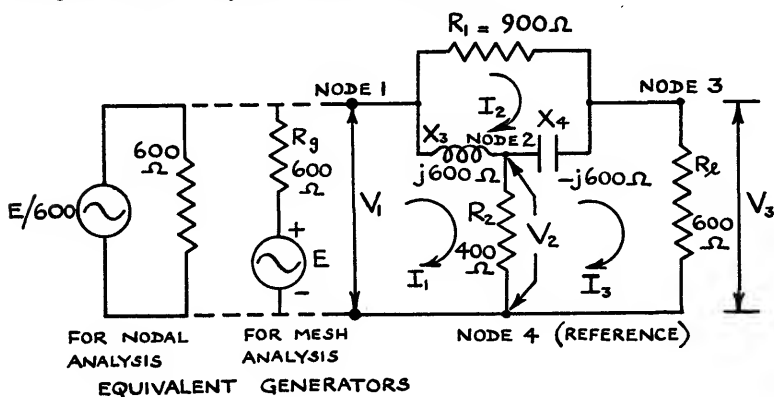
[*Ans.* $V_1 = E(0.499 + j \, 0.214)$
$V_2 = E(0.143 - j \, 0.143)$
$V_3 = 0.284E$]

30. Write down the nodal equations for the circuit illustrated.
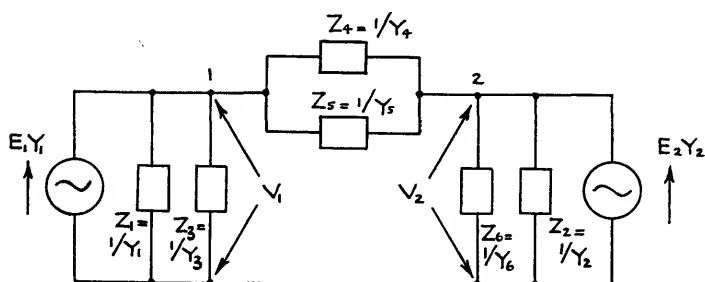
[*Ans.* $-E_1 Y_1 + V_1(Y_1 + Y_3 + Y_4 + Y_5) - V_2(Y_4 + Y_5) = 0$;
$-E_2 Y_2 + V_2(Y_2 + Y_4 + Y_5 + Y_6) - V_1(Y_4 + Y_5) = 0$]

TRANSIENTS AND OSCILLATORY CIRCUITS

31. For the series circuit shown, prove that the current after closing the switch S is oscillatory, of gradually decreasing amplitude and of frequency 159 c/s. Plot the current-time wave from the instant the switch is closed.

R = 0·5Ω   L = 0·005H

C = 200μF

S   200V
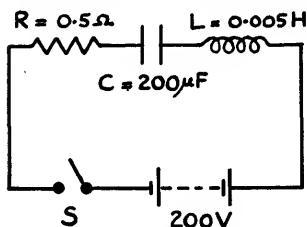
The 0·5-Ω resistor is now replaced by a 10-Ω one. Obtain the new expression for the current which flows after closing the switch and plot the current-time wave.

Repeat the calculation for the case where the resistor has a value of 20 Ω.

[*Ans.* 40,000$t\, e^{-1000t}$ amperes; 11·54 ($e^{-268t} - e^{-3732t}$) amperes]

32. Derive an expression for the current which flows in the circuit shown, immediately after closing switch S. At the instant at which S is closed the applied voltage is zero. Plot curves showing the variation of the two components of the current wave, and of the resultant current, with time. (Observe that after a time corresponding to about three complete cycles of the supply voltage the transient term has become relatively small.)

If the resistance in the circuit is reduced to zero show that the current never becomes negative, and that the voltage and current waves pass through zero values simultaneously.

L = 0·02H   R = 2Ω

S   200 sin 628t VOLTS
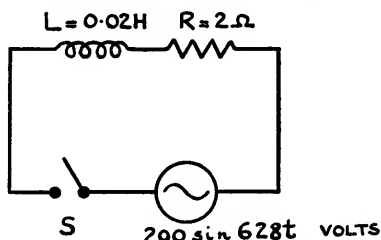
[*Ans.* 15·52 e^{-100t} + 15·71 sin (628t − 81°) amperes]

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33. Derive an expression for the current which flows in the circuit shown, immediately after closing the switch S. At the instant at which S is closed the applied voltage is zero. Plot graphs showing how the two components of the current wave, and the resultant current, vary with time. Observe that (a) about 0·05 sec after closing S the amplitude of the transient wave is less than 10% of its maximum amplitude of 41·3 A; (b) the frequency of the transient current is 159 c/s.
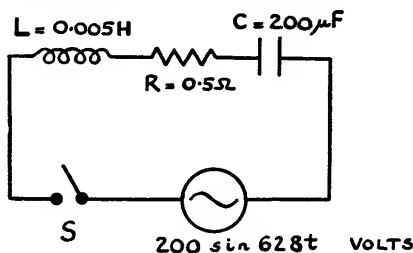
$$[Ans.\ 41\cdot5\ e^{-50t} \cos{(1{,}000t + 173\cdot5°)}$$
$$+ 41\cdot3 \cos{(628t - 6°)}\ \text{amperes}]$$

34. Repeat Question 33 for the case where $R = 0\cdot25\ \Omega$ and the supply frequency is 159 c/s, the other constants remaining the same.
$$[Ans.\ -800\ e^{-25t} \sin{1{,}000t} + 800 \sin{1{,}000t}\ \text{amperes}]$$

35. Repeat Question 33 for the case where $R = 10\cdot1\ \Omega$ and the supply frequency is 50 c/s, the other constants remaining the same. Assume that when the switch is closed the applied voltage is at its maximum positive value.
$$[Ans.\ -131\cdot6\ e^{-1{,}151t} + 125\ e^{-868t} + 11\cdot4 \cos{(314t + 54\cdot7°)}$$
$$\text{amperes}]$$

36. An inductor-capacitor-resistor series circuit has the following constants: $C = 2\cdot5\ \mu\text{F}$, $L = 2\ \text{mH}$, $R = 40\ \Omega$. The applied voltage, which is sinusoidal, has a peak value of 100 V and the frequency is 1,000 c/s. The mains switch is closed when the voltage is at half its peak value and the capacitor is initially uncharged. Derive an expression for the current at a time t after closing the switch.
$$[Ans.\ 1\cdot73\ e^{-10^4t} \sin{(10^4t - 61°\ 44')}$$
$$+ 1\cdot54 \sin{(6{,}283t + 80°\ 23')}]$$

37. A 2·5-μF capacitor, a 2-mH coil and an 80-Ω resistor are connected in series with a switch. The capacitor is initially charged to a voltage of 100 V. Derive expressions for the current, the capacitor voltage and the charge on the capacitor at a time t after closing the switch. Calculate, also, the maximum value of the current.

If the 80-Ω resistor is replaced by a 40-Ω one derive an expression for the current at a time t after closing the switch.

[*Ans.* $1 \cdot 77\{e^{-5,858t} - e^{-34,142t}\}$ A;
$\{120 \cdot 8\ e^{-5,858t} - 20 \cdot 8\ e^{-34,142t}\}$ V;
$10^{-4}\{3 \cdot 02\ e^{-5,858t} - 0 \cdot 52\ e^{-34,142t}\}$ coulombs;
$1 \cdot 02$ A; $5\ e^{-10^4 t} \sin (10^4 t)$ A]

38. A steady voltage of 100 V is applied to the circuit shown. The capacitor is initially uncharged. Derive expressions for the currents i_1 and i_2 at time t after closing the switch.
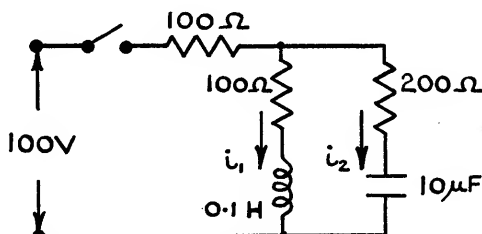
[*Ans.* $i_1 = \{- 0 \cdot 395\ e^{-1,575t} - 0 \cdot 105\ e^{-424t} + 0 \cdot 5\}$ A;
$i_2 = \{0 \cdot 168\ e^{-1,575t} + 0 \cdot 1653\ e^{-424t}\}$ A]

39. In the circuit illustrated, $R_1 = 20\ \Omega$, $R = R_2 + R_L = 100\ \Omega$, $L_1 = 2$ H, $L = L_2 + L_L = 4$ H and $M = 2$ H. A sinusoidal voltage of peak value 100 V and frequency 50 c/s is suddenly applied to the primary at the instant when it is a maximum. Derive expressions for the currents i_1 and i_2 at any time t after the voltage is applied.
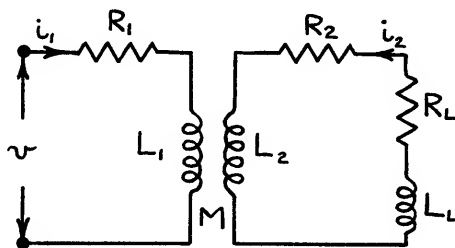
[*Ans.* $i_1 = \{- 0 \cdot 2176\ e^{-61 \cdot 9t} + 0 \cdot 173\ e^{-8 \cdot 1t}$
$+ 0 \cdot 287 \cos (314t - 81° 3')\}$ A;
$i_2 = \{- 2 \cos 314t + 0 \cdot 182\ e^{-61 \cdot 9t} + 0 \cdot 041\ e^{-8 \cdot 1t}$
$+ 0 \cdot 1146 \cos (314t - 81° 3') - 1 \cdot 81 \sin (314t - 81° 3')\}$ A]

40. Find the Laplace transforms of:

(a) $y(t) = e^{-at} t^{n-1}/(n-1)!$ and
(b) $y(t) = e^{-bt} \sin at$.

Obtain the Laplacian subsidiary equation for a series circuit containing resistance (R), inductance (L) and capacitance (C) and show how the transient response of such a circuit may be investigated no matter what the form of the applied voltage.

Calculate the current in the series circuit, at time t, from the subsidiary equation if there is no initial current and no initial charge on the capacitor and the applied voltage E is constant. Assume that $R = 2\sqrt{L/C}$.

$$[Ans.\ 1/(p+a)^n;\ a/\{(p+b)^2 + a^2\};$$
$$(Lp + R + 1/Cp)\bar{I} = \bar{V} + LI_0 - Q_0/Cp;$$
$$Et\ e^{-at}/L]$$

41. (a) Use the Laplace-transformation procedure to solve the following simultaneous differential equations:

$$\left. \begin{array}{c} (D^2 + 2)x - Dy = 1 \\ Dx + (D^2 + 2)y = 0 \end{array} \right\} t > 0$$

given that at $t = 0$, $x = x_0$, $y = Dx = Dy = 0$.

(b) For the circuit illustrated write down the various Laplacian subsidiary equations and show that:

$$\bar{V}_a = \frac{\bar{V}Z_4 Z_2(r_a - \mu Z_3)}{Z_4 Z_1 Z_2(1 + \mu) + (Z_4 + Z_1)\{Z_2 Z_3 + r_a(Z_2 + Z_3)\} + r_a Z_4 Z_1}$$

where V is applied at time $t = 0$ when steady conditions prevail.
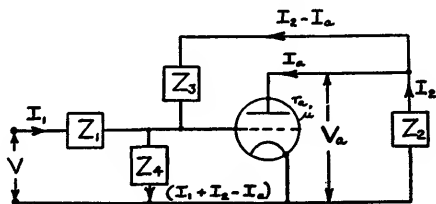
(c) A voltage $v(t)$ is applied to the primary circuit of the figure illustrated at time $t = 0$ with zero initial conditions. Show how the

secondary current $i_2(t)$ can be found by the Laplace-transformation method. Neglect the primary resistance.
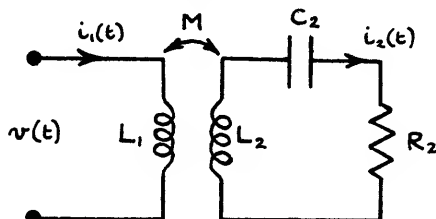
$$\left[\begin{array}{l} Ans. \ (a) \ x = \tfrac{1}{2} + \tfrac{1}{3}(2x_0 - 1) \cos t + \tfrac{1}{6}(2x_0 - 1) \cos 2t; \\ \qquad\quad y = \tfrac{1}{3}(2x_0 - 1) \sin t - \tfrac{1}{6}(2x_0 - 1) \sin 2t; \\ (c) \ \bar{i}_2 = \dfrac{-Mp\bar{v}}{(L_1L_2 - M^2)p^2 + R_2L_1p + L_1/C_2} \end{array}\right]$$

42. (*a*) An oscillatory circuit tuned for a wavelength of 300 m has a coil inductance 150 μH and an effective resistance of 10 Ω. Calculate the logarithmic decrement.

[*Ans.* 0·033]

(*b*) Prove that in an oscillatory circuit for which the logarithmic decrement is 0·1 there are 47 oscillations in a wave train before the amplitude of the current has fallen to 1 % of its initial value.

43. Find the exact natural frequency of free oscillations in an oscillatory circuit in which the capacitance is 0·055 μF, the inductance is 2 μH and the resistance is 1 Ω.

Determine also the minimum value of the circuit resistance which would make the discharge of the capacitor unidirectional.

[*Ans.* 478 kc/s; 12·1 Ω]

WAVEFORM ANALYSIS

44. Find the Fourier-series representation of the output-voltage wave of a single-phase half-wave rectifier.

Hence, deduce the corresponding expression for a full-wave rectifier.

Determine the r.m.s. value of the full-wave rectifier waveform from the Fourier-series formula.

$$\left[Ans.\ E\left\{ \frac{1}{\pi} + \frac{1}{2}\sin\theta - \frac{2}{\pi}\sum_{n=2,4,6\ ..}\frac{\cos n\theta}{(n^2-1)}\right\};\right.$$

$$\left. E\left\{ \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=2,4,6\ ...}\frac{\cos n\theta}{(n^2-1)}\right\};\ E/\sqrt{2} \right]$$

45. Show that the Fourier series of the square waveform illustrated is

$$y = \frac{4E}{\pi}\left(\cos x - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} - \frac{\cos 7x}{7} + \ldots \right)$$
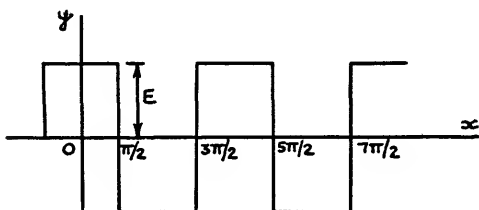
46. Show that the Fourier series of the sawtooth waveform illustrated is

$$y = \frac{2E}{\pi}\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} \cdots\right)$$

Find, graphically, the result of adding the first four terms and determine, graphically, some of the coefficients of the Fourier series.
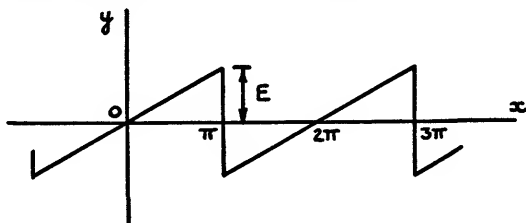
47. Show that the Fourier series of the short square pulse illustrated is

$$y = E\left\{n + \frac{2}{\pi}\left(\sin n\pi \cos x + \frac{\sin 2n\pi \cos 2x}{2}\right.\right.$$
$$\left.\left. + \frac{\sin 3n\pi \cos 3x}{3} + \cdots\right)\right\}$$
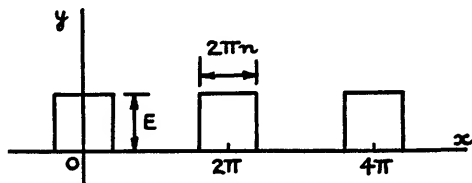
48. Show that the Fourier series of the short triangular pulse illustrated is

$$y = E\left[\frac{n}{2} + \sum_{N=1}\left\{\frac{2}{N\pi}\sin N\pi n - \frac{2}{N^2\pi^2 n}\right.\right.$$
$$\left.\left.\left(N\pi n \sin N\pi n - 2\sin^2\frac{N\pi n}{2}\right)\right\}\cos Nx\right]$$
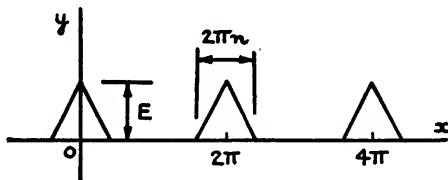
49. Find the Fourier series for the waveform illustrated which is produced by an *m*-phase rectifier.
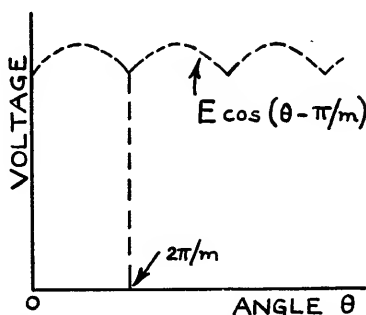
Hence, show that the percentage of sixth harmonic in a six-phase rectifier waveform is 5·71 %.

$$\left[Ans. \; (mE/\pi) \sin (\pi/m) \left\{ 1 - \sum_{n=1}^{n=\infty} \frac{2}{(m^2n^2 - 1)} \cos mn\theta \right\} \right]$$

50. One of the Fourier transform pair can be written

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t)^{-j\omega t} dt$$

Write down the corresponding formulae for $f(t)$. How are these formulae related to the Fourier series formulae?

Calculate $g(\omega)$ for the following time function:

$$f(t) = 0, \; t < 0$$
$$f(t) = e^{-\alpha t}, \; t \geqslant 0$$

Show how this spectrum can be used to calculate the response of an amplifier of complex gain $G(\omega)$ to a step voltage input.

$$[Ans. \; f(t) = \int_{-\infty}^{\infty} g(\omega)e^{j\omega t} \, d\omega;$$
$$1/2\pi(\alpha + j\omega)]$$

SEMICONDUCTOR FUNDAMENTALS

51. (*a*) What are the first four permissible energy levels for a hydrogen atom?

(*b*) An electron in a hydrogen atom makes a transition from energy level 2 to the ground state. Determine the energy released by the electron. What is the frequency of the resulting radiation from the atom?

[*Ans.* (*a*) -13.6 eV; -3.4 eV; -1.51 eV; -0.85 eV,
(*b*) 10.2 eV; 2.465×10^{15} c/s]

52. Calculate the conductivity and resistivity at $300°K$ of (*a*) pure germanium, (*b*) pure silicon. For germanium at $300°K$ assume that the density of carriers is $2.5 \times 10^{13}/cm^3$ and for silicon $1.6 \times 10^{10}/cm^3$. The carrier mobilities at $300°K$ are given in the Table below.

Carrier mobilities for germanium and silicon
at $300°K$ (cm^2/(volt-sec))

	Germanium	Silicon
Electrons	3,600	1,500
Holes	1,700	500

[*Ans.* (*a*) 0.0212 mho/cm; 47.2 ohm-cm
(*b*) 5.12×10^{-6} mho/cm; $195,300$ ohm-cm]

53. A specimen of intrinsic germanium at $300°K$, for which the density of carriers is $2.5 \times 10^{13}/cm^3$, is doped with impurity atoms such that there is one impurity atom for every 10^6 germanium atoms. All the impurity atoms may be assumed ionized. The density of germanium atoms is $4.4 \times 10^{22}/cm^3$. Determine the resistivity of the doped material. [*Ans.* 0.039 ohm-cm]

54. For the doped material in Problem 53 calculate the electron and hole densities.

$$[Ans. \ 4{\cdot}4 \times 10^{16}/cm^3; \ 1{\cdot}41 \times 10^{10}/cm^3]$$

55. Determine the drift velocities of holes and electrons at 300°K for an electric field of 100 V/cm in (a) germanium; (b) silicon. The carrier mobilities are given in Problem 52.

[Ans. (a) 17×10^4 cm/sec; 36×10^4 cm/sec (b) 5×10^4 cm/sec;
$$15 \times 10^4 \ cm/sec]$$

56. Calculate the diffusion constants for holes and electrons at 300°K in (a) germanium; (b) silicon. The carrier mobilities are given in Problem 52. [Ans. (a) 44 and 93·1 cm/sec,
$$(b) \ 12{\cdot}9 \ and \ 38{\cdot}8 \ cm/sec]$$

57. Assuming that the diffusion length for both holes and electrons in germanium at 300°K is 0·1 cm and using the results of Problem 56 determine the average lifetimes of the holes and electrons.

$$[Ans. \ 227 \ \mu s; \ 107 \ \mu s]$$

58. Obtain an expression for the contact potential developed across a p–n junction. The hole and electron densities in the p-type material are p_p and n_p respectively and in the n-type material are p_n and n_n.
$$[Ans. \ kT \log_e (p_p/p_n)/e = kT \log_e (n_n/n_p)/e]$$

59. Derive an expression for the current produced by the application of a voltage V across a p–n semiconductor junction in terms of the saturation current I_s.

If the saturation current density at 300°K is 25 μA/cm² calculate the voltage which would have to be applied across the junction to cause a forward current density of 5A/cm².

$$[Ans. \ I_s \{\exp (eV/kT) - 1\}; \ 0{\cdot}316 \ V]$$

60. Derive a formula for the Hall coefficient of a semiconductor specimen possessing n electrons and p holes per unit volume. The electron and hole densities are n and p respectively and the corresponding mobilities are μ_n and μ_p.

Hence, find the values of the coefficient for (a) an intrinsic semiconductor with carrier density n_i; (b) a highly-doped n-type material.

[Ans. $(p\mu_p{}^2 - n\mu_n{}^2)/e(p\mu_p + n\mu_n)^2$; (a) $(\mu_p - \mu_n)/n_i e(\mu_p + \mu_n)$;
$$(b) \ -1/ne]$$

3

VALVE AND TRANSISTOR CHARACTERISTICS

61. A diode valve has the following I_a/V_a characteristic:

I_a (mA)	0·52	1·17	1·90	2·78	3·85	5·15	6·50
V_a (V)	25	50	75	100	125	150	175

This valve is placed in series with a resistor of 20,000 Ω and a battery of 200 V. A resistor of 60,000 Ω is connected between the anode and cathode of the diode. Determine the current through the diode.

[*Ans.* 3 mA]

62. The anode-voltage/anode-current characteristic of a certain diode is given by the following figures:

Voltage V_a (V)	0	5	10	15	20	25	30	35
Current I_a (mA)	0	3·1	8·9	17·0	26·8	38	51·4	66

Plot the dynamic characteristic curve if the load has a resistance of 2,500 Ω. Hence find the load current, and the voltage across the load when the supply voltage is 50 V.

[*Ans.* 14·5 mA; 36·25 V]

63. The anode-voltage/anode-current characteristic of a certain diode is given by the following figures:

Voltage V_a (V)	2	5	10	15	20
Current I_a (mA)	2·3	6·3	13·7	22·0	31·0

Show that the relationship between I_a and V_a is of the form $I_a = KV_a{}^n$ and find the values of n and K.

[*Ans.* $\simeq$ 1·14; $\simeq$ 1]

64. In tests on a certain thyratron, with a steady value of negative grid voltage applied to the valve, the anode voltage was gradually

raised until the valve conducted. The corresponding grid and anode voltages at the point of conduction were:

Grid voltage (V) .	− 8·7	− 8·0	− 7·0	− 6·0	− 5·0	− 4·0	− 3·0	− 2·0	− 1·0	− 0·5
Anode voltage (V)	191	176	152	131	109	88	66	46	34·2	33·8

Plot the control characteristic, and estimate the control ratio over that portion of the graph which is approximately linear.

[*Ans.* 21·8]

65. A thyratron just becomes conducting with 200 V on the anode when the negative grid voltage is held at − 8 V. The control ratio is 35. Find the minimum anode voltage for conduction when the grid is held at − 20 V. Determine also the critical grid voltage, when the anode voltage is 340 V.

[*Ans.* 620 V; − 12 V]

66. The linear parts of the control characteristics of a certain thyratron are given by the following expressions:

$V_a = [− 120 \ V_g − 160]$, when the temperature is 40°C

and $V_a = [− 70 \ V_g − 130]$, when the temperature is 70°C,

where V_a is the anode voltage in volts, and V_g is the grid voltage in volts.

Evaluate, for an anode voltage of 400 V, the change in critical grid voltage when the temperature of the valve rises from 40°C to 70°C.

[*Ans.* 2·8 V]

67. The thyratron of Question 66 is used as a controlled rectifier on a sinusoidal a.c. supply of peak value 350 V. Determine the striking angles for a d.c. grid bias of − 4 V (*a*) when the temperature is 40°C, and (*b*) when it is 70°C.

[*Ans.* 66° 5′; 25° 22′]

68. Three triodes having amplification factors of 10, 20 and 30, and with mutual conductances 2, 5 and 3 mA/V respectively, are operated in parallel. Calculate the equivalent mutual conductance, the anode resistance and the amplification factor of the combination.

[*Ans.* 10 mA/V, 1,818 Ω, 18·18]

69. A certain thoriated-tungsten filament operating at 1,900°K gave a saturation current of 85 mA. Calculate the corresponding current for a pure-tungsten filament of the same area operating at 2,500°K. The Dushman constants are:

For thoriated tungsten
$$A = 3\!\cdot\!0 \text{ amps/(cm)}^2(°K)^2, \ b = 30,500°K.$$
For pure tungsten
$$A = 60\!\cdot\!2 \text{ amps/(cm)}^2(°K)^2, \ b = 52,400°K.$$

[*Ans.* 21·8 mA]

70. A certain filament operating at 2,100°K was thought to give a saturation current 10,000 times as great as that when the operating temperature was 1,600°K. If this had been true what would the work function of the filament material have been?

[*Ans.* 5 V]

71. Calculate the space-charge-limited current density between parallel plates 2 mm apart when the voltage across them is 200 V.

[*Ans.* 165 mA/sq cm]

72. Two diodes each have an anode 4 mm in diameter and 2 cm long, but one of them has a filament 0·1 mm in diameter, while the other has an indirectly-heated cathode 1·5 mm in diameter. Calculate the space currents flowing in each valve when the anode voltage is 25 V.

[*Ans.* 17 mA; 41 mA]

73. Explain the conditions under which the current in a diode is given by the law $I = kV^{3/2}$. Show that the shape of the electrodes only affects the constant k.

74. Calculate the operating characteristics and life for a 10% evaporation of mass of an ideal tungsten filament having a length of 2 cm and a diameter of 2·5 mm when operated at a temperature of 2,600°K. Use the data given in the Table which has been published by Jones and Langmuir* for an ideal tungsten filament 1 cm long and 1 cm diameter. The density of the material is 19.

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* H. A. Jones and I. Langmuir, 'The Characteristics of Tungsten Filaments as Functions of Temperature,' *Gen. Elec. Rev.*, **30**, pp. 312–13, 1927.

Filament Temperature	Filament Details					
	Heating watts W'	Resistance R_f'	Current I_f'	Voltage V_f'	Emission current I_e'	Evaporation M' in grams/ cm²/sec
2,600°K	263·0 W	$98·66 \times 10^{-6}\ \Omega$	1,632 A	$161·1 \times 10^{-3}$ V	2·25 A	$2·76 \times 10^{-8}$

[*Ans.* Power radiated 13·17 W; Resistance 0·3155 Ω;
Filament current 6·45 A; Voltage drop 2·04 V;
Emission current 0·1125 A; 376 hr]

75. A diode with parallel-plane electrodes 4 cm apart has an anode voltage which is 8 V negative with respect to the cathode. Calculate the maximum distance which an electron can travel from the cathode surface if it leaves it with an energy of 2 eV.

[*Ans.* 1 cm]

76. In a cylindrical diode the electric-field intensity at the cathode surface is 10^6 V/m, and the cathode temperature is 2,600°K. Determine the percentage increase in the zero-external-field thermionic-emission current because of the Schottky effect.

[*Ans.* 18·4%]

77. (*a*) A low-mu, plane-electrode triode has a grid-anode spacing of 0·19 cm, a grid-wire spacing of 0·127 cm and a grid-wire radius of 0·0064 cm. Estimate the amplification factor.

(*b*) In a cylindrical-electrode triode with an amplification factor of 20 the anode radius is 1·05 cm. The evenly-spaced grid wires are each of 0·04 cm radius, and are arranged to form a squirrel cage around a grid-wire circle of radius 0·5 cm. Determine the total number of grid wires.

(*c*) Derive the Vodges–Elder expressions* for the amplification factors of both high-mu, plane-electrode and cylindrical-electrode triode valves.

[*Ans.* (*a*) $\simeq$ 8, (*b*) $\simeq$ 10, (*c*) see the solution for the expressions]

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* See F. B. Vodges and F. R. Elder, 'Formulas for the Amplification Constant for Three-element Tubes,' *Phys. Rev.*, **24**, pp. 683–9, 1924.

78. The characteristics of a junction transistor are given in the following Table:

Collector Voltage V_{ce} (volts)	Collector Current (I_c) in mA		
	$I_b = 0$	$I_b = 40\,\mu A$	$I_b = 80\,\mu A$
1	0·2	1·90	3·7
4	0·3	2·05	4·0
7	0·4	2·20	4·3

The transistor is connected in a common-emitter stage with a collector load of 1,500 Ω, to supply voltage of 6V and a d.c. bias of 40 μA. Plot the characteristics, draw the appropriate load line and calculate the power dissipated in the transistor.

What will be the total voltage swing at the collector for an a.c. input signal current of 40 μA peak in the base?

[*Ans.* 6 mW; $\simeq 4.9$ V]

79. The output characteristics of a certain *p-n-p* transistor for common-base and common-emitter connections are illustrated in the figures. Determine, from these curves, the values of $\alpha = -(\partial i_c/\partial i_e)_{V_{cb}}$ and $\alpha' = (\partial i_c/\partial i_b)_{V_{cb}}$.
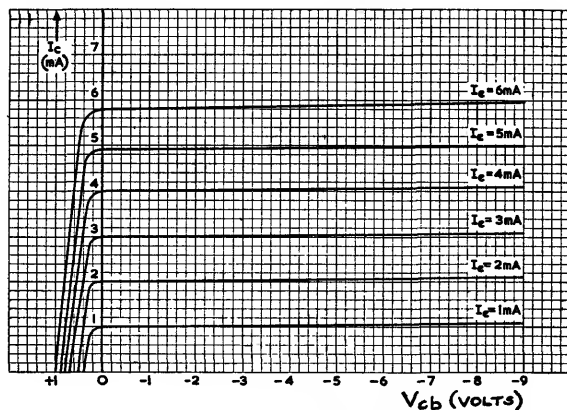
[By courtesy of Mullard Ltd.]

Show how α' may be expressed in terms of α and vice-versa.
[*Ans.* 0·98; $\simeq$ 57; $\alpha' = \alpha/(1 - \alpha)$; $\alpha = \alpha'/(1 + \alpha')$]
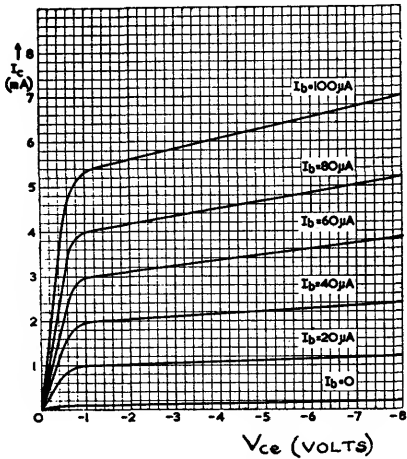
[By courtesy of Mullard Ltd.]

80. For a transistor used in the common-emitter configuration the relationship between collector current and collector voltage, with various fixed values of base current, are given in the following Table.

Collector Voltage (V)	Collector Current (mA)				
	Base Current $-30\,\mu A$	Base Current $-50\,\mu A$	Base Current $-70\,\mu A$	Base Current $-90\,\mu A$	Base Current $-110\,\mu A$
-2	$-0\cdot9$	$-1\cdot55$	$-2\cdot2$	$-2\cdot85$	$-3\cdot55$
-4	$-0\cdot92$	$-1\cdot65$	$-2\cdot4$	$-3\cdot05$	$-3\cdot77$
-6	$-0\cdot95$	$-1\cdot77$	$-2\cdot55$	$-3\cdot25$	$-4\cdot0$
-8	$-0\cdot98$	$-1\cdot90$	$-2\cdot75$	$-3\cdot5$	$-4\cdot2$

Draw the static characteristics of the transistor and use these to determine the current gain when the collector voltage is -5 V.

The transistor is to be used as a common-emitter amplifier with a load resistance of 1,800 Ω and a collector battery voltage of −9 V. Draw the load line and use this to find the base current for a collector voltage of −4 V.

[*Ans.* 37; 82 μA]

EQUIVALENT CIRCUITS OF VALVES
AND TRANSISTORS

81. In the circuit illustrated the input signal e is 1 V r.m.s. and the frequency is 2,000 c/s. Calculate the reading of the a.c. voltmeter if it has a resistance of 10,000 Ω and negligible reactance. The amplification factor of the valve is 20 and its anode resistance is 8,000 Ω.
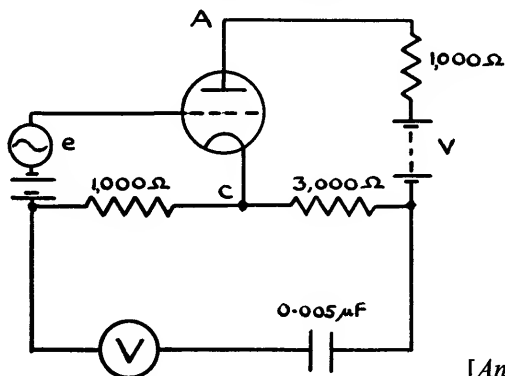
[*Ans.* 2·06 V]

82. Draw the equivalent circuit of the arrangement shown and find the r.m.s. a.c. anode current in the valve. The valve constants are $\mu = 20$ and $r_a = 5,000\ \Omega$.

[*Ans.* 35·9 μA]

29
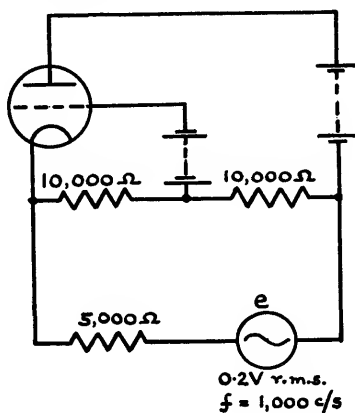
83. Draw the equivalent circuit for the network illustrated, and write down the equations from which the various alternating currents and voltages may be calculated. Assume that, with the reference potentials shown, the voltage e_2 is 30° ahead of the voltage e_1, and that the r.m.s. values of e_1 and e_2 are 1 V and 2 V, respectively.
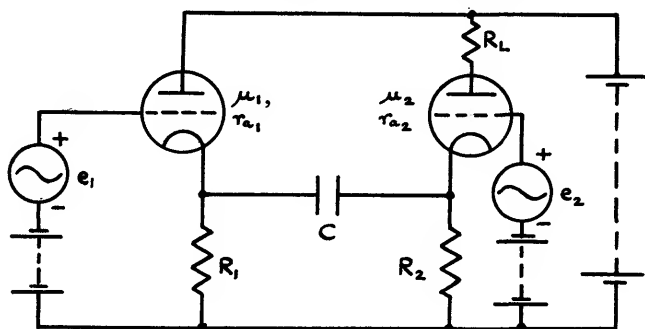
84. Derive the current-source equivalent circuit for a triode valve, where the valve is supposed to be replaced by a current generator which supplies a current $g_m V_g$ flowing from anode to cathode within the valve, and which has the anode resistance r_a connected across the generator terminals. g_m is the mutual conductance of the valve and V_g the grid-cathode voltage.

85. Two triode valves have amplification factors of 20 and 40, and anode resistances of 5,000 and 10,000 Ω, respectively. The two anodes are joined together, and a load of resistance 20,000 Ω is connected between the anodes and the h.t. supply line. An alternating voltage of 4 V is applied between the grid and cathode of the first valve, and a voltage of 2 V, of the same frequency and phase, is applied between the grid and cathode of the second valve. Calculate the alternating voltage across the load resistor and the alternating component of the anode current in each valve.

[*Ans.* 68·6 V; 2·3 mA; 1·1 mA]

86. (*a*) Draw the equivalent circuit of a tetrode, including all interelectrode capacitances, with an impedance Z_L in the anode circuit. Neglecting the grid-anode capacitance, simplify the circuit.

(*b*) Draw the equivalent circuit of a pentode, including all interelectrode capacitances, with an impedance Z_L in the anode

circuit. Show that, to a very good approximation, the input capacitance is equal to the control-grid/cathode and control-grid/screen-grid capacitances in parallel, and that the output capacitance is equal to the anode/cathode, anode/screen-grid and anode/suppressor-grid capacitances in parallel.

87. The hybrid parameters of a certain transistor are:
$h_{11} = 35\ \Omega$, $h_{21} = -0.976$, $h_{22} = 1.0\ \mu$mhos and $h_{12} = 7 \times 10^{-4}$. Calculate the values of r_{11}, r_{12}, r_{21}, r_{22}, α, r_e, r_b, r_c and r_m.
[*Ans.* 718·2 Ω; 700 Ω; 976 kΩ; 1 MΩ; 0·976; 18·2 Ω;
700 Ω; $\simeq$ 1MΩ; 975·3 kΩ]

88. Draw three equivalent circuits which can be used to represent the transistor under small-signal conditions. Show how analyses of these circuits permit all the elements of the equivalent networks of the common-emitter and common-collector circuits to be expressed in terms of the elements of the common-base arrangement.

89. Sketch common-cathode, common-emitter and common-collector transistor amplifier circuits and then draw their equivalent triode-valve configurations.

90. Show that the arrangement illustrated, which is frequently used as a transistor equivalent circuit, does not, in general, satisfy the reciprocity condition.
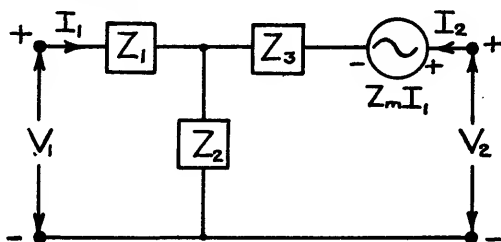
Determine the condition that must be satisfied for reciprocity to apply.
[*Ans.* $Z_m = 0$]

91. A transistor has a current amplification factor of 0·96 at low frequencies and the alpha cut-off frequency is 5 Mc/s. Determine the

current amplification factor at 10 Mc/s and calculate the frequency at which the current amplification factor falls to 0·6.

[*Ans.* 0·43; 6·25 Mc/s]

92. The current amplification factor α of a common-base junction transistor, operating at a frequency f is given by:

$$\alpha = \alpha_0 \left\{ \frac{1}{1 + j(f/f\alpha)} \right\}$$

where α_0 is the low-frequency value of α and $f\alpha$, called the alpha cut-off frequency, is that frequency where $\alpha = \alpha_0/\sqrt{2}$.

Derive a corresponding expression for the current amplification factor when the transistor is used in the common-emitter configuration and give the corresponding cut-off frequency in terms of α_0 and $f\alpha$.

$$\left[Ans. \quad \frac{\alpha_0}{(1 - \alpha_0) + j(f/f\alpha)}; f\alpha(1 - \alpha_0) \right]$$

ELECTRONIC COMPUTING CIRCUITS

93. A common-cathode difference amplifier is illustrated in the diagram. Draw the equivalent circuit for the arrangement and prove that, if

$$(R_{l_1} + r_a)/(\mu + 1) \ll R_c$$

and
$$(R_{l_2} + r_a)/(\mu + 1) \ll R_c$$
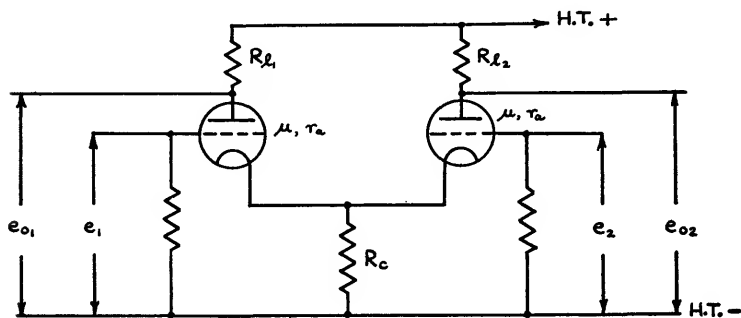
the output voltages e_{o1} and e_{o2} are

$$-\mu R_{l_1}(e_1 - e_2)/(R_{l_1} + R_{l_2} + 2r_a)$$

and
$$\mu R_{l_2}(e_1 - e_2)/(R_{l_1} + R_{l_2} + 2r_a)$$

respectively.

Show also that if $e_2 = 0$ and $R_{l_1} = R_{l_2}$, the circuit may be used to produce push-pull signals from a single source of voltage.

94. A cascode type of difference amplifier is shown in the diagram. Analyse the operation of the circuit and show that the output voltage $e_0 = \mu(e_1 - e_2)/2$.

33
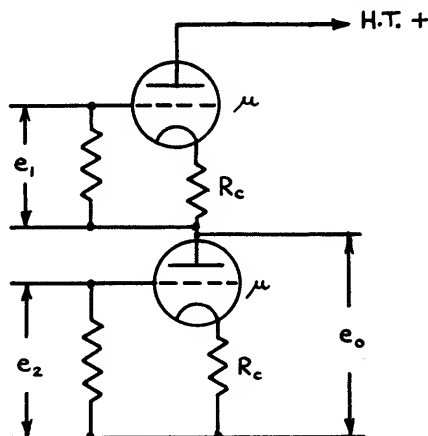
95. A simple feedback summing amplifier is illustrated in the diagram. Show that the output voltage e_o is nearly equal to $-(e_1 + e_2 + e_3)$, if the gain of the stage, as measured between the grid and anode terminals of the valve, is high.
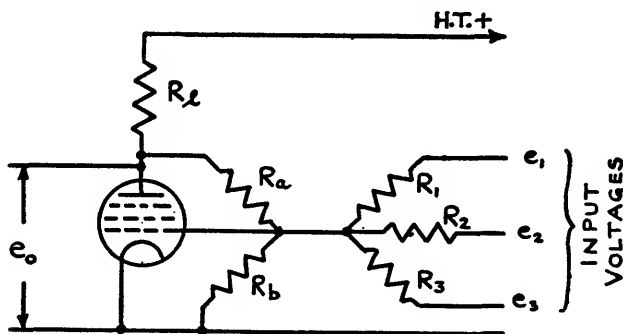
96. An arrangement for the addition of voltages in the cathode circuit of a chain of identical valves is shown in the diagram. Draw the equivalent circuit and apply Millman's network theorem* to show that the output voltage $\simeq \mu \Sigma e_n / n(\mu + 1)$ where n is the number of stages.

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* J. Millman, 'A Useful Network Theorem,' *Proc. I.R.E.*, **28**, p. 413, 1940. See also, F. A. Benson, *Electrical Engineering Problems with Solutions*, Spon, 1954, pp. 17–18 and pp. 104–109.
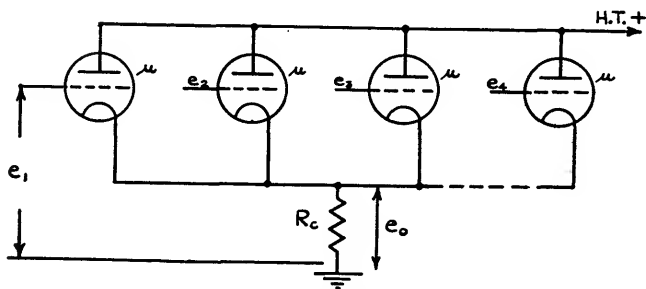
97. A difference amplifier, an ordinary amplifier of gain A and a squaring circuit are connected as illustrated in the block diagram. Show that, if A is large, the output voltage e_o is $k\sqrt{e_i}$ where k is a constant.
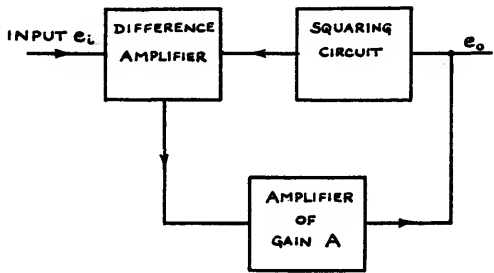
98. A difference amplifier, an ordinary amplifier of gain A and a multiplying circuit are connected as illustrated in the block diagram. Show that, if A is large, the output voltage e_o is $k\,e_{i_1}/e_{i_2}$ where k is a constant.
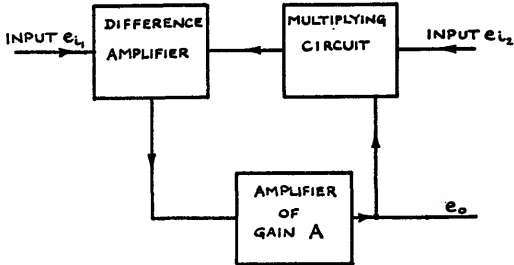
RECTIFICATION

99. A high-vacuum diode, with an internal resistance of 150 Ω, supplies power to a 1,000-Ω load from a 300-V r.m.s. source. Find:

(*a*) the mean load current,

(*b*) the r.m.s. alternating load current,

(*c*) the d.c. power supplied to the load,

(*d*) the input power to the anode circuit,

(*e*) the rectification efficiency,

(*f*) the ripple factor.

[*Ans.* (*a*) 117 mA; (*b*) 184 mA; (*c*) 13·8 W; (*d*) 39·1 W;
(*e*) 35·3%; (*f*) 1·21]

100. A gas diode, for which the striking and extinction voltages may both be taken as 10 V, supplies power in a half-wave rectifier circuit to a 1,000-Ω load from a 300-V r.m.s. source. Calculate:

(*a*) the mean load voltage,

(*b*) the d.c. power supplied to the load,

(*c*) the input power to the anode circuit,

(*d*) the rectification efficiency,

(*e*) the ripple factor.

[*Ans.* (*a*) 130 V; (*b*) 16·9 W; (*c*) 43·7 W;
(*d*) 38·7%; (*e*) 1·225]

101. Calculate the regulation and efficiency for a half-wave rectifier circuit, from no load to 80 mA. The transformer r.m.s. secondary voltage is 230 V and the internal resistance of the diode is 500 Ω. Find also the current at which maximum power is obtained.

[*Ans.* 40 V; 40·6 to 24·9%; 103·5 mA]

102. A full-wave single-phase rectifier employs a double-diode valve, the internal resistance of each element of which may be assumed constant at 500 Ω. The transformer r.m.s. secondary voltage

from the centre tap to each anode is 300 V and the load has a resistance of 2,000 Ω. Evaluate:

(a) the mean load current,
(b) the r.m.s. alternating load current,
(c) the d.c. output power,
(d) the input power to the anode circuit,
(e) the rectification efficiency,
(f) the ripple factor,
(g) the regulation from no-load to the given load.

[*Ans.* (a) 108 mA; (b) 120 mA; (c) 23·3 W; (d) 36 W;
(e) 64·8%; (f) 0·482; (g) 54 V]

103. A moving-iron ammeter and a simple moving-coil ammeter are placed in series with the load in a half-wave rectifier circuit. The reading on the a.c. instrument is 5 A. What is the reading of the other ammeter?

Calculate the instrument readings if the other half-wave is also rectified. Assume sinusoidal waveforms.

[*Ans.* 3·18 A; 7·07 A; 6·37 A]

104. A metal rectifier has the voltage-current characteristic shown. A sinusoidal alternating voltage, with a maximum value of 2 V, is applied to the rectifier, in series with a non-inductive resistor of value 80 Ω and a moving-coil ammeter of negligible resistance. Calculate the reading of the instrument.

VOLTAGE (V)   [*Ans.* 3·32 mA]

4
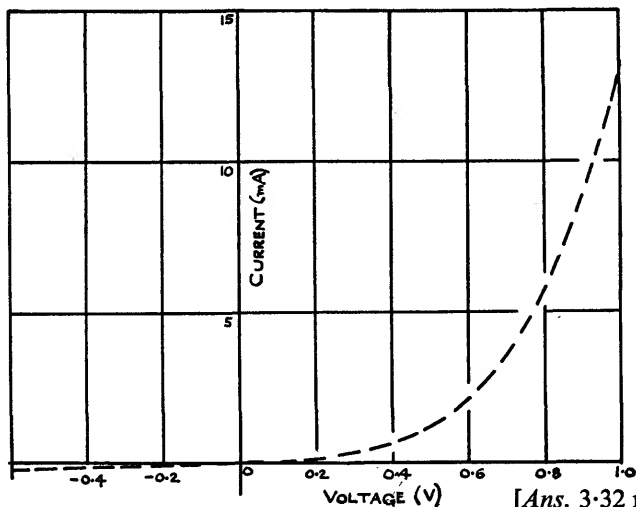
105. The rectifying element of a single-phase half-wave rectifier circuit has a resistance of $10\,\Omega$ in the forward direction and its resistance in the reverse direction may be taken as infinite. A resistor and a capacitor in parallel form the load and the capacitance is so great that the voltage across it is practically constant during both the charging and discharging periods. The resistance of the load is such that current flows through the rectifying element for one-sixth of each cycle of the a.c. supply voltage.

Determine the resistance of the load and the efficiency of rectification.

[*Ans.* $585\,\Omega$; 89.4%]

106. The grid voltage of the thyratron shown in the diagram is such that conduction begins $60°$ after the start of each cycle. Calculate:

(*a*) the r.m.s. value of the load current,

(*b*) the r.m.s. value of the voltage across the thyratron,

(*c*) the total power delivered by the a.c. supply.

The drop in the valve during conduction is 10 V.
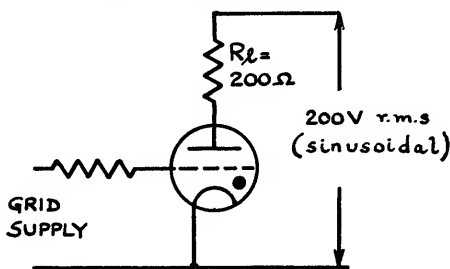
[*Ans.* (*a*) 0.63 A; (*b*) 155 V; (*c*) 77 W]

107. A single-phase full-wave rectifier circuit, employing a single L-type filter, is to supply 120 mA at 300 V with a ripple that must not exceed 10 V. Design a suitable filter if the supply frequency is (*a*) 50 c/s, (*b*) 60 c/s.

[*Ans.* A 10-H choke and a 4-μF capacitor are suitable in both cases]

108. A full-wave rectifier is used to supply power to a 2,000-Ω load. Two 20-H chokes and two 16-μF capacitors are available for filtering purposes. Calculate, approximately, the ripple factors for the following cases:

(*a*) one choke only in series with the load,

(b) two chokes in series with the load,

(c) one capacitor only in parallel with the load,

(d) two capacitors in parallel with the load,

(e) a single, L-type filter using one choke and one capacitor,

(f) a single, L-type filter using two chokes in series and two capacitors in parallel,

(g) a double, L-type filter, each section consisting of one choke and one capacitor.

The supply frequency should be taken as (i) 50 c/s, (ii) 60 c/s.

[Ans. (i) (a) 0·074; (b) 0·037; (c) 0·090; (d) 0·045; (e) 0·0037;
(f) 0·0009; (g) 2·95 × 10⁻⁵

(ii) (a) 0·062; (b) 0·031; (c) 0·075; (d) 0·0375; (e) 0·0025;
(f) 0·0006; (g) 1·42 × 10⁻⁵]

109. Outline the design of a power supply, from a single-phase full-wave rectifier, using a π-section filter, to give a d.c. output of 250 V at 50 mA and with a ripple factor not exceeding 0·01 %. The supply frequency should be taken as (i) 50 c/s, (ii) 60 c/s.

[There is no unique solution to this problem]

110. In a full-wave rectifier circuit, employing a 20-H choke in a π-section filter, what would be the power dissipated in a resistor R which replaced the choke and gave the same ripple factor, with an output current of 100 mA? The supply frequency should be taken as (i) 50 c/s, (ii) 60 c/s.

Repeat the calculation for the case where the output current is only 10 mA.

[Ans. (i) 125·7 W; 1·257 W;
(ii) 150·8 W; 1·508 W]

VOLTAGE AND CURRENT STABILIZATION

111. A stabilized power supply to give 280 V at 40 mA employs a 'Stabilovolt' tube. The tube current at full load is 20 mA and the d.c. supply voltage is 420 V. Determine the value of the series resistor.

Calculate also the variation of output voltage if the input voltage varies by $\pm$ 5%, and the variation of output voltage if the load current varies by $\pm$ 10 mA. The 'Stabilovolt' has four gaps, and the impedance of each gap may be taken as 40 Ω.

[*Ans.* 2,333 Ω; $\pm$ 1·43 V; $\pm$ 1·6 V]

112. The d.c. input voltage to a simple glow-discharge stabilizer is V_i, and the limiting resistor has a resistance R. The resistance of the load is R_L. Discuss how the values of V_i and R are chosen when the tube and load are specified.*

A certain tube has a maximum allowable current of 40 mA, and a minimum specified current of 5 mA. The working voltage of the tube is 150 V, and may be assumed constant. If the input voltage V_i varies by 10%, plot curves showing (*a*) the relation between the maximum and minimum values of load current I_{Lmax} and I_{Lmin}, if $I_{Lmax} + I_{Lmin} = 30$ mA, and (*b*) the relation between the minimum value of V_i and $(I_{Lmax} - I_{Lmin})$.

113. Two glow-discharge tubes in series, each having a running voltage of 100 V, are connected to a d.c. supply of voltage 400 V. They supply a load taking a current of 20 mA. The normal tube current is 30 mA. Calculate the resistance of the series resistor.

If the specified current range of each tube is 10 to 50 mA, find the range of input voltage over which stabilization is effective, and the range of load resistance.

[*Ans.* 4 kΩ; 320 V to 480 V; 5 kΩ to ∞]

114. Derive an expression for the ratio of the percentage change of output voltage to the percentage change of input voltage for the simple parallel-valve voltage stabilizer shown, if the load resistor

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* Similar calculations can be made for a Zener-diode shunt stabilizer, e.g. see J. A. Chandler, 'The Characteristics and Applications of Zener (Voltage Reference) Diodes,' *Electronic Engineering*, **32**, p. 78, 1960.

R_l is constant. Hence show that stabilization in such a circuit is impossible without a reference voltage and that the larger the value of the mutual conductance of the valve the better the stabilization.

Assume that the valve characteristics are linear and that the heater voltage of the valve remains constant.
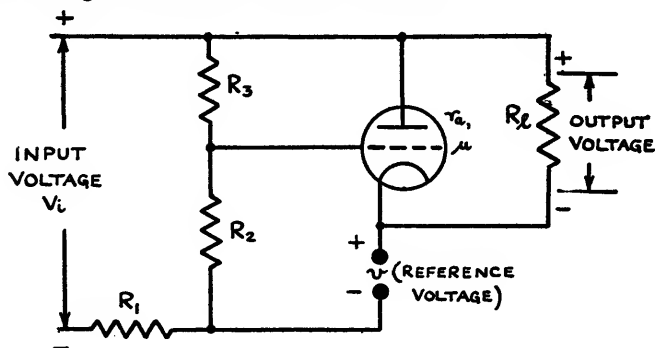
$$\left[Ans. \ \cfrac{1}{1 + \cfrac{v}{V_i} \left[\cfrac{R_1(\mu R_3 - r_a)}{r_a(R_2 + R_3)} - 1 \right]} \right]$$

115. The equivalent circuit of a series-valve stabilizer is illustrated. Find the change in output voltage for a 10% change of input voltage if $R_1 = 10$ kΩ, $R_2 = 1$ MΩ, $R_3 = 16$ MΩ, $V_i = 2{,}700$ V, $v = 100$ V, $\mu = 300$, $r_a = 100$ kΩ and $R_l = 260$ kΩ.

Calculate also the change in output voltage produced by a 10% change of R_l.

Make the same assumptions as in the previous problem.
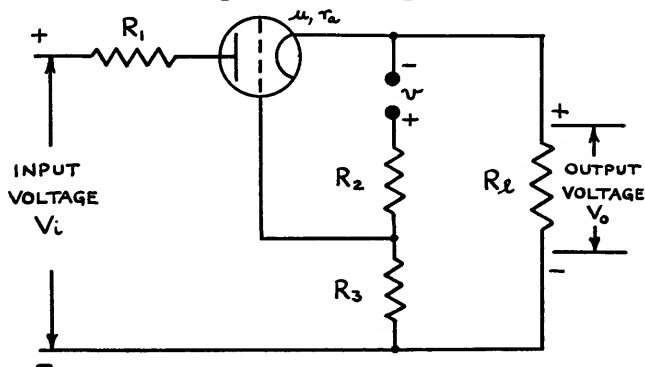
[*Ans.* 14 V; 3·5 V]

116. The diagram shows one form of thermionic-valve voltage stabilizer. Derive an expression for the ratio of the percentage change of output voltage to the percentage change of input voltage of the stabilizer if the load resistor is constant. Hence, show that theoretically, perfect stabilization is obtained when

$$R_3\mu_2(r_{a_1} + \mu_1 R_4) = (R_2 + R_3)(R_4 + R_5 + r_{a_2})$$

where μ_1 and μ_2 are the amplification factors of valves 1 and 2 respectively and r_{a_1} and r_{a_2} are the corresponding anode resistances.

Make the same assumptions as in the previous two problems.
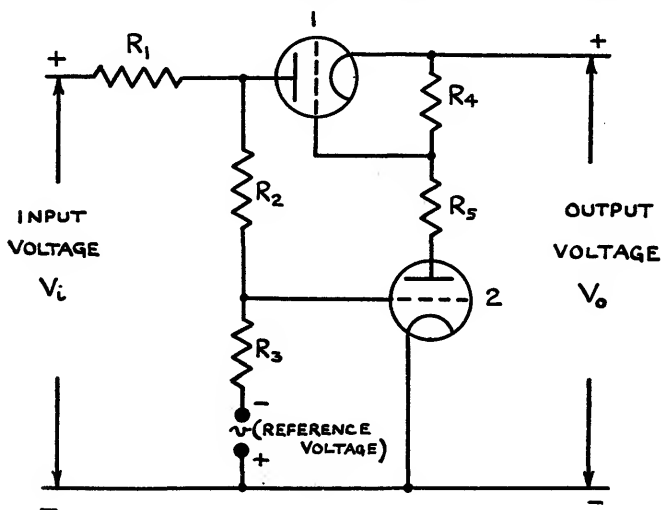
[*Ans.* $1/(1 - Av/BV_i)$

where $A = \mu_2\{(R_1 + R_2 + R_3)(R_1 + r_{a_1} + \mu_1 R_4) - R_1^2\}$
$\qquad - R_3\mu_2(R_1 + r_{a_1} + \mu_1 R_4)$
$\qquad - R_1(R_4 + R_5 + r_{a_2})$
and $B = R_3\mu_2(r_{a_1} + \mu_1 R_4) - (R_2 + R_3)(R_4 + R_5 + r_{a_2})$]

117. The anode voltage V_a, the grid voltage V_g and the anode current I_a of a triode are related by the expression

$$V_a = r_a I_a - \mu V_g - c r_a{}^*,$$

where c is a constant which is normally small.

Show that a 10% change of the heater voltage of the triode in

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* See F. A. Benson, G. V. G. Lusher and M. S. Seaman, 'Variations of Triode Characteristics with Heater Voltage,' *Electrical Journal*, **151**, p. 481, 1953.

Question 115 causes the output voltage of the stabilizer to change by about 4·2 V (assume that c is originally zero, and that it changes by 0·8 mA for a 10% change of heater voltage).

118. The voltage/current characteristic of a certain barretter is given by the following figures:

Voltage (V) .	80	100	120	140	160	180
Current (A) .	0·476	0·495	0·498	0·500	0·502	0·514

A number of valve heaters having a total resistance of 100 Ω are connected in series with this barretter. Find the current through the circuit when the input voltage is 200 V.

When the input voltage changes by ± 10% calculate the corresponding current variation.

[*Ans.* 0·5 A; 0·004 A]

AMPLIFIERS

119. A certain triode operates with an anode voltage of 250 V and a grid voltage of − 8 V. The anode current is then 9 mA. If the valve is used with a load having a resistance of 10,000 Ω, what is the value of the supply voltage required?

If the h.t. supply voltage is fixed at 430 V, calculate the resistance of the load to keep the valve working at the same operating point.

[*Ans.* 340 V; 20 kΩ]

120. A voltage amplifier employs a valve operating with an anode current of 9 mA and a negative bias of 8 V. Find the value of the resistance of a cathode resistor to give the required bias.

Determine also a suitable value for the cathode by-pass capacitance, if the signal frequency is (*a*) 1,000 c/s, or (*b*) 100 c/s.

[*Ans.* 889 Ω; 2 μF; 20 μF]

121. A low-frequency amplifier has a gain of 60 db. The input circuit is of 600 Ω resistive impedance and the output is arranged for a load of 10 Ω. What will be the current in the load when an alternating voltage of 1 V is applied at the input?

Express the gain of the amplifier in nepers.

[*Ans.* 12·9 A; 6·9 nepers]

122. A triode valve with an amplification factor of 20 and an anode resistance of 8,000 Ω is used as an amplifier with an inductive load of inductance 0·8 H and resistance 1,000 Ω. Determine the gain and phase shift of the amplifier at a frequency of 300 c/s and sketch the vector diagram of the arrangement. The input voltage is 5 V.

By calculating the gain and phase shift of the amplifier at a frequency of 2,000 c/s, show that both frequency distortion and phase-shift distortion occur.

[*Ans.* 3·92/− 133°; 14·97/− 143·8°]

123. By analysing the equivalent circuit of the RC-coupled amplifier correlate the sinusoidal and pulse responses of the amplifier.

124. A triode amplifier operating at a frequency of 10,000 c/s has a resistive load of 90,000 Ω. Calculate the voltage gain. The valve has an amplification factor of 60 and an anode resistance of 40,000 Ω. The interelectrode capacitances are $C_{ga} = 3 \cdot 0 \, \mu\mu F$, $C_{gc} = 3 \cdot 0 \, \mu\mu F$ and $C_{ac} = 3 \cdot 6 \, \mu\mu F$.

Find the gain of this stage when it forms the first section of a two-stage amplifier. The two stages are identical. Make any reasonable assumptions.

[*Ans.* 41·6; 39·1$\underline{/160 \cdot 2°}$]

125. Calculate the gain, the input capacitance and the input resistance of a triode amplifier when the load is a coil having an inductance of 20 mH and a resistance of 2,500 Ω, and the frequency is 10,000 c/s. The amplification factor of the triode is 20 and the anode resistance is 7,700 Ω. The interelectrode capacitances are $C_{ga} = 3 \cdot 4 \, \mu\mu F$, $C_{gc} = 3 \cdot 4 \, \mu\mu F$ and $C_{ac} = 3 \cdot 6 \, \mu\mu F$.

[*Ans.* 5·4$\underline{/- 160 \cdot 4°}$; 24·2 $\mu\mu F$; $- 2 \cdot 564$ MΩ]

126. A valve with an amplification factor of 80 and an anode resistance of 50,000 Ω has a load resistor of 100,000 Ω connected between the anode and the positive h.t. supply terminal. Between the cathode and the negative h.t. terminal is a resistor of 2,000 Ω with a capacitor of 1 μF connected in parallel. An alternating voltage v_i is applied between the grid and the negative h.t. terminal, and the output voltage v_o is measured across the anode load. Calculate the maximum and minimum values of the ratio v_o/v_i.

At what frequency is the magnitude of v_o/v_i equal to 0·707 of its maximum value?

[*Ans.* 53·3; 25·6; 121·2 c/s]

127. The first stage of a resistor-capacitor coupled amplifier employs a valve with an amplification factor of 20 and an anode resistance of 7,700 Ω. The resistance of the load is 50,000 Ω, the coupling capacitor has a capacitance of 0·01 μF, and the grid leak (including the resistive component of the input impedance of the next stage) has a resistance of 500,000 Ω. The input capacitance of the next stage is 200 $\mu\mu F$. Evaluate the gain of the stage at intermediate frequencies.

Find also the frequencies at which the gain falls to $1/\sqrt{2}$ of its intermediate-frequency value and calculate the frequency range over which the gain is greater than 14.

[*Ans.* $-$ 17·1; 31 c/s; 121,000 c/s; 44 to 84,960 c/s]

128. A two-stage resistor-capacitor coupled amplifier is to be designed with an overall mid-frequency gain of at least 6,000 and with the gain only 5 % below the mid-frequency value at a frequency of 100 kc/s. Pentodes are available which have

$$C_{gc} + C_{ac} = 10 \cdot 5 \,\mu\mu\text{F}$$

and g_m is $5 \cdot 2$ mA/V. It may be assumed that $10 \,\mu\mu\text{F}$ of stray capacitance shunts the equivalent circuit of one stage. Determine the value of the load resistance required and the actual overall gain at mid frequency.   [*Ans.* $17 \cdot 2$ kΩ; 7,992]

129. Draw the equivalent circuit for one stage of an inductor-capacitor coupled amplifier, and explain how the frequency-response characteristic of the amplifier may be examined.

130. By considering an RC-coupled amplifier employing pentodes show that it is not possible to increase bandwith without a commensurate sacrifice in gain and vice-versa.

Calculate the maximum figure of merit (gain × bandwidth) for a pentode which has $g_m = 5 \cdot 7$ mA/V, $C_{gc} = 6 \cdot 6 \,\mu\mu\text{F}$ and $C_{ac} = 2 \cdot 6 \,\mu\mu\text{F}$.

[*Ans.* $98 \cdot 6$]

131. Three non-identical RC-coupled valve amplifier stages are cascaded. The bandwidth limits f_1 and f_2 for the individual amplifiers are given in the Table below.

Amplifier	Frequency f_1 (c/s)	Frequency f_2 (kc/s)
1	100	250
2	80	350
3	50	550

Obtain equations from which the overall bandwidth limits could be calculated.

[See the solution for the equations]

132. The circuit illustrated provides an anode-load impedance that rises with decrease of frequency and can compensate for the reduction in gain at low frequencies caused by capacitor C.
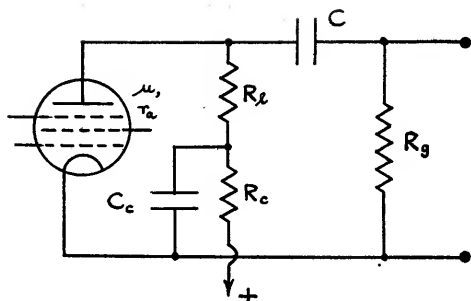
Assuming that R_g and r_a are large with respect to R_l, that the shunting effect of C and R_g on the load is negligible and that R_c is large compared with the reactance of C, show that if $C_c R_l = C R_g$ the low-frequency gain is independent of frequency.

133. The circuit illustrates one method of extending the upper limit of the frequency range of an RC-coupled amplifier where an inductor L counteracts the effect of C_g in reducing the load impedance at high frequencies.
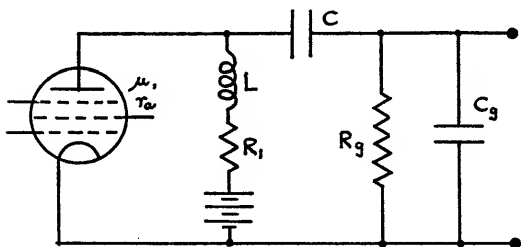

Analyse the circuit to find a desirable relation between L, C_g and R_1.

[*Ans.* For best flatness of the response curve
$L = 0.414 \, C_g R_1^2$ but a single value of L is not satisfactory for simultaneous flat gain and constant time delay and a compromise is necessary.]

134. Assuming the characteristic curves for a triode valve to be equidistant straight lines, prove that the maximum possible anode-circuit efficiency for a class-A amplifier, coupled to a resistive load through an ideal transformer, is 50%.

Show also that the theoretical maximum efficiency for the simple series-fed, class-A, power amplifier is 25%.

135. A transformer-coupled amplifier has the following constants:
Amplification factor of valve $= 10$.
Anode resistance of valve $= 8,000\,\Omega$.
Ratio of secondary to primary turns of transformer $= 3$.
Effective leakage inductance of transformer referred to primary $= 0 \cdot 5$ H.
Total effective shunt capacitance of transformer referred to primary $= 1,000\,\mu\mu$F.
Total effective resistance of transformer referred to primary $= 15,000\,\Omega$.
Resistance of primary winding of transformer $= 3,500\,\Omega$.
Inductance of primary winding of transformer $= 70$ H.
Obtain a curve showing how the gain of the amplifier varies with frequency.

136. A triode in an amplifier has an anode resistance of $8,000\,\Omega$ and an amplification factor of 16. It is coupled to the following stage by a transformer with a step-up ratio of 3. The secondary of the transformer is loaded with a resistance of $450\,k\Omega$. Calculate the stage gain at a frequency where the primary reactance is $5\,k\Omega$.

[*Ans.* 24·3]

137. A triode valve operates from a 300-V supply and its load is a resistor of $2,000\,\Omega$ coupled through an ideal transformer of ratio $1:1$. When a sinusoidal voltage is applied between the grid and the cathode of the valve, the maximum and minimum values of anode current are 150 mA and 20 mA. When the alternating grid voltage is zero, the anode current is 80 mA. Determine the power delivered to the load, the efficiency and the approximate percentage of second-harmonic current.

[*Ans.* 4·23 W; 0·17; 3·85%]

138. Determine the mean current, fundamental gain and second-harmonic distortion for a triode valve which has the anode-current/anode-voltage characteristics shown in the figure, when it is operating with a grid bias of -8 V, an anode supply voltage of 400 V and a load resistor of $8,000\,\Omega$. The peak input signal is 6 V.
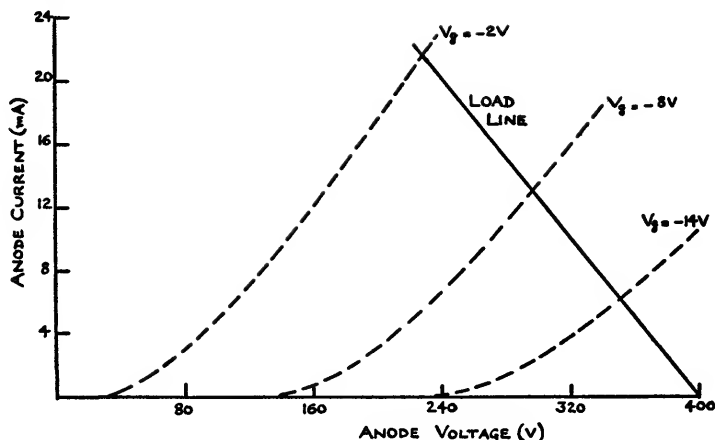
[*Ans.* 13·575 mA; 10·2; 4·9 %]

139. The anode current of a triode can be expressed as $[I_a + B_o + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + B_4 \cos 4\omega t]$ when the input voltage to the grid is sinusoidal, and of the form $v_g = V_g \cos \omega t$. Obtain a 5-point schedule by the Espley* method for determining B_o, B_1, B_2, B_3 and B_4 in terms of the anode currents for $\omega t = 0$, $\pi/3$, $\pi/2$, $2\pi/3$ and π.

$$[Ans.\ B_o = (I_{\max} + 2I' + 2I'' + I_{\min})/6 - I_a$$
$$B_1 = (I_{\max} + I' - I'' - I_{\min})/3$$
$$B_2 = (I_{\max} - 2I_a + I_{\min})/4$$
$$B_3 = (I_{\max} - 2I' + 2I'' - I_{\min})/6$$
$$B_4 = (I_{\max} - 4I' + 6I_a - 4I'' + I_{\min})/12$$

where $I_{\max}$, I', I_a, I'' and $I_{\min}$ are the anode currents for $\omega t = 0$, $\pi/3$, $\pi/2$, $2\pi/3$ and π respectively]

140. The following figures refer to a certain 25-W triode valve which delivers power to a resistive load by means of a choke-capacitor coupling.

Grid voltage $V_g = 0$.

Anode voltage V_a (V) . .	24	60	100	150	180
Anode current I_a (mA) . .	10	30	68	120	154

* D. C. Espley, 'Harmonic Production in Thermionic Valves with Resistive Loads,' *Proc. I.R.E.*, **21**, pp. 1439-1446, 1933.

Grid voltage $V_g = -10.7$ V.

Anode voltage V_a (V) . .	123	150	200	250
Anode current I_a (mA) . .	10	25	66	120

Grid voltage $V_g = -21$ V.

Anode voltage V_a (V) . .	220	250	300	350
Anode current I_a (mA) . .	10	27	67	119

Grid voltage $V_g = -32$ V.

Anode voltage V_a (V) . .	312	350	400
Anode current I_a (mA) . .	10	29	68

The h.t. supply voltage is 300 V. Determine the approximate resistance of the load for maximum undistorted power output. Calculate this maximum value of power and the efficiency.

[*Ans.* 1·95 kΩ; 4 W; 16%]

141. (*a*) Show mathematically that in a push-pull amplifier circuit employing two identical valves all even harmonics are suppressed in the output.

(*b*) Two triodes in a push-pull amplifier each have an anode-voltage/anode-current characteristic passing through the quiescent point (250 V, 30 mA) which is given by the following figures:

Anode voltage (V) . .	150	175	200	225	250	275	300	325
Anode current (mA) .	0	3	9	18	30	43	58	73

Draw the two curves with one inverted, and then obtain the composite characteristic. Find, from the curves, the anode resistance at the quiescent point of (i) each valve, (ii) the composite valve.

[*Ans.* (i) 2,000 Ω; (ii) 1,000 Ω]

142. Two power triodes, each having characteristics as defined by the figures in the following Tables, operate in class-A push-pull. Draw the composite characteristics if the quiescent point is at

$V_a = 200$ V, $V_g = -20$ V. Draw also the composite load line for an anode-to-anode load of 5 kΩ.

Determine the power output of this push-pull amplifier when the peak input to each valve is 20 V.

Grid voltage $V_g = 0$.

Anode voltage V_a (V) . .	0	40	80	120
Anode current I_a (mA) . .	0	13	32	52

Grid voltage $V_g = -20$ V.

Anode voltage V_a (V)	0	40	80	120	160	200	240
Anode current I_a (mA)	0	0	0	5	15	30	47

Grid voltage $V_g = -40$ V.

Anode voltage V_a (V) .	0	40	80	120	160	200	240	280	320
Anode current I_a (mA) .	0	0	0	0	0	3	9	19	33

[*Ans.* ≃ 1·53 W]

143. (*a*) Show that the maximum possible efficiency of a class-B audio-frequency amplifier, for sinusoidal signals, is 78·5%.

(*b*) A class-B amplifier operates from a 500-V, h.t. supply. The relation between the maximum permissible peak anode current (in amperes) and the minimum anode voltage is $I_{max} = 10^{-3} V_{min}$. If transformer losses are neglected, what is the maximum a.c. power which can be obtained, and what is the efficiency?

(*c*) Show that for a class-C amplifier where the angle of flow is 120° the maximum efficiency is 89·6%.

[*Ans.* (*b*) 31·25 W; 39·3%]

144. The anode current in a class-C amplifier may be regarded as triangular pulses having a peak value of 2·5 A and an angle of flow of 90°. The grid voltage varies sinusoidally and the anode-current/grid-voltage characteristic is linear. If the h.t. supply is 2·5 kV and the r.m.s. current delivered to a 750-Ω load is 0·8 A, what is the efficiency of the amplifier?

If the peak-to-peak amplitude of the anode voltage is 4 kV find the instantaneous anode voltage when anode current commences to flow.

[*Ans.* 61·9%; 1,086 V]

145. A triode valve with an amplification factor of 50 and an anode resistance of 30,000 Ω has for its anode load a parallel resonant circuit, of 'Q' = 45, which contains a resistor R of 15,000 Ω. The circuit resonates at 20,000 c/s. A second coil L_2 is coupled magnetically to the resonant-circuit coil L, the mutual inductance M between the coils being 1 mH. Calculate the voltage between the terminals of L_2 when a voltage of 1 V at a frequency of 20,500 c/s is applied between the grid and cathode of the valve.

[*Ans.* 3·485 V]

146. A pentode in a tuned-amplifier circuit has an anode resistance of 500 kΩ and a mutual conductance of 5 mA/V. In its anode circuit is a 20-μH coil with a 'Q' of 50, and this is tuned to parallel resonance at 1,592 kc/s. The output voltage is fed to a second stage of input resistance 500 kΩ through a coupling capacitor of negligible reactance. Calculate the gain of the stage at resonance.

[*Ans.* 48]

147. An amplifier has a gain of 20, without feedback. If 10% of the output voltage is fed back by means of a resistive negative-feedback circuit, determine the actual amplification.

[*Ans.* 6·67]

148. An amplifier employing a pentode with an amplification factor of 1,000 and a mutual conductance of 5 mA/V has a 200-kΩ load resistor. Calculate the voltage amplification (*a*) without feedback, (*b*) with 5% negative voltage feedback.

Determine also the effective constants of the valve when feedback is used.

[*Ans.* 500; 19·2; $\mu' = 19\cdot6$; $r_a' = 3\cdot92$ kΩ; $g_m' = 5$ mA/V]

149. A certain audio-frequency amplifier has a nominal gain of 120 and gives an output voltage of 60 V to its output transformer, with 10% second-harmonic distortion. How much feedback must be used to reduce the distortion to 1%? Find also the additional gain required ahead of the feedback amplifier, in order to give the same output voltage.

[*Ans.* Feedback factor = − 0·075; 10]

150. A multistage amplifier, when operated without feedback and with normal supply voltage, has a gain of 24,000. When the supply voltage falls by 25% the amplification is only 16,000. Show that if a negative-feedback potentiometer across the output is used to feed back 1/1,000 of the output voltage to the input, the amplification is nearly independent of variations in supply voltage.

[*Ans.* Gain for normal supply voltage = 960; gain when supply voltage falls by 25% from its normal value = 941]

151. An amplifier employing pentodes with anode slope resistance r_a and mutual conductance g_m has three identical resistor-capacitor coupled stages, each coupling circuit having capacitance C and resistance R_g. The anode load resistance in each stage is R_l. A fraction β of the amplifier output voltage is fed back to the input without phase-shift.

Use Nyquist's criterion to find the maximum value of β which may be employed without causing instability.

Assume that R_g and r_a are large compared with R_l and neglect interelectrode and stray capacitances.

[*Ans.* $8/(g_m R_l)^3$]

152. Show that the output impedance of a cathode-follower stage, which employs a valve with a mutual conductance of 4 mA/V, is about 250 Ω but the input impedance is high.

153. Draw the equivalent circuit of the feedback arrangement illustrated in the diagram and use Millman's network theorem* to find the overall gain. Obtain also, expressions for the output and input admittances of the network. If a pentode is used in the circuit, for which the anode resistance r_a is 1 MΩ and the mutual conductance g_m is 2 mA/V, and $Y_1 = Y_f = 2 \times 10^{-6}$ mho and $Y_g = 0.2 \times 10^{-6}$ mho, show that the gain is approximately unity. Under the same conditions, prove that the value of the output impedance is approximately $[2/g_m]$, and that the input impedance is approximately Y_1.

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* J. Millman, 'A Useful Network Theorem,' *Proc. I.R.E.*, **28**, p. 413, 1940. See also, F. A. Benson, *Electrical Engineering Problems with Solutions*, Spon, 1954, pp. 17–18 and pp. 104–109.
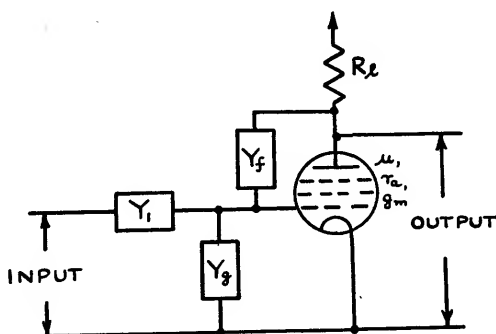
Compare the results with the corresponding ones for a cathode follower.

$$[Ans. \; Y_1(Y_f - g_m)/$$
$$\{(Y_1 + Y_f + Y_g)(Y_a + Y_l) + Y_f(Y_1 + Y_g + g_m)\} = A \text{ say,}$$
$$\text{where } Y_l = 1/R_l \text{ and } Y_a = 1/r_a;$$
$$Y_a + Y_f(Y_1 + Y_g + g_m)/(Y_1 + Y_f + Y_g);$$
$$Y_1\{Y_g + (1 - A)Y_f\}/(Y_1 + Y_g + Y_f)]$$

154. (a) Show that, for an amplifier which possesses voltage feedback, the output impedance is reduced in a ratio numerically equal to the voltage gain without feedback. Repeat the calculation for a circuit provided with current feedback, and thus prove that the output impedance is increased in the same ratio as the reduction in gain.

(b) Plot a Nyquist diagram* for a single-stage amplifier having a resistive load and negative voltage feedback, and investigate the stability of the circuit.

155. A junction transistor has the following constants: $r_{11} = 550 \, \Omega$, $r_{12} = 500 \, \Omega$, $r_{21} = 1 \cdot 9 \, M\Omega$, $r_{22} = 2 \, M\Omega$. Determine the input resistance of a common-base amplifier stage using this transistor as the load resistance varies from zero to infinity.

If the resistance of the source at the input of the amplifier is zero find the output resistance of the arrangement.

Calculate, also, the maximum possible voltage gain.

$$[Ans. \; 75 \text{ to } 550 \, \Omega; \; 2 \cdot 72 \times 10^5 \, \Omega; \; 3,454]$$

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* See H. Nyquist, 'Regeneration Theory,' *Bell System Tech. J.*, **11**, p. 126, 1932.

156. Derive expressions for the voltage amplification, the current amplification, the input resistance, the output resistance and the power gain of a common-base transistor-amplifier stage.

[*Ans.* $\dfrac{(r_m + r_b)R_l}{r_b(r_c - r_m + R_l + r_e) + r_e(r_c + R_l)}$;

$(r_m + r_b)/(r_b + r_c + R_l)$;

$(r_e + r_b) - r_b(r_b + r_m)/(r_b + r_c + R_l)$;

$r_c - \dfrac{r_b(r_m - R_g - r_e)}{R_g + r_e + r_b}$

$\dfrac{(r_m + r_b)^2\, R_l}{(r_b + r_c + R_l)\{r_b(r_c - r_m + R_l + r_e) + r_e(r_c + R_l)\}}$,

where r_b, r_c, r_e and r_m are the usual transistor parameters, R_g is the internal resistance of the source, and R_l is the load resistance]

157. Derive expressions for the voltage and current gains and the input and output resistances of the following transistor amplifiers:

(a) a common-emitter circuit, (b) a common-collector circuit.

[*Ans.* (a) $\dfrac{R_l}{r_b + \{(r_b + r_e)(r_c + R_l)/(r_e - r_m)\}}$;

$(r_c - r_m)/(r_e + r_c - r_m + R_l)$

$r_b + \dfrac{r_e(r_c + R_l)}{r_c + r_e - r_m + R_l}$

$(r_c - r_m) + \dfrac{r_e(r_b + r_m + R_g)}{r_b + r_e + R_g}$

(b) $\dfrac{r_c R_l}{r_b(r_c - r_m + r_e + R_l) + r_c(r_e + R_l)}$;

$r_c/(r_e + r_c - r_m + R_l)$;

$r_b + \dfrac{r_c(r_e - R_l)}{r_e + r_c - r_m + R_l}$

$r_e + \dfrac{(r_c - r_m)(r_b + R_g)}{r_b + r_c + R_g}$

where r_b, r_c, r_e and r_m are the usual transistor parameters, R_g is the internal resistance of the source, and R_l is the load resistance]

158. A junction transistor whose parameters are $r_{11} = 820\Omega$, $r_{12} = 800\ \Omega$, $r_{21} = 1{\cdot}98\ M\Omega$ and $r_{22} = 2M\Omega$ is used in a single-stage, common-emitter amplifier, with a load resistance of 430 Ω.
Calculate the voltage gain, the current gain and the input resistance.
$$[Ans.\ -15{\cdot}2;\ \simeq 99;\ \simeq 2{,}755\ \Omega]$$

159. Derive the following expressions for the common-base transistor amplifier:

Voltage gain $= \dfrac{-h_{21}}{h_{11}h_{22} - h_{12}h_{21} + h_{11}/R_l}$

Current gain $= h_{21}/(h_{22}R_l + 1)$

Input resistance $= \dfrac{(h_{11}h_{22} - h_{12}h_{21}) + h_{11}/R_l}{h_{22} + 1/R_l}$

where h_{11}, h_{12}, h_{21} and h_{22} are the usual hybrid parameters and R_l is the load resistance.

160. The hybrid parameters for a common-emitter transistor amplifier circuit are $h_{11}' = 800\ \Omega$, $h_{21}' = 47$, $h_{12}' = 5{\cdot}4 \times 10^{-4}$ and $h_{22}' = 80\ \mu$mhos. The load resistance is 20 kΩ. Calculate the voltage and current gains.
$$[Ans.\ -598;\ 18]$$

161. Determine the response of an RC-coupled common-emitter transistor amplifier at low and intermediate audio frequencies by analyzing the circuit illustrated.
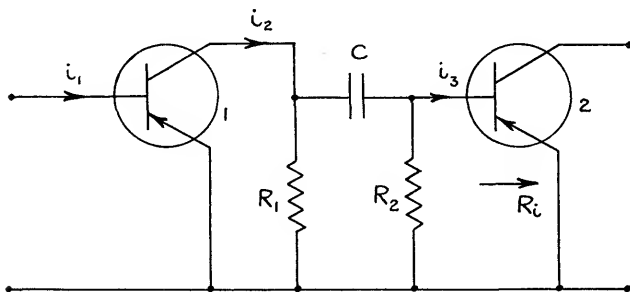

Draw an approximate high-frequency equivalent circuit for the common-base transistor arrangement and show how the high-frequency response of an RC-coupled common-emitter amplifier may be calculated.
[See the solution for the response expressions]

162. A transistor biasing circuit giving a measure of stabilization of the working point is illustrated. Obtain a relationship between the stability factor S, the parameters R_b and R_c and the current amplification α of the transistor.
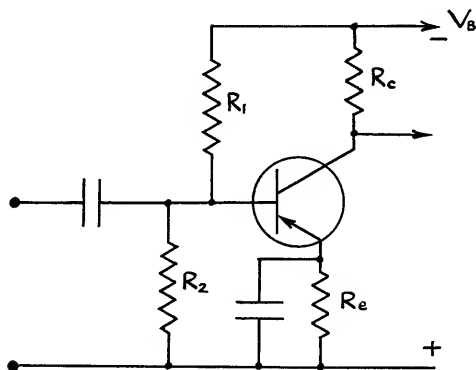
In a typical case $R_c = 10 \text{ k}\Omega$, $R_b = 100 \text{ k}\Omega$ and $\alpha = 0.98$. Determine S.
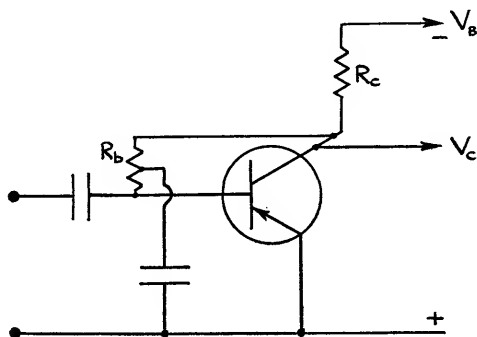
$$[Ans. \quad S = \frac{R_b + R_c}{R_c + (1 - \alpha)R_b}; \ 9\cdot2]$$

163. A transistor bias stabilization circuit is illustrated. Show that the stability factor S is given by $(R_b + R_e)/\{R_e + R_b(1 - \alpha)\}$ where $R_b = R_1 R_2/(R_1 + R_2)$.

In a typical circuit $R_1 = 50 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $R_e = 2\cdot5 \text{ k}\Omega$ and $\alpha = 0.98$. Find S.

$$[Ans. \ 6]$$

OSCILLATORS

164. A screen-grid valve has a negative anode resistance r_a of 90,000 Ω with suitable anode and screen voltages, and it is to be used with a coil L of 150 μH and a capacitor C of 500 $\mu\mu$F capacitance to form a dynatron oscillator.* Find the maximum coil resistance R to permit oscillation, and the corresponding frequency.

[*Ans.* 3·33 Ω; 581 kc/s]

165. A certain triode with an amplification factor of 9 has an anode resistance of 11,000 Ω when the anode and grid voltages are 90 V and − 6 V respectively, and an anode resistance of 9,000 Ω when the anode and grid voltages are 135 V and − 9 V respectively. A tuned-anode circuit has $L = 175 \mu$H, $C = 220 \mu\mu$F and $R = 18$ Ω. The grid coil has an inductance of 60 μH. With the higher anode voltage, what coefficient of coupling is required between the coils to make the circuit oscillate? How much must the coupling be if the anode voltage is dropped to 90 V and the bias adjusted accordingly?

[*Ans.* 0·228; 0·237]

166. A triode, with an amplification factor of 8 and an anode resistance of 6,000 Ω, has a tuned anode circuit of $L = 200 \mu$H, $R = 8$ Ω and $C = 0·0005 \mu$F. The grid coil has an inductance of 35 μH. The maximum available coupling is 40%. To the tuned circuit is coupled a tuned aerial having a resistance of 24 Ω. Determine the maximum permissible mutual inductance between the aerial and the tuned circuit if oscillations are to be maintained.

[*Ans.* 5·94 μH]

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* A point-contact transistor is capable of operation in a negative-resistance oscillator. Although this has rather limited application the analysis is frequently given as it is instructive, e.g. see L. M. Krugman, *Fundamentals of Transistors*, 2nd Revised Edition, Rider and Chapman and Hall, 1959 or J. D. Ryder, *Electronic Fundamentals and Applications*, 2nd Edition, Pitman, 1960.

167. Using a certain triode, having an amplification factor of 5 and an anode resistance of 1,800 Ω, it is desired to generate a frequency of 25 c/s in a tuned-anode circuit. Two coils, each of 0·6 H inductance and 11 Ω resistance, are available, the maximum attainable coupling between them being 32%. Can the required oscillations be produced with this arrangement?

What is the lowest frequency at which the circuit will oscillate?

[*Ans.* No; 48·3 c/s]

168. A triode, with an amplification factor of 9 and an anode resistance of 11,000 Ω, has a tuned-grid circuit with constants of values $L = 180 \mu H$, $R = 26 \Omega$ and $C = 0.0012 \mu F$. The coil in the anode circuit has an inductance of 50 μH and a coupling of 30% to the grid circuit. Will the circuit oscillate?

[*Ans.* No]

169. Determine approximately the condition necessary for maintaining oscillations in a tuned-grid circuit, using a triode with an amplification factor of 10 and an anode resistance of 10,000 Ω, when the tuning capacitance is 0·01 μF and the grid-coil resistance is 100 Ω.

[*Ans.* Mutual inductance = 1 mH]

170. Prove that for all valve oscillators

$$[Z + r_a/(1 + \mu N)] = 0$$

where Z is the vector impedance of the whole external circuit connected between the anode and cathode of the valve, N is the complex ratio $[V_g/V_a]$ between the grid and anode voltage vectors, μ is the amplification factor of the valve and r_a the anode resistance of the valve.

Use this theorem to determine the frequency of oscillation of a simple tuned-anode oscillator. Determine also the condition necessary for continuous oscillation.

$$[Ans. \ \frac{1}{2\pi} \sqrt{\frac{r_a + R}{r_a LC}}; \ M = (L + r_a RC)/\mu \ (R, L \text{ and } C \text{ are}$$

the oscillatory circuit constants, and M is the mutual inductance between grid and anode coils)]

171. In a certain Hartley oscillator the inductance of each coil is 20 mH and the capacitor has a capacitance of 0·1 μF. Determine the

frequency of oscillation if there is no mutual inductance between the coils. Neglect losses. Find also the coefficient of coupling which will reduce the frequency to 2 kc/s.

[*Ans.* 2·517 kc/s; 0·584]

172. In a class-A Hartley oscillator the two sections of the coil have inductances of 45 mH and 15 mH, the latter being in the grid circuit. The capacitor has a capacitance of 0·2 μF. The amplification factor of the valve is 20. Neglecting losses, calculate the critical mutual inductance for maintaining oscillations.

[*Ans.* 13·42 mH]

173. In a class-A Hartley oscillator the two sections of the coil have inductances L_1 and L_2 and resistances R_1 and R_2 respectively, the latter being in the grid circuit. The capacitor has a capacitance of C. The mutual inductance between the sections of the coil is M. The amplification factor of the valve is μ and the anode resistance of the valve is r_a. Derive expressions for the frequency of oscillation, and the condition for continuous oscillations.

[*Ans.*

$$f = \frac{1}{2\pi} \sqrt{\frac{1 + R_1/r_a}{C\left[(L_1 + L_2 + 2M) + \dfrac{(R_1 L_2 + R_2 L_1)(1 + \mu) - \mu M R_1}{r_a}\right]}};$$

$$(r_a + R_1)(R_1 + R_2) - \omega^2(L_1 - \mu M)(L_1 + L_2 + 2M) + (L_1 - \mu M)/C$$
$$= R_1(R_1 - \mu R_2) - \omega^2(L_1 + M)(L_1 + M - \mu L_2 - \mu M)$$

where $\omega = 2\pi f$]

174. The coil in the tuned circuit of a Colpitts oscillator has an inductance L and a resistance R. The two capacitors have capacitances C_1 and C_2. The valve has an amplification factor μ and an anode resistance r_a. Determine the frequency of oscillation, and the condition for steady oscillations.

[*Ans.* $f = \omega/2\pi = \dfrac{1}{2\pi} \sqrt{\dfrac{1}{L}\left\{\dfrac{1}{C_2} + \dfrac{1}{C_1}(1 + R/r_a)\right\}}$;

$(1 + \mu)/\omega^2 C_1 C_2 = r_a R + L/C_1$]

175. A triode valve has a coil in its anode circuit of inductance L_2 and resistance R_2. In the grid-cathode circuit of the valve is a second coil of inductance L_1 and resistance R_1. There is no magnetic

coupling between the coils, but a capacitor of capacitance C is connected between the anode and grid of the valve. The valve has an amplification factor μ and an anode resistance r_a. Determine the condition for maintenance of oscillations in this circuit, and the corresponding frequency. Neglect valve-electrode capacitances.

$$\left[Ans.\ r_a(R_1 + R_2) + R_1 R_2(1 + \mu) + L_2/C = \omega^2 L_1 L_2(1 + \mu); \right.$$

$$\left. f = \omega/2\pi = \frac{1}{2\pi} \sqrt{\frac{1 + R_2/r_a}{C[L_1 + L_2 + (R_1 L_2 + R_2 L_1)(1 + \mu)/r_a]}} \right]$$

176. A crystal-oscillator circuit is illustrated. Determine the frequency of oscillation if the equivalent circuit of the crystal is assumed loss-free.
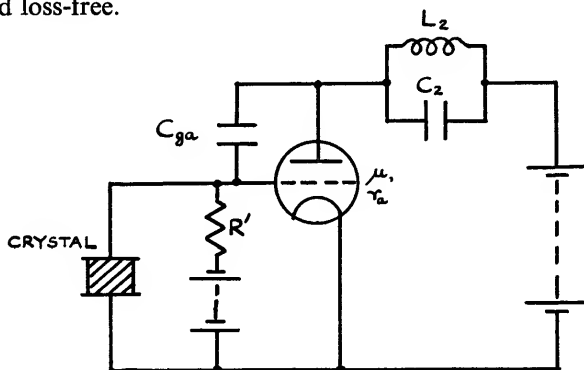
$$\left[Ans.\ \frac{1}{2\pi} \sqrt{\frac{1}{LC} \left[\frac{1 + (1 + C_1/C_{ga} + C/C_{ga})/\mu}{1 + (1 + C_1/C_{ga})/\mu} \right]} \right.$$

where L, C and C_1 are the crystal constants]

177. In a certain Wien-bridge type of oscillator* the frequency-selective network employs $120{,}000\ \Omega$ resistors and $0{\cdot}001\text{-}\mu\text{F}$ capacitors. Find the frequency of oscillation.   [*Ans.* 1,326 c/s]

* A good account of such oscillators is given by T. P. Flanagan, 'Resistor-Capacitor Oscillator Design,' *Marconi Instrumentation*, 3, p. 82, May, 1952. An oscillator using the Wien-bridge Network and valves has been described by F. E. Terman, R. R. Buss, W. R. Hewlett and F. C. Cahill, 'Some Applications of Negative Feedback with Particular Reference to Laboratory Equipment,' *Proc. I.R.E.*, **27**, p. 649, 1939.

The ladder phase-shift networks were first fully described by E. L. Ginzton and L. M. Hollingsworth, 'Phase-shift Oscillators,' *Proc. I.R.E.*, **29**, p. 43, 1941.

Complete circuit diagrams of some transistor R-C ocsillators of the Wien-bridge type and employing ladder networks have been given in the Mullard Reference Manual on Transistor Circuits, 1st Edition, 1960.

178. A three-section ladder phase-shift oscillator* has three similar phase-advancing sections, each consisting of a 100-kΩ resistor and a 0·0005-μF capacitor. Calculate the frequency of oscillation, and show that the attenuation ratio of the network is 29.

[*Ans.* 1,300 c/s]

179. Repeat the calculation of Question 178 for a similar oscillator having three phase-retarding sections.

[*Ans.* 7,800 c/s]

180. A fourth similar section is added to the phase-shift network of Question 178. Calculate the new frequency of oscillation and the attenuation ratio of the network.

[*Ans.* 2,663 c/s; 18·39]

181. Show that the period of oscillation of the simple discharge-tube relaxation oscillator of the type illustrated is

$$CR \log_e \{(V - V_e)/(V - V_s)\}$$

where V_s is the striking voltage of the tube and V_e is the extinction voltage.
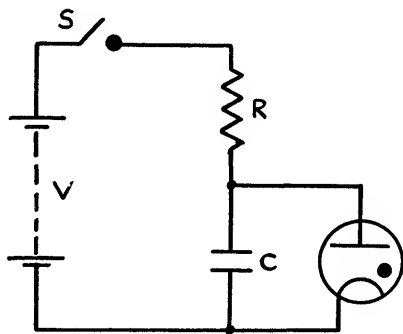
182. A glow-discharge tube relaxation oscillator is supplied at 200 V. The striking and extinction voltages of the tube are 160 V and 120 V, respectively. Calculate the resistance to be used with a 0·04-μF capacitor for a frequency of oscillation of 100 c/s.

Determine also the percentage change in frequency if the supply voltage drops by 1 %.

[*Ans.* 360·7 kΩ; − 3·66%]

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* See footnote on p. 61.

183. In a certain thyratron relaxation oscillator the capacitor has a capacitance of 0·01 μF and the resistor has a resistance of 500 kΩ. The supply voltage is 250 V. The thyratron has a control ratio of 30 and its extinction voltage is 20 V. Find the grid bias required to give an oscillation amplitude of 100 V.

What is the period of oscillation under these conditions?

[*Ans.* $-$ 4 V; 2·83 $\times$ 10^{-3} sec]

184. In a symmetrical multivibrator each valve has an anode voltage of 110 V with the coupling capacitor removed, and the static cut-off grid bias is 20 V with full anode voltage. The h.t. supply voltage is 250 V. Calculate the frequency of oscillation when each capacitor has a capacitance of 0·005 μF and each grid resistor has a resistance of 50 kΩ.

[*Ans.* 1,027 c/s]

185. Determine the beat frequency when a signal of 300-m wavelength is combined with the output of a 1·3-Mc/s oscillator.

If a second signal of 400-m wavelength is now received to what frequency must the oscillator be tuned to provide the same beat frequency as before?

[*Ans.* 300 kc/s; 1,050 kc/s]

186. A voltage source with a frequency range of 50 to 10,000 c/s is required. Calculate the ratio of the maximum/minimum values of capacitance required (*a*) in a single feedback type of oscillator, (*b*) using the beat-frequency method, where the oscillator with the lower frequency operates at 100 kc/s.

[*Ans.* 4 $\times$ 10^4; 1·2]

187. Show that for maintenance of oscillations in a transistor Colpitts oscillator circuit*

$$C_1/C_2 = \{r_m \pm \sqrt{r_m{}^2 - 4(r_c - r_m)(r_b + r_e)}\}/2(r_b + r_e),$$

where C_1 and C_2 are the tuned-circuit capacitances and r_b, r_c, r_e and r_m are the usual transistor parameters. It must be assumed in deriving the expression that the frequency of oscillation is given by

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* Transistor feedback oscillators can be designed as equivalents for all the known types of valve oscillator and in appearance look very similar to their valve counterparts. Tuned-collector, Hartley, Colpitts and crystal-controlled circuits are some examples of transistor devices frequently described in books, e.g. see L. M. Krugman, *Fundamentals of Transistors*, 2nd Revised Edition, Rider and Chapman and Hall, 1959. R. B. Hurley, *Junction Transistor Electronics*, Wiley, 1958. W. D. Bevitt, *Transistors Handbook*, Prentice-Hall, 1956. J. D. Ryder, *Engineering Electronics*, Chapter 12, McGraw-Hill, 1957.

$1/2\pi\sqrt{LC}$ and $r_e \ll r_m$, where L is the inductance of the tuned circuit.

Determine, also, the exact frequency of oscillation for the circuit and show that changes of the transistor parameters r_c and r_e are important when considering frequency stability.

[*Ans.* $(1/2\pi)\sqrt{1/LC + 1/xC_1C_2}$,
where $C = C_1C_2/(C_1 + C_2)$ and
$x = r_br_c + r_br_e + r_cr_e - r_br_m$]

188. The essential parts of a Hartley transistor oscillator are as illustrated.
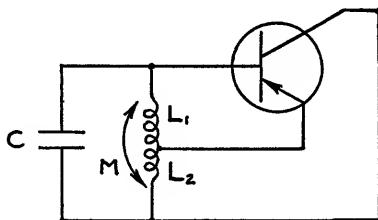
Show that the frequency of oscillation (f) is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{C(L_1 + L_2 + 2M) - (L_1L_2 - M^2)/k}}$$

where $k = (r_b + r_e)r_c + r_b(r_e - r_m)$, r_b, r_c, r_e and r_m being the usual transistor parameters.

Prove, also, that for maintenance of oscillations the oscillator requires that:

$$(L_2 + M)/(L_1 + M) \geqslant r_m/(r_b + r_e)$$

NOISE

189. Calculate the r.m.s. value of the noise voltage developed across a 1,000-Ω resistor at a temperature of 17°C in a frequency band of 10 Mc/s.

[*Ans.* 12·66 μV]

190. Show that the noisy resistor of Question 189 can be regarded, as far as the effect of thermal-agitation noise is concerned, as a noise-free resistor in parallel with a generator supplying a constant current of 12·66 $\times$ 10^{-9} A.

191. Evaluate the r.m.s. component of noise current in a 20-kc/s bandwidth due to a random current of 1 mA.

[*Ans.* 2·53 $\times$ 10^{-9} A]

192. The emission current of a temperature-limited diode is 10 mA. Determine the r.m.s. value of the fluctuation components of the current in a 20-kc/s bandwidth.

[*Ans.* 7·98 $\times$ 10^{-9} A]

193. A resistor of R_1 ohms is placed in parallel with a capacitor of C farads. Find the noise voltage across the combination in the frequency band 0 to ∞.

[*Ans.* $\sqrt{kT/C}$ where k is Boltzmann's constant and T is the absolute temperature]

194. It is convenient to interpret the noise in a triode as being due to a noisy resistor in series with the grid of a noise-free valve. Calculate the value of this noisy resistor for a triode with a mutual conductance g_m equal to 2·6 mA/V.

[*Ans.* 961 Ω]

195. The noise of a pentode can be supposed to be due to a noisy resistor in series with the control grid of a noise-free valve. Estimate the value of this noisy resistor for a pentode in which the anode

current is 10 mA, the screen-grid current is 2·5 mA and the mutual conductance g_m is 9 mA/V.

[*Ans.* 716 Ω]

196. The positive-ion control-grid current in a valve containing some gas is 0·01 μA. The mutual conductance of the valve is 5 mA/V, the anode current is 1 mA and the shunt resistance of the grid circuit is 100,000 Ω. Calculate the value of the equivalent noise-generating grid resistor.

[*Ans.* 2,003·2 Ω]

197. A network consists of a resistor R_1 at temperature T_1 in parallel with a further resistor R_2 at temperature T_2. What is the maximum noise power per unit bandwidth available from this network? Under these conditions calculate the value of the load resistance connected across the network.

[*Ans.* $k(T_1R_2 + T_2R_1)/(R_1 + R_2)$;
$R_1R_2/(R_1 + R_2)$, where k
is Boltzmann's constant]

198. The following statement often simplifies noise calculations on a network when there are several elements present, all at different temperatures:

"The effective noise temperature of a complex two-terminal network is equal to the summation over all the elements of the product (temperature of element) $\times$ (fraction of total power dissipated in that element when the network is regarded as a passive load)."

Verify this statement in the case of a network consisting of two resistors R_1 and R_2 at temperatures T_1 and T_2 respectively when they are connected (*a*) in series, (*b*) in parallel. The Johnson formula may be assumed.

Use the statement to show that the effective temperature of a length l of transmission line at temperature T_1 terminated by its matched load at temperature T_2 is $T_1\{1 - \exp(-2\alpha l)\} + T_2 \exp(-2\alpha l)$ where α is the voltage attenuation constant.

199. Three resistors, having resistances R_1, R_2 and R_3 ohms are maintained at temperatures T_1, T_2 and T_3 respectively. They are initially all connected in parallel. Show, starting from the Johnson

noise generated in each resistor, that this parallel combination is equivalent to a single resistance R at temperature T where

$$R = R_1 R_2 R_3/(R_1 R_2 + R_2 R_3 + R_1 R_3)$$

and $T = (T_1 R_2 R_3 + T_2 R_1 R_3 + T_3 R_1 R_2)/(R_1 R_2 + R_2 R_3 + R_1 R_3).$

Calculate the values of R and T if the resistors are now connected in series.

[*Ans.* $R_1 + R_2 + R_3$; $(T_1 R_1 + T_2 R_2 + T_3 R_3)/(R_1 + R_2 + R_3)$]

200. To perform some noise measurements on a receiver a resistor contained in a variable-temperature bath is used to replace the aerial. The resistor is matched to the input impedance of the receiver. Show that the noise power per unit bandwidth delievered to the receiver is kT watts where k is Boltzmann's constant and T is the temperature of the bath.

The noise output from the receiver is first measured with the bath at room temperature (300°K) and is then found to have doubled itself when the temperature of the bath is raised to 900°K. Prove that the noise figure of the receiver is about 3 dB.

201. A signal generator whose output impedance is $500\ \Omega$ is calibrated in terms of the power it will deliver to a matched load (i.e. the maximum available signal power). It is connected to a receiver with a bandwidth of 10 kc/s and whose first stage consists of a triode with negligible shot noise and which has a $1\ \mathrm{k}\Omega$ resistor connected between cathode and grid.

What will be the signal-generator reading when it is adjusted so that the signal output from the receiver is equal to the noise output? The temperature should be taken as 300°K.

[*Ans.* $6{\cdot}2 \times 10^{-17}$ W]

202. A galvanometer is stated by its manufacturer to have a sensitivity of 75 mm/μA. The effective mirror-scale distance is 1 m. The specific couple of the suspension is 10^{-10} newton-m/radian.

Calculate the r.m.s. deflection due to thermal agitation at 300°K. Estimate also the minimum detectable current.

[*Ans.* $0{\cdot}0129$ mm; $0{\cdot}172$ mμA]

203. A galvanometer has a 50-turn rectangular coil of area 1 cm². The coil is situated in a radially directed magnetic field of strength

1 Wb/m². The specific couple of the suspension is 10^{-10} newton-m/radian.

Assuming the usual optical system, calculate the deflection of the light spot on a scale one metre distant from the mirror attached to the coil when a current of 10^{-10} A passes through the coil.

At what current is the deflection equal to the r.m.s. random deflection due to thermal fluctuations at a temperature of 300°K?

[*Ans.* 1 cm; 0·129 $\mu\mu$A]

MODULATION, DETECTION AND FREQUENCY CHANGING

204. The tuned circuit of an oscillator in an amplitude-modulated transmitter employs a 50-μH coil and a 0·001-μF capacitor. If the oscillator output is modulated up to 10,000 c/s, what is the frequency range occupied by the carrier and sidebands?

[*Ans.* 702 to 722 kc/s]

205. The aerial current of a transmitter is 8 A when the carrier only is transmitted, but it increases to 8·93 A when the carrier is sinusoidally modulated. Find the percentage modulation.

Determine the aerial current when the depth of modulation is 0·8.

[*Ans.* 70%, 9·19 A]

206. A certain transmitter radiates 9 kW of power with the carrier unmodulated, and 10·125 kW when the carrier is sinusoidally modulated. Calculate the depth of modulation.

If another audio wave, modulated to 40%, is also transmitted, determine the radiated power.

[*Ans.* 0·5; 10·845 kW]

207. Several frequencies simultaneously modulate a carrier. Show that the total power of all the sidebands will always be less than half of the carrier power.

208. A signal voltage $E_s = 1 \cdot 5 \sin (1,000t)$ V and a carrier voltage $E_c = 5 \sin (4 \times 10^6 t)$ V are applied to the grid of a triode whose anode-current/grid-voltage characteristic can be represented by the expression $I_a = [10 + 2V_g + 0 \cdot 02V_g{}^2]$ mA. Determine the amplitude and frequency of the various components of the anode current, and evaluate the depth of modulation.

[*Ans.* $I_a = 10 \cdot 2725 + 3 \sin 10^3 t + 10 \sin (4 \times 10^6 t)$
$- 0 \cdot 25 \cos (8 \times 10^6 t) - 0 \cdot 0225 \cos (2 \times 10^3 t)$
$+ 0 \cdot 15 \cos (4 \times 10^6 - 10^3)t - 0 \cdot 15 \cos (4 \times 10^6 + 10^3)t$ mA;
0·03]

209. If a radio-frequency carrier wave is amplitude-modulated by a band of frequencies, 300 c/s to 3·4 kc/s, what will be the bandwidth of the transmission and what frequencies will be present in the transmitted wave if the carrier frequency is 104 kc/s?

[*Ans.* 6·8 kc/s; 100·6 to 103·7 kc/s, 104 kc/s, 104 to 107·4 kc/s]

210. A balanced modulator employs two triodes which have slightly different characteristics. If their characteristics are $i_{a_1} = [I_a + aV_g + b_1V_g^2]$ and $i_{a_2} = [I_a + aV_g + b_2V_g^2]$, find the output spectrum when the carrier frequency is f_c and the audio frequency is f_a.

$$[Ans. \text{ Output} = 2aE_a \sin \omega_a t + \left(\frac{b_1 - b_2}{2}\right)(E_c^2 + E_a^2)$$

$$- \left(\frac{b_1 - b_2}{2}\right)(E_c^2 \cos 2\omega_c t + E_a^2 \cos 2\omega_a t)$$

$$+ (b_1 + b_2)E_aE_c\{\cos(\omega_c - \omega_a)t - \cos(\omega_c + \omega_a)t\}]$$

211. The following particulars refer to a certain anode-modulated, class-C amplifier:

D.C. anode-supply voltage, $E_a = 2,000$ V.

Average anode current, $\bar{I}_a = 200$ mA.

Modulation voltage on the secondary side of the modulation transformer, $e_i = 1,400 \sin 2,000 \pi t$ volts.

Anode-circuit efficiency $= 0·8$.

Determine:

(*a*) the depth of modulation m,

(*b*) the approximate maximum value of the anode-cathode voltage of the valve,

(*c*) the power delivered by the d.c. supply,

(*d*) the power delivered by the modulation transformer,

(*e*) the r.f. output power without modulation,

(*f*) the r.f. output power with modulation,

(*g*) the impedance into which the modulation transformer works.

[*Ans.* (*a*) 0·7; (*b*) 6,460 V; (*c*) 400 W; (*d*) 98 W; (*e*) 320 W; (*f*) 398 W; (*g*) 10,000 Ω]

212. A frequency-modulated wave can be represented by the expression $i = I \sin(ct + M \sin at)$, where M is the frequency-deviation ratio, a is the angular frequency of the modulating signal

and c is the angular frequency of the carrier. Show that the spectrum consists of a carrier and an infinite number of sidebands, all of whose amplitudes are various-order Bessel functions of M.

213. A frequency-modulated wave, resulting from modulation by an audio-frequency wave of frequency $f_a = 5{,}000$ c/s, has a frequency deviation of 50 kc/s. If this wave when radiated produces an unmodulated field of 1 mV/m at a certain point, what is the strength of the carrier and the sidebands at the same point when the wave is modulated?

[*Ans.* Carrier, $240\ \mu$V/m; sidebands, $50\ \mu$V/m,
$260\ \mu$V/m, $50\ \mu$V/m, etc.]

214. Show that the reactance-valve circuit illustrated, neglecting the effect of the alternating anode voltage on the valve current, is equivalent to a resistance $R_{AB} = \{1 + (R\omega C)^2\}/g_m$ in parallel with an inductance $L_{AB} = \{1 + (R\omega C)^2\}/g_m R\omega^2 C$, where g_m is the mutual conductance of the valve.

Find the equivalent resistances and reactances of the circuit for the three other combinations of resistance, inductance and capacitance elements which may be used for Z_1 and Z_2.

[*Ans.*

Z_1 Z_2	C R	R L	L R
R_{AB}	$\{1 + (R\omega C)^2\}/g_m(R\omega C)^2$	$(R^2 + \omega^2 L^2)/g_m\omega^2 L^2$	$(R^2 + \omega^2 L^2)/g_m R^2$
X_{AB}	$C_{AB} = [g_m RC/\{1 + (R\omega C)^2\}]$	$C_{AB} = [g_m RL/(R^2 + \omega^2 L^2)]$	$L_{AB} = [(R^2 + \omega^2 L^2)/g_m R\omega^2 L]$

215. A diode detector employs a 220-kΩ resistor and a 100-$\mu\mu$F capacitor. Calculate the maximum value of the depth of modulation if peak clipping is to be avoided, and the maximum modulating frequency which the detector is designed to handle is 6,000 c/s.

[*Ans.* 0·77]

216. Show mathematically that additive frequency changing can be obtained by applying the signal and oscillator voltages to the grid-cathode circuit of a valve with a non-linear anode-current/ grid-voltage characteristic; and also show that a valve having two control grids can be used as a multiplicative frequency changer.

217. An amplitude-modulated superheterodyne receiver has an intermediate frequency of 465 kc/s and the desired-signal frequency is 700 kc/s. Prove that undesired-signal frequencies of 351, 816, 1,867 and 2,797 kc/s present at the grid of the frequency changer, produce 2-kc/s interference whistles if the I_a/V_g characteristic of the valve has a third-power term.

218. (*a*) Design an oscillator tuned circuit containing only two pre-set components for a receiver covering the frequency band 550 to 1,500 kc/s, which has to gang with a signal circuit containing a 156-μH inductance coil. The intermediate frequency is 465 kc/s.

(*b*) Design an oscillator tuned circuit containing three preset components for the receiver.

[*Ans.* (*a*) Padding capacitance = 288 $\mu\mu$F,
Tuning inductance   = 117·3 μH,
(*b*) Padding capacitance = 601 $\mu\mu$F,
Trimmer capacitance = 36·5 $\mu\mu$F,
Tuning inductance   = 77·4 μH]

MOTION OF ELECTRONS IN ELECTRIC AND MAGNETIC FIELDS

219. An electric field of 10^4 V/m is parallel with but opposed to a magnetic field of 5 mWb/m². Electrons travelling with a velocity of $1\cdot19 \times 10^7$ m/sec enter the region of the fields at an angle of 30° with the direction of the electric field. Determine the motion of an electron.

[*Ans.* The electron path is helical]

220. A magnetic field B and an electric field E are at right-angles to one another as illustrated. Determine the path of an electron which starts at rest at the origin O.
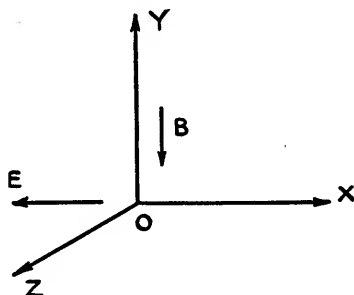
[*Ans.* The path is cycloidal: the path generated by a point on the circumference of a circle which rolls along the Z axis]

221. In a certain cathode-ray tube there is a magnetic field of $0\cdot01$ Wb/m² along the axis and an electric field of 10^4 V/m applied to the deflector plates which are $0\cdot02$ m long. Calculate how far from the axis an electron will be when it leaves the region between the deflector plates if it was travelling initially along the axis with a velocity of 10^6 m/sec.

[*Ans.* $2\cdot04 \times 10^{-2}$ m]

222. A magnetic field B of 1 mWb/m² and an electric field E of 5 kV/m are at angle of 20° with each other as illustrated. Determine the path of an electron which starts at rest at the origin.
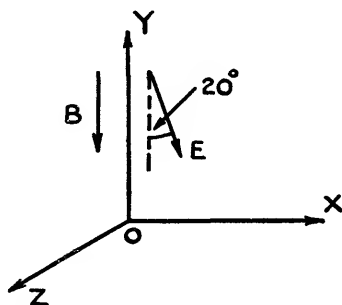
[*Ans*. The projection of the path in the *XZ* plane is a common cycloid]

TIME BASES AND ELECTRON-BEAM
DEFLECTION IN CATHODE-RAY TUBES

223. In the time-base circuit shown the supply frequency is 50 c/s and the capacitance of C is $2\,\mu$F. Calculate the resistance of R to give a circular time base for equal sensitivities of the X and Y plates.

Repeat the calculation for sensitivities of the X and Y plates equal to 0·45 and 0·55 mm/V, respectively.

[*Ans.* 1,592 Ω; 1,946 Ω]
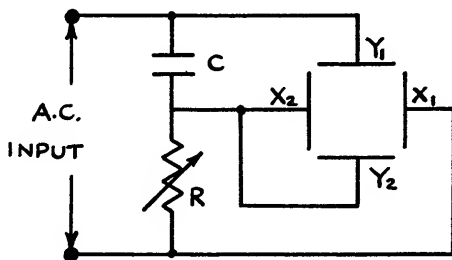
224. In a thyratron time-base circuit the control ratio of the valve is 30, and its extinction voltage is 20 V. The grid bias is set at − 5 V. The capacitor has a capacitance of 0·01 μF and it is charged at a constant rate of 1·5 mA. The output from the circuit is applied to a cathode-ray tube which has a sensitivity of 0·8 mm/V. Find the length of the time base and the frequency of sweep.

[*Ans.* 10·4 cm; 1,154 c/s]

225. A certain linear time base employs a saturated diode and a thyratron. The striking and extinction voltages of the thyratron are 280 V and 30 V respectively, and the diode charging current is constant at 5 mA. Find the range of capacitance values required for the capacitor if the sweep frequency is to be variable from 20 c/s to 20 kc/s.

[*Ans.* 0·001 to 1 μF]

226. A series circuit consisting of a 2,500-Ω resistor and a 4-μF capacitor is connected to a 300-V d.c. supply. Across the capacitor is a glow-discharge tube which is triggered to strike every 20 ms. The tube on striking discharges the capacitor completely in a negligibly short time. Determine the r.m.s. value of the charging current. Find also the voltage across the capacitor when the tube strikes.

[*Ans.* 60 mA; 260 V]

227. A cathode-ray tube has plane-parallel deflector plates 1·5 cm long spaced 0·3 cm apart. The screen is 20 cm from the ends of the deflector plates. Before entering the space between the plates the electron beam is accelerated by a voltage of 1,500 V. Determine the sensitivity of the tube in volts/cm. Neglect any fringing at the ends of the deflector plates.

[*Ans.* 28·9]

228. A cathode-ray tube is constructed with internally-mounted plane-parallel magnetic poles 1·5 cm long and the ends of these are a distance of 20 cm from the screen. Before entering the field due to the poles the electrons are accelerated by a voltage of 1,500 V. Determine the sensitivity of the tube in Wb/m²/m. Neglect any fringing of the magnetic field.

[*Ans.* 0·042]

KINETIC THEORY OF GASES

229. Calculate the root-mean-square speed of nitrogen molecules (N_2) in metres/sec at 273°K and 373°K. Take the mass of a proton as 1.67×10^{-27} kg and the atomic weight of nitrogen as 14.

[*Ans.* 492; 575]

230. What fraction of the molecules per cubic metre in a gas have speeds lying within $\pm 1\%$ of the most probable speed?

[*Ans.* 0.0166]

231. Show that the distribution function for gas molecules having x directed velocity components in the range v_x to $v_x + dv_x$ and with any velocity in the y and z directions is:

$$dN_x = N \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \exp\left(-\frac{mv_x^2}{2kT}\right) dv_x$$
$$= Ne^{-\omega^2} d\omega / \sqrt{\pi}$$

where $\omega = v_x / v_p$ and v_p is the most probable speed.

232. Use the results of Problem 231 to show that the number of gas molecules that strike unit area of the container walls in unit time with a normal velocity greater than v is $Nv_p e^{-\omega^2} / 2\sqrt{\pi}$.

233. Use the expression given in Problem 231 to show that the pressure exerted by a gas on the walls of its containing vessel is equal to NkT.

234. A metal may be thought to consist of a box containing many electrons. If these electrons behave like molecules of a gas and require an energy greater than ϕ in a direction normal to one face of the box to escape from it, show using the results of Problem 232 that the electron current emission from the box has a temperature

dependence of the form $A\sqrt{T}\,\mathrm{e}^{-\phi/kT}$ where T is the absolute temperature and A is a constant for the metal.

235. For a gas having a Boltzmann distribution of energies the fractional number having energy in the range E to $E + dE$ is given by:

$$2\pi \left(\frac{1}{\pi kT}\right)^{3/2} E^{\frac{1}{2}}\,\mathrm{e}^{-E/kT}\,dE.$$

Use this expression to determine the most probable energy and the mean energy of the system.

Calculate the temperature to which a gas would have to be raised if its mean energy was 1 electron volt.

[*Ans.* $kT/2$; $3kT/2$; 7,740°K]

CONDUCTION IN GASES

236. (*a*) Determine the minimum velocity which an electron must have to excite an argon molecule.

(*b*) An electron travelling with the same velocity as the one in (*a*) ionizes a mercury molecule by collision. If the excess energy is assumed to be shared equally between the colliding and liberated electrons, what is the final velocity of each?

(*c*) The mean free path of an electron in neon gas is 0·079 cm at room temperature and a pressure of 1 mm of mercury. Find the minimum field strength for an electron, starting at rest, to acquire the ionization energy in its mean free path.

(*d*) A mercury atom is excited to an energy of 7·93 V. It returns to the normal 'ground' state in two steps, first falling to an energy level of 6·71 V and then to zero. Calculate the wavelengths of the emitted radiations.

[*Ans.* (*a*) $2·02 \times 10^6$ m/sec; (*b*) $0·459 \times 10^6$ m/sec;
(*c*) 272 V/cm; (*d*) 10,160 Å; 1,848 Å]

237. A discharge tube, with electrodes 1 cm apart producing a uniform field, contains a gas for which

$$\alpha/p = 15e^{-350p/E},$$

where p is the pressure in mm Hg and E the field in volts/cm.

Find the maximum multiplication, the pressure at which it occurs and the average energy required per ion-pair, if the applied voltage is 100 V.

[*Ans.* 4·81; 0·29 mm Hg; 64 eV]

238. In a discharge tube containing gas at a pressure of 5 mm of mercury the following readings of current I and electrode separation x were obtained with a constant voltage gradient:

x (cm)	0·2	0·4	0·6	0·8	1·6
$I(\mu\mu A)$	1·8	3·3	6·0	11·0	200

Estimate the values of the primary ionization coefficient in the gas, the secondary emission coefficient at the cathode and the electrode spacing at which breakdown may be expected.

[*Ans.* 3/cm; 0·0033; 1·91 cm]

239. In a uniform-field discharge gap at a certain gas pressure the effective value of γ for the cathode is 0·02. Breakdown is found to occur at 400 V when the gap is 5 mm long. Find the value of the first ionization coefficient and the multiplication obtained with 200 V across a 2·5 mm gap in the same gas.

[*Ans.* 7·82 cm^{-1}; 8·06]

240. In a discharge tube with electrodes 20 cm apart the positive column of a glow discharge is about 15 cm long and a Langmuir probe of effective surface area 0·033 cm^2 is inserted at a point 12 cm from the anode. The following values were obtained for the probe potential relative to the anode V_p and indicated current I_p:

V_p (volts)	−6	−8	−10	−12	−14	−16
I_p (mA)	37·0	36·6	36·4	18·8	5·0	1·32

V_p (volts)	−18	−20	−22	−24	−26
I_p (mA)	0·30	0·02	−0·06	−0·08	−0·08

Estimate the electron temperature and concentration and find the voltage gradient in the plasma if the anode fall of potential is 5 V positive. If the random current density is six times the drift current density determine the mobility of the electrons in the plasma.

[*Ans.* 17,800° K; 3·3 × 10^{11}/c.c.; 0·5 V/cm;
6·8 × 10^6 cm/sec/V/cm]

241. A Langmuir probe of surface area 2·5 mm^2 can draw a saturated electron current of 18 mA from a glow-discharge plasma.

When the probe is 2·2 V negative to the surrounding plasma the electron current is one-quarter of its saturated value. Find the electron concentration and temperature.

[*Ans.* 2·15 × 10^{17} m^{-3}; 18,200°K]

242. A cathode-ray tube has an anode voltage of 500 V and the distance from anode to screen is 20 cm. What is the maximum allowable pressure in the tube if not more than 10% of the electrons in the beam are to be scattered in their passage from the anode to the screen?

The total collision cross-section for the gas in the tube at a pressure of 1 mm of mercury and an energy of 500 eV is 10^{-16} cm^2. Assume Loschmidt's number is 2·7 × 10^{19} molecules/c.c. at N.T.P.

[*Ans.* 1·405 × 10^{-3} mm of mercury]

243. In a pulsed discharge experiment microwave beams of wavelengths 8 mm and 3 cm respectively are directed towards the discharge tube. It is found that transmission of the beams resumes at times 5·7 and 81 μsec respectively after the end of the current pulse. Estimate the effective recombination coefficient of the plasma.

[*Ans.* $\simeq 10^{-8}$]

PHOTOELECTRICITY

244. (*a*) Show that for photoelectric emission to be possible over the whole visible region, 4,000 to 8,000 Å, the work function of the photosensitive surface must be less than 1·55 eV.

Determine the threshold wavelength in the case of a caesium surface for which the work function is 1·8 eV.

(*b*) Determine the maximum velocity of the emitted photo-electrons when a molybdenum surface, having a work function of 4·3 eV, is irradiated with the mercury line, 2,537 Å.

[*Ans.* (*a*) 6,890 Å;   (*b*) 4·56 × 10⁵ m/sec]

245. A photoelectric cell has a semi-cylindrical cathode of radius 1 cm and an anode of radius 0·05 cm situated along the axis of the cathode. It is filled with argon at a pressure of 1 mm of mercury for which gas the number of ion pairs produced per cm path of an electron is given approximately by $Ape^{-Bp/E}$, where $A = 13·6$, $B/A = 17·3$, p is the pressure in mm of mercury and E is the field strength in volts/cm. Show that, neglecting fringing effects in the electrostatic field, the naperian logarithm of the gas amplification factor is given approximately by $(V/52)e^{-35·3/V}$, where V is the voltage between the electrodes.

246. Find the sensitivity of a photomultiplier which has six stages, if the gain/stage is 5 and the cathode sensitivity is 24 μA/lumen. If the maximum safe output current is 3 mA calculate the maximum safe illumination.

[*Ans.* 375 mA/lumen; 8 millilumens]

247. The characteristics of a certain photovoltaic cell are illustrated in the diagram. The cell is placed 91·4 cm from a lamp of

180 c.p. The window area is 19·35 sq. cm. Find the reading of a micro-ammeter, having a resistance of 600 Ω, which is connected to the cell.
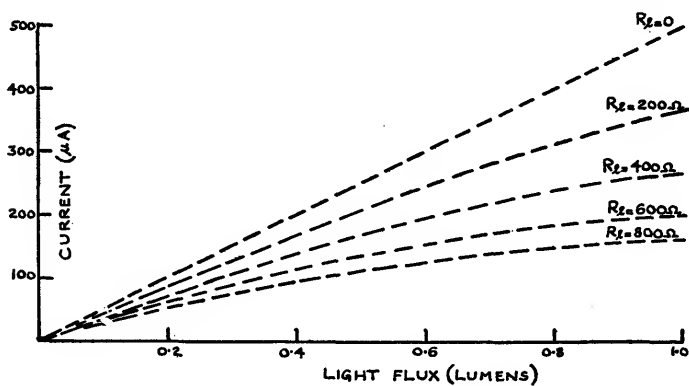
[*Ans.* 120 μA]

U.H.F. EFFECTS AND U.H.F. VALVES

248. A valve has a cathode-lead inductance of 10^{-8} H and a mutual conductance of 2 mA/V. What is the component of input conductance due to this at a frequency of 200 Mc/s if the input capacitance is $10\,\mu\mu$F?

[*Ans.* 316 micromhos]

249. A cylindrical magnetron has a cathode of radius r_c and a coaxial anode of radius r_a which is maintained at a positive potential V with respect to the cathode. A magnetic field is applied parallel to the common axis of the electrodes. The critical flux density for the electrons just to graze the anode is B_c. Assuming that the electrons produced at the cathode have zero initial velocities, prove that the value of B_c is given by

$$B_c = [\sqrt{8\ Vm/e}/r_a(1 - r_c^2/r_a^2)],$$

where m and e are the mass and charge of an electron respectively.

250. Show that maximum efficiency and maximum output voltage of a reflex klystron cannot be obtained together.

251. A schematic diagram of a klystron is shown. The effect of the superimposed buncher alternating voltage on the accelerating voltage
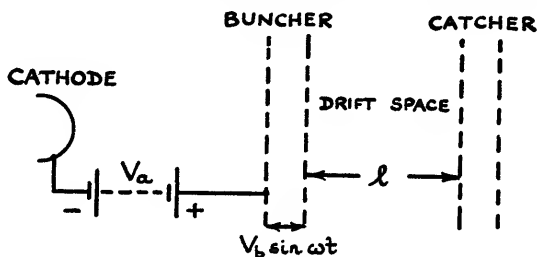
V_a is to give the electrons a speed v. Assuming that the buncher grids are very close together show that the velocity of an electron beyond

the buncher is given by $v = \sqrt{2\,eV_a/m}(1 + V_b \sin \omega t/V_a)$, and thus that the electrons are velocity modulated at the buncher frequency. m and e are the mass and charge of an electron respectively.

If the time of arrival, at the catcher, of an electron that passed through the buncher at time t_b is t_c, show that

$$\theta_c = [\theta_b + \theta_o - k \sin \theta_b] \text{ radians}$$

where $\theta_b = \omega t_b$ is the departure angle, $\theta_c = \omega t_c$ is the arrival angle, $\theta_o = [\omega l/\sqrt{2\,eV_a/m}]$ and k is the bunching parameter

$$[\omega V_b l/2V_a \sqrt{2\,eV_a/m}]$$

252. An electron beam with an average charge density ρ_o is moving with a velocity v_o. A wave of small amplitude is superimposed, so that the charge density and velocity may then be written as $\rho = [\rho_o + \rho_1 e^{-Pz+j\omega t}]$ and $v = [v_o + v_1 e^{-Pz+j\omega t}]$ respectively. Show that the possible values for the propagation constant P are $[j(\omega \pm \omega_P)/v_o]$, where ω_P is the electron plasma frequency.

Hence, explain the operation of a klystron amplifier by supposing that a pair of grids, between which there is an alternating voltage of known frequency, is placed at the point $z = 0$ in a uniform electron beam.

TRANSMISSION LINES AND NETWORKS

253. Calculate the attenuation constant, the wavelength (or phase) constant, the characteristic impedance, the velocity of propagation and the wavelength for a line having a resistance of 10·4 Ω/mile, a capacitance of 0·00835 μF/mile, an inductance of 3·67 mH/mile and a conductance of 0·8 micromho/mile at a frequency of 796 c/s.
[*Ans.* 0·00785 neper/mile; 0·0287 radian/mile; 711/— 14° 14′ Ω;

174,300 miles/sec; 219 miles]

254. A line having the constants given in the preceding problem is 300 miles long, and is terminated by its characteristic impedance. A 2-V generator with an internal resistance of 600 Ω is connected to the sending end. Determine the voltage and current at the receiving end.
[*Ans.* 0·1036/— 499° 30′ V; 0·0001458/— 485° 16′ A]

255. Repeat the calculations of Question 253 for the case where loading coils, each of resistance 7·3 Ω and inductance 246 mH, are added at intervals of 7·88 miles.
[*Ans.* 0·0036 neper/mile; 0·085 radian/mile; 2,038/— 1° 19′ Ω;

58,800 miles/sec; 74 miles]

256. Repeat the calculations of Question 254 for the case where the line length is reduced to 100 miles and the load impedance is (353·5 + *j*353·5) Ω.

[*Ans.* 0·472 V; 944 mA]

257. An alternating voltage of 10 V and of frequency 1,000 kc/s is connected to the sending end of a transmission line which can be represented by a resistance of 70 Ω with a shunt capacitance of 0·001 μF at the receiving end. An inductive load of inductance 0·002 H and resistance 100 Ω is connected across the receiving end of the line. Determine the load current. The internal resistance of the supply is 10 Ω.

[*Ans.* 0·71 mA]

258. A transmission line of length l has resistance, inductance and leakage conductance per unit length R, L and G. Its capacitance is negligible.

It is short-circuited at both ends and at time $t = 0$ a voltage distribution $V = V_0 \sin (\pi x/l)$ is set up on it.

Calculate the subsequent variation of voltage.

$$[Ans. \ V = V_0 \sin (\pi x/l) e^{-\gamma t}$$
$$\text{where } \gamma = R/L + \pi^2/LGl^2]$$

259. A transmission line, of length 5 m, is tested at a frequency of 20 Mc/s. When the far end of the line is short-circuited the impedance measured at the sending end is 4·61 Ω resistive, and when the far end is open-circuited the impedance becomes 1,390 Ω resistive. Calculate the characteristic impedance of the line, the attenuation constant in db/m, the velocity of propagation and the permittivity of the dielectric.

$$[Ans. \ 80 \ \Omega; \ 0·1; \ 2 \times 10^8 \ \text{m/sec}; \ 2·25]$$

260. A loss-free transmission line of characteristic impedance 70 Ω is terminated by an impedance $R + jX$. The standing-wave ratio (expressed so as to be a quantity greater than unity) is 2, and the position of the first voltage maximum is one-twelfth of a wavelength from the termination. Calculate R and X.

$$[Ans. \ R = 80 \ \Omega; \ X = 52 \ \Omega]$$

261. A transmission line of length l, having a propagation constant P and a characteristic impedance Z_o, is terminated in an impedance Z_r. The input impedance is given by:

$$Z_1 = Z_o \left[\frac{Z_r \cosh Pl + Z_o \sinh Pl}{Z_r \sinh Pl + Z_o \cosh Pl}\right]$$

Develop from this expression the theory of the cartesian-grid form of circle-diagram, for solving transmission-line problems, explaining clearly how 'u' circles, 'v' arcs and 'n' arcs are derived.

Explain, also, how the Smith polar form of transmission-line calculator is developed.

262. A length of transmission line, 0·30 of a wavelength long and of characteristic impedance 75 Ω, is terminated by an impedance $Z = (37·5 + j52·5) \ \Omega$. Use both the cartesian-grid form of circle

diagram and the Smith Chart to find the input impedance, assuming the line to be loss-free.

What is the voltage standing-wave ratio existing in the line?

Determine also the new input impedance for a loss in the line of 1·15 db.

$$[\textit{Ans. } (31 \cdot 5 - j41 \cdot 2) \,\Omega; \; 0 \cdot 315; \; (42 - j36) \,\Omega]$$

263. At radio frequencies the resistance per unit length R of a concentric line of copper, with air dielectric, is given by:

$$R = 41 \cdot 6\sqrt{f}[1/a + 1/b]10^{-7} \,\Omega/\text{m},$$

and the characteristic impedance $Z_o = 138 \log_{10} (b/a) \,\Omega$, where

a is the outer radius of the inner conductor in cm,

b is the inner radius of the outer conductor in cm,

and f is the frequency in c/s.

Calculate the input impedance of a quarter-wavelength, short-circuited line of this type at a frequency of 500 Mc/s, if $b = 1$ cm and $b/a = 9 \cdot 2$.

$$[\textit{Ans. } 248{,}600 \,\Omega]$$

264. Calculate the value of 'Q', for the line in Question 263.

$$[\textit{Ans. } 1{,}468]$$

265. Show that the input impedance of a short-circuited transmission line is equal to

$$[Rc(l/\lambda)/2f \cos^2 \{2\pi(l/\lambda)\} + jZ_o \tan (2\pi l/\lambda)]$$

where l/λ is the line length in wavelengths, f is the frequency, c is the velocity of light, Z_o is the characteristic impedance and R is the resistance of both conductors of the line per unit length. Hence, prove that for such a line, where l/λ is 0·2, the ratio of the selectivity of the line reactance to the selectivity of a lumped reactance is 4·28.

266. Calculate the attenuation and the characteristic impedance, at a frequency of 8,800 Mc/s, of an air-filled coaxial line having the following properties:

Diameter of inner conductor $= 0 \cdot 113$ cm

Inner diameter of outer conductor $= 0 \cdot 794$ cm

Resistivity of inner conductor $= 1 \cdot 78 \times 10^{-6}$ ohm-cm

Resistivity of outer conductor $= 6 \cdot 5 \times 10^{-6}$ ohm-cm

$$[\textit{Ans. } 0 \cdot 331 \text{ db/m}; \; 117 \,\Omega]$$

267. Repeat the calculations in Question 266 for a line entirely filled with a dielectric having a permittivity of 2·25 and a power factor of 0·0004.

[*Ans.* 0·977 db/m; 78 Ω]

268. A polythene-filled coaxial cable is terminated with a short-circuiting piston, and its input admittance is measured by the location of a point of minimum voltage, and the determination of the voltage standing-wave ratio, for a measuring line of characteristic impedance 75 Ω. The resulting admittance circle obtained by moving the piston over half a wavelength is as shown. Calculate the attenuation and apparent characteristic impedance of the cable at this frequency. Y_o is the characteristic admittance of the slotted line.
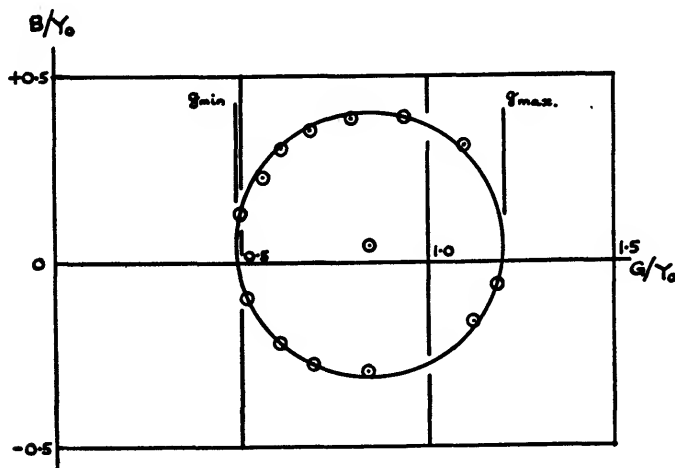
[*Ans.* 6·55 db; 97·8 Ω]

269. Assuming that the zero-susceptance points were accurately located in Question 268, calculate, by using the two-point system of Blackband and Brown,* the values of attenuation and characteristic impedance which would be obtained by this method of measurement.

[*Ans.* 6·60 db; 97·7 Ω]

* See W. T. Blackband and D. R. Brown, 'The Two-point Method of Measuring Characteristic Impedance and Attenuation of Cables at 3,000 Mc/s,' *J.I.E.E.*, **93**, Part IIIA, p. 1383, 1946.

270. It is desired to connect a transmission line of characteristic impedance 75 Ω to a load of impedance $(150 + j0)$ Ω using a quarter-wavelength transformer. Determine the characteristic impedance of the transformer for perfect matching.

[*Ans.* 106 Ω]

271. A load of impedance $(100 + j100)$ Ω is to be matched to an open-wire transmission line of characteristic impedance 500/0 Ω, using a short-circuited stub and a quarter-wave line inserted between the 500-Ω line and the load. Calculate the characteristic impedance of the quarter-wave line, and the minimum length of stub required. Assume that the stub and quarter-wave line have the same characteristic impedance, that the stub is at the load end of the quarter-wave line and that the lines are loss-free. The frequency is 100 Mc/s.

[*Ans.* 316 Ω; 1·23 m]

272. The diagram shows the arrangement of a double-stub tuner. The positions of the stubs on the transmission line are fixed, but the lengths l_1 and l_2 are adjustable. If the line and stubs have the same known characteristic impedance Z_o, and the load impedance Z_l is also known, explain how a Smith Chart can be employed to determine the shortest values of l_1 and l_2 which will give no reflected wave at the point P.
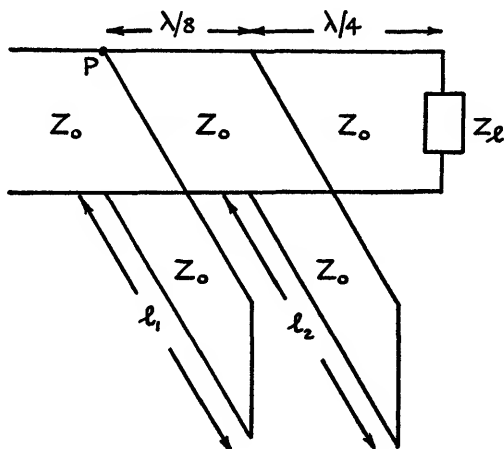
273. In a linear passive four-terminal network, the input voltage and current, V_1 and I_1, can respectively be expressed in the forms

$$V_1 = [AV_2 + BI_2] \text{ and } I_1 = [CV_2 + DI_2];$$

where V_2 and I_2 are the output voltage and current respectively. Find the values of A, B, C and D for the network shown.
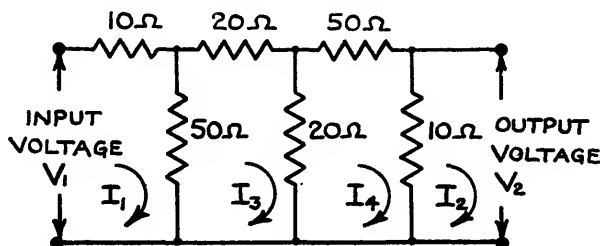
[*Ans.* $A = 20.8$, $B = 179\ \Omega$, $C = 0.68$ mho, $D = 5.9$]

274. A symmetrical four-terminal network has an image impedance of $600\ \Omega$ resistive, and an image attenuation constant of 0.5 neper. The image phase constant is zero. A resistive load of $1,000\ \Omega$ is connected between one pair of terminals, and the other two terminals are connected to a generator of e.m.f. $10\ V$ and an internal impedance of $200\ \Omega$ resistive. Determine the current through the load.

[*Ans.* 5.44 mA]

275. Find the frequency at which the output of the twin-T network shown is zero, when the input voltage is finite.
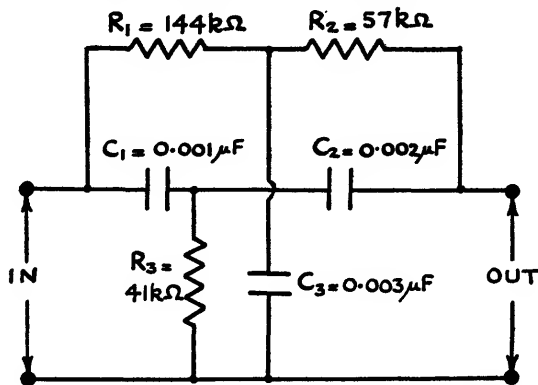
[*Ans.* $1,240$ c/s]

276. (*a*) Find the values of the resistors of the T-section attenuator which will have an iterative impedance of 600 Ω and a loss of (i) 10 db, (ii) 20 db.
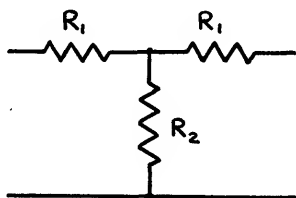
(*b*) In an H-type attenuator, matched to a transmission line of 600-Ω impedance, each of the series resistances is 240 Ω. Determine the attenuation and the value of the shunt resistance.

$$[\textit{Ans.} \ (a) \ (i) \ R_1 = 311 \cdot 8 \ \Omega, \ R_2 = 421 \cdot 6 \ \Omega;$$
$$(ii) \ R_1 = 491 \ \Omega, \ R_2 = 121 \ \Omega;$$
$$(b) \ 19 \cdot 1 \ \text{dB}; \ 135 \ \Omega]$$

277. Open- and short-circuit tests are performed on the four-terminal network illustrated. An impedance of $(250 + j100) \ \Omega$ is measured between terminals 1 and 2 with terminals 3 and 4 open-circuited. With 3 and 4 short-circuited the impedance between 1 and 2 is $(400 + j300) \ \Omega$. An impedance of $(200 + j0) \ \Omega$ is measured between 3 and 4 with 1 and 2 open. Evaluate the impedances of the equivalent T-network.
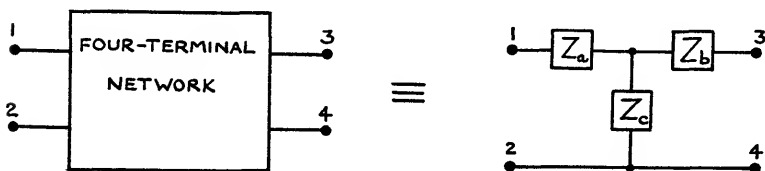
$$[\textit{Ans.} \ Z_a = (150 + j300) \ \Omega;$$
$$Z_b = (100 + j200) \ \Omega;$$
$$Z_c = (100 - j200) \ \Omega]$$

278. Determine the Z, Y, A, B, C, D and image parameters Z_{i1}, Z_{i2} and θ of the network shown.
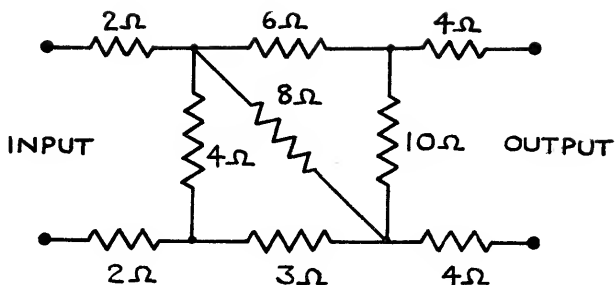
$[Ans.\ Z_{11} = 6{\cdot}704\ \Omega,\qquad Z_{12} = 1{\cdot}078\ \Omega,$
$\quad\ Z_{22} = 13{\cdot}006\ \Omega,\qquad Y_{11} = 0{\cdot}1513\ \text{mho},$
$\quad\ Y_{21} = 0{\cdot}0126\ \text{mho},\quad Y_{22} = -\,0{\cdot}0784\ \text{mho},$
$\quad\ A = 6{\cdot}222,\qquad\qquad B = 79{\cdot}37\ \Omega,$
$\quad\ C = 0{\cdot}928\ \text{mho},\qquad D = 12,$
$\quad\ Z_{i1} = 6{\cdot}66\ \Omega,\qquad\ Z_{i2} = 12{\cdot}84\ \Omega,$
$\quad\ \theta = 2{\cdot}846]$

279. Starting with the equations relating the input voltage and current with the output voltage and current of a linear passive four-terminal network, show that the transfer matrices of the simple circuits (a) and (b) illustrated are given by

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\ \text{and}\ \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

respectively.

(a)   (b)
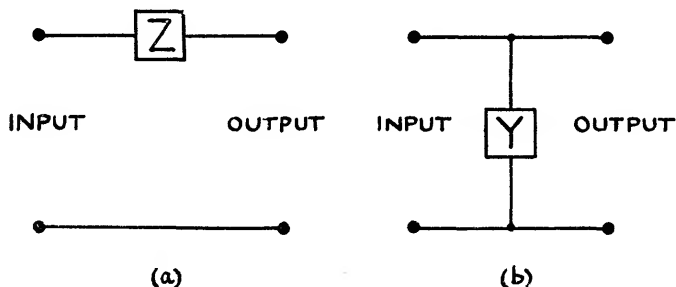
Hence, show that the transfer matrix of a network which has general parameters A, B, C and D, and a load Z_l in cascade is

$$\begin{bmatrix} A + B/Z_l & B \\ C + D/Z_l & D \end{bmatrix}$$

Use these results to find the voltage gain and the input resistance of a common-base transistor amplifier stage in terms of the transistor parameters r_{11}, r_{12}, r_{21} and r_{22} and the load resistance R_l.

$$[Ans.\ \frac{r_{21}R_l}{r_{11}R_l + r_{11}r_{22} - r_{12}r_{21}};$$

$$r_{11} - r_{12}r_{21}/(r_{22} + R_l)]$$

280. Show that the transfer matrices for the arrangements (a), (b), (c) and (d) illustrated are respectively,

$$\begin{bmatrix} (1+ZY) & Z \\ Y & 1 \end{bmatrix},\ \begin{bmatrix} 1 & Z \\ Y & (1+ZY) \end{bmatrix},$$

$$\begin{bmatrix} (1 + Z_1/Z_{12}) & (Z_1 + Z_2 + Z_1Z_2/Z_{12}) \\ 1/Z_{12} & (1 + Z_2/Z_{12}) \end{bmatrix},$$

$$\begin{bmatrix} (1 + Z_{12}/Z_2) & Z_{12} \\ \{1/Z_1 + (1 + Z_{12}/Z_1)(1/Z_2)\} & (1 + Z_{12}/Z_1) \end{bmatrix}.$$

(a)

(b)

(c)

(d)
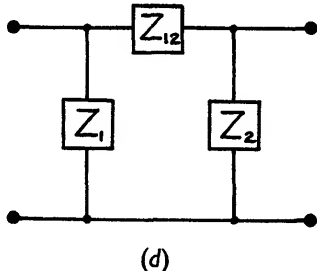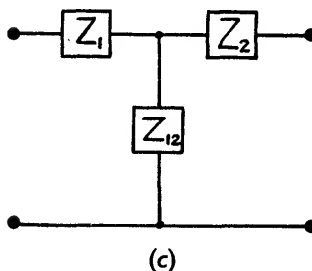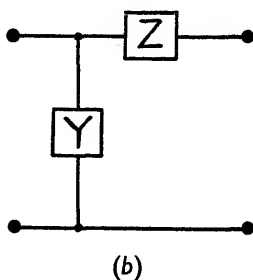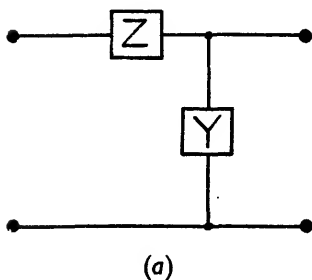
281. Use the results of Question 279 to find the ratios V_2/V_1 for the circuits shown.
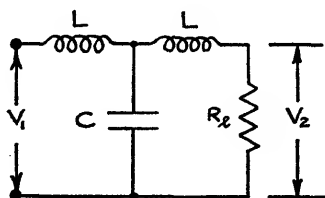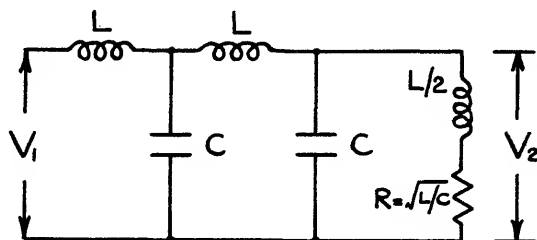
[*Ans.*
$$\frac{1}{1 - 3\omega^2 LC + \omega^4 L^2 C^2 + j\omega L(2 - \omega^2 LC)/(\sqrt{L/C} + j\omega L/2)};$$

$$\frac{R_l}{(1 - \omega^2 LC)R_l + j(2\omega L - \omega^3 L^2 C)},$$

where $\omega = 2\pi \times$ supply frequency.]

282(*a*) Show that the transfer matrix for a combination of two four-terminal networks in cascade is equal to the product of the transfer matrices of the individual networks.

(*b*) Two networks have general A, B, C, D parameters as follows:

	Network 1	Network 2
A	1·50	1·66
B	11 Ω	4 Ω
C	0·25 mho	1 mho
D	2·5	3·0

If the networks are connected in cascade in the order 1, 2, show that the transfer matrix of the combination is:

$$\begin{bmatrix} 13\cdot5 & 39\cdot0 \\ 2\cdot92 & 8\cdot5 \end{bmatrix}.$$

If the two networks are connected with their inputs and outputs in parallel, prove that the resulting admittance matrix is:

$$\begin{bmatrix} 43/44 & -15/44 \\ 15/44 & -73/132 \end{bmatrix}.$$

283. Show that the $[Z]$ and $[A]$ matrices of the symmetrical lattice network illustrated are:

$$\begin{bmatrix} (Z_1 + Z_2)/2 & (Z_2 - Z_1)/2 \\ (Z_2 - Z_1)/2 & (Z_1 + Z_2)/2 \end{bmatrix}$$

and

$$\begin{bmatrix} (Z_1 + Z_2)/(Z_2 - Z_1) & 2Z_1Z_2/(Z_2 - Z_1) \\ 2/(Z_2 - Z_1) & (Z_1 + Z_2)/(Z_2 - Z_1) \end{bmatrix}$$
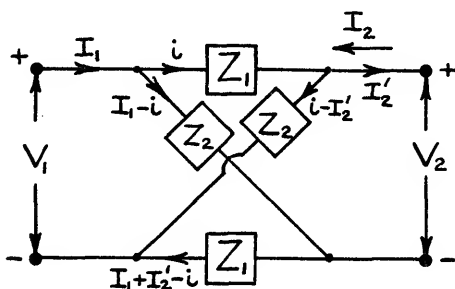
284. The admittance matrices of the networks (a), (b), (c) and (d) illustrated are respectively:

$$\begin{bmatrix} 0 & 0 \\ -g_m & -1/r_a \end{bmatrix}, \quad \begin{bmatrix} Y_g & 0 \\ -g_m & -1/r_a \end{bmatrix},$$

$$\begin{bmatrix} Y_g + j\omega(C_{ga} + C_{gc}) & -j\omega C_{ga} \\ (j\omega C_{ga} - g_m) & -\{1/r_a + j\omega(C_{ga} + C_{ac})\} \end{bmatrix}$$

and

$$\begin{bmatrix} Y_g & 0 \\ -g_m/\{1 + Z_c(g_m + 1/r_a)\} & -(1/r_a)/\{1 + Z_c(g_m + 1/r_a)\} \end{bmatrix}.$$
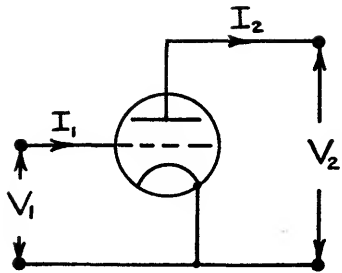
(a)
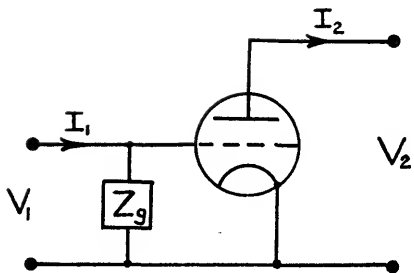
(b)
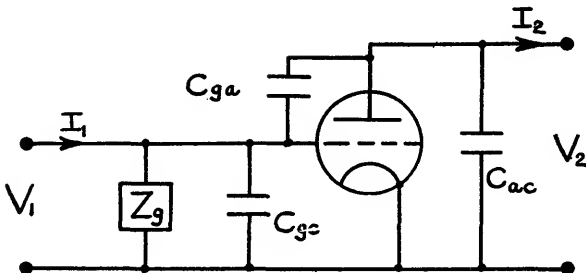
(c)
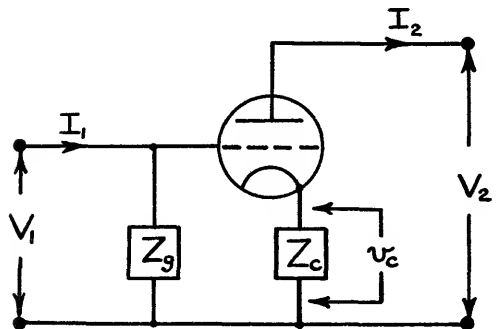
(d)

Obtain these matrices from first principles. g_m and r_a are the mutual conductance and the anode slope resistance of the valve in each case, $Y_g = 1/Z_g$ and $\omega = 2\pi \times$ the frequency.

285. Derive the transfer matrices of the various elements of the circuit illustrated and then show that the transfer matrix for the whole arrangement is:

$$[A] = \begin{bmatrix} n(1 + Z_1 Y) & nZ_2(1 + Z_1 Y) + Z_1/n \\ nY & nZ_2 Y + 1/n \end{bmatrix}$$
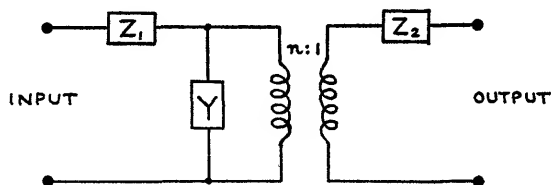
WAVEGUIDES

286. State Maxwell's equations, and use them to derive an expression giving the critical wavelength for the H_{mn} mode in a rectangular waveguide, of internal dimensions a and b cm.

[*Ans.* $2/\{(m/a)^2 + (n/b)^2\}^{\frac{1}{2}}$ cm]

287. Use Maxwell's equations to derive expressions for the E and H components of the two basic types of wave which can be propagated in a rectangular waveguide. Sketch the field patterns for the normal H_{o1} mode.

288. Calculate the critical and guide wavelengths in an air-filled rectangular waveguide, with internal dimensions 7·62 cm × 2·54 cm, for the normal H_{o1} mode at a frequency of 3,000 Mc/s.

[*Ans.* 15·2 cm; 13·3 cm]

289. Derive expressions for the field components of both the E and H waves in a circular waveguide, and show that the critical wavelengths for the E_o and H_o modes are 2·61a and 1·64a respectively, where a is the guide radius.

290. A cylindrical attenuator, operated in the E_o mode at 1,000 Mc/s, is required to provide 100 db attenuation over a 10 cm length. Calculate the radius of the cylinder required.

[*Ans.* 2·06 cm]

291. Calculate the attenuation per metre in an air-filled rectangular copper waveguide, with internal dimensions 2·54 cm × 1·27 cm for the H_{o1} mode at a free-space wavelength of 3·1 cm.

Repeat the calculation for a free-space wavelength of 3·2 cm.

[*Ans.* 0·0801 db/m; 0·0814 db/m]

292. Repeat the working of Question 291 for a copper guide with internal dimensions 7·62 cm × 2·54 cm, at a frequency of 3,000 Mc/s.

[*Ans.* 0·022 db/m]

293. A rectangular copper waveguide, with internal dimensions 4·8 cm × 1·6 cm, carrying the H_{o1} wave is filled with a dielectric which has a dielectric constant of 2·55, and a loss angle δ given by $\tan \delta = 0{\cdot}0006$. Determine the total attenuation per metre, due to the losses in the wall metal and the dielectric, at a frequency of 3,000 Mc/s.

[*Ans.* 0·399 db/m]

294. Calculate the attenuation in an air-filled rectangular waveguide, having internal dimensions 2·54 cm × 1·27 cm for the H_{o1} mode at a free-space wavelength of 3·2 cm, (*a*) assuming perfectly smooth walls, and (*b*) with surface-roughness coefficients* $K_{T1} = 1{\cdot}036$, $K_{T2} = 1{\cdot}067$ and $K_p = 1{\cdot}049$. The resistivity of the waveguide wall metal is $6{\cdot}266 \times 10^{-6}$ ohm-cm.

[*Ans.* (*a*) 0·157 db/m; (*b*) 0·165 db/m]

295. The attenuation in an air-filled rectangular nickel waveguide carrying the H_{o1} mode and having internal dimensions 2·54 cm × 1·27 cm, at a frequency of 9,645 Mc/s, is found to be 0·378 db/m. The surface-roughness coefficients* are $K_{T1} = 1{\cdot}119$, $K_{T2} = 1{\cdot}109$ and $K_p = 1{\cdot}090$. Determine the apparent permeability of the nickel at this frequency.

[*Ans.* 3·35]

296. The following information was obtained from standing-wave measurements on a short-circuited, low-loss, waveguide component:

Position of first minimum   = 3·749 cm.

Position of second minimum = 5·731 cm.

Distance between two points of equal field strength, on either side of a minimum, at which the value of the field strength is K times the minimum, is $w = 0{\cdot}088$ cm when $K^2 = 2{\cdot}12$.

Find the guide wavelength, the voltage standing-wave ratio and the loss in the component.

[*Ans.* 3·964 cm; 15·25; 0·581 db]

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* For definitions of these coefficients see: J. Allison and F. A. Benson, 'Surface Roughness and Attenuation of Precision-drawn, Chemically-polished, Electropolished, Electroplated and Electroformed Waveguides,' *Proc. I.E.E.*, **102**, Part B, p. 251, 1955.

FILTERS

297. A constant-k, low-pass filter is designed to cut off at a frequency of 1,000 c/s, and the resistance of the load circuit is 50 Ω. Calculate the values of the components required, and the attenuation constant per section at a frequency of 1,500 c/s.

[*Ans.* $L = 15 \cdot 92$ mH; $C = 6 \cdot 37 \, \mu$F; $1 \cdot 928$]

298. (*a*) In a constant-k, band-pass filter the ratio of the capacitances in the shunt and series arms is 100:1, and the resonant frequency of both arms is 1,000 c/s. Calculate the bandwidth of the filter.

(*b*) A constant-k band-pass filter terminated with a 600-Ω resistor has lower and upper cut-off frequencies of 120 kc/s and 123 kc/s, respectively. Calculate, for a T-section, the values of each series inductance, each series capacitance, the shunt inductance and the shunt capacitance. [*Ans.* (*a*) 200 c/s;

(*b*) $31 \cdot 8$ mH; $53 \cdot 9 \, \mu\mu$F; $9 \cdot 7 \, \mu$H; $0 \cdot 177 \, \mu$F]

299. A constant-k, high-pass filter is required for a cut-off frequency of 2,500 c/s. The resistance of the load circuit is 600 Ω. Determine the values of the components required.

[*Ans.* $L = 19 \cdot 1$ mH; $C = 0 \cdot 053 \, \mu$F]

300. A wave filter can be built up of either T-sections of the type illustrated in diagram (*a*) or of π-sections as shown in diagram (*b*). Determine the iterative impedances of the two sections.

(a) (b)
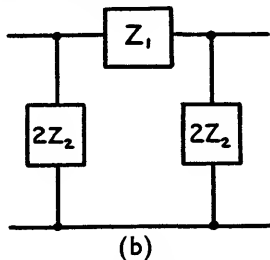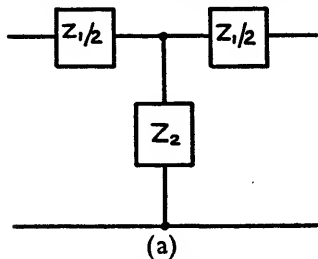
[*Ans.* $Z_{oT} = \sqrt{Z_1 Z_2 (1 + Z_1/4Z_2)}$; $Z_{o\pi} = 2Z_2 \sqrt{Z_1/(Z_1 + 4Z_2)}$]

301. Draw the series-derived T-section and the shunt-derived π-section corresponding to the prototype sections illustrated in the previous problem. Show clearly how the impedances in the series and shunt arms of the derived sections are determined.

302. Prove that, for a derived low-pass filter, a T-section of which is illustrated, the ratio of the frequency for infinite attenuation to the cut-off frequency is $\sqrt{1 + L_{1m}/4L_{2m}}$.
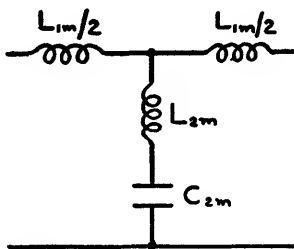
303. Show that the image impedances of the series-derived and shunt-derived half sections of a filter are respectively equal to the mid-series and mid-shunt iterative impedances of the corresponding full sections.

304. Design a constant-k, low-pass filter to have a cut-off frequency of 796 c/s and a load impedance of $600\,\Omega$ using (a) a T-section, or (b) a π-section.

> [*Ans.* (a) Series inductances, 120 mH each,
> Shunt capacitance, $0.666\,\mu$F;
> (b) Series inductance, 240 mH,
> Shunt capacitances, $0.333\,\mu$F each]

305. Design an m-derived, low-pass filter to have the same characteristics as that of the filter in the preceding problem, using (a) a T-section, or (b) a π-section. Take m as 0·6.

> [*Ans.* (a) Inductance of each series arm, 72 mH,
> Inductance in shunt arm, 64 mH,
> Capacitance in shunt arm, $0.4\,\mu$F;
> (b) Inductance in series arm, 144 mH,
> Capacitance in series arm, $0.178\,\mu$F,
> Capacitance of each shunt arm, $0.2\,\mu$F]

AERIALS

306. Evaluate the radiation resistance of a single-turn circular loop with a circumference of a quarter of a wavelength.

[*Ans.* 0·77 Ω]

307. Determine the radiation resistance of a dipole aerial 1/12 wavelength long.

Find also the directivity of a short dipole.

[*Ans.* 5·5 Ω; 1·5]

308. An aerial having a directivity of 90 is operating at a wavelength of 2 m. Calculate the maximum effective aperture of the aerial.

[*Ans.* 28·6 m²]

309. If 100 kW of energy are radiated from an aerial of 100 m effective height, at a frequency of 60 kc/s, what is the strength of the electric field at a distance of 100 km? Assume that no absorption effects are present.

[*Ans.* 0·03 V/m]

310. A certain aerial has an effective height of 100 m and the current at the base is 450 A r.m.s. at a frequency of 40 kc/s. Calculate the power radiated. The total resistance of the aerial circuit being 1·12 Ω, determine the efficiency of the aerial.

[*Ans.* 56·9 kW; 25·1 %]

311. An aerial array consists of 10 vertical aerials in a straight line spaced half a wavelength apart and equally energized in phase. Deduce the angular width of the forward beam in the horizontal plane.

[*Ans.* 23° 4′]

312. Calculate the voltage induced by a plane wave of field strength 0·01 V/m and wavelength 300 m in a frame aerial 1 m square having 12 turns, the plane of the frame being in the plane of propagation of the wave.

[*Ans.* 25·14 × 10⁻⁴ V]

MEASUREMENTS

313. A Schering bridge is used for measuring the power loss in dielectrics. The specimens are in the form of discs 0·3 cm thick and have a dielectric constant of 2·3. The area of each electrode is 314 cm² and the loss angle is known to be 9′ for a frequency of 50 c/s. The fixed resistor of the network has a value of 1,000 Ω and the fixed capacitance is 50 $\mu\mu$F. Determine the values of the variable resistor and capacitor required.

[*Ans.* 4,260 Ω; 0·00196 μF]

314. The diagram shows Anderson's bridge for measuring the inductance L and resistance R of an unknown impedance. Find the values of L and R if balance is obtained when $Q = S = 1,000\,\Omega$, $P = 500\,\Omega$, $r = 200\,\Omega$ and $C = 2\,\mu$F.
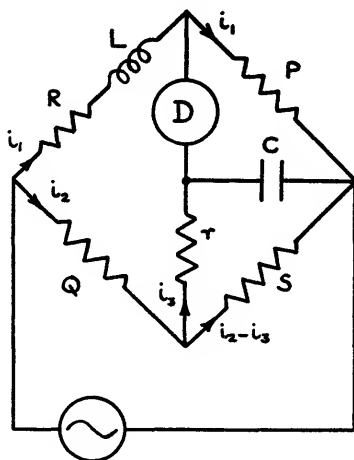
[*Ans.* 1·4 H; 500 Ω]

315. The arms AB, AD of a bridge are of inductances L_1, L_2 and of resistances R_1, R_2 respectively; the arms DC, BC contain capacitors

of capacitances C_1, C_2 and are of resistances R_3, R_4 respectively. BD contains the detector and AC a source of alternating voltage. Show that the bridge will not be balanced for all frequencies unless $L_1R_3 = L_2R_4$, $C_1R_2 = C_2R_1$, and either $R_1R_3 = R_2R_4$ or $L_1 = C_1R_2R_4$.

316. The series-resistance bridge network is used for the comparison of capacitances. A capacitor of capacitance C_1 and equivalent series resistance ρ_1 is compared with a standard air capacitor of capacitance 0·023 μF and zero equivalent resistance. When a balance is obtained the resistor in series with C_1 has a value of 11·4 Ω, and that in series with the standard capacitor has a value of 10 Ω. The non-inductive resistors have values of 1,000 Ω and 1,250 Ω, one end of the latter being connected to one side of C_1. Calculate ρ_1 and C_1.

[*Ans.* 1·1 Ω; 0·0184 μF]

317. The diagram gives the connections of Hay's bridge for the measurement of large self-inductances. L is the inductance to be measured, R_1 its resistance, C is a variable standard capacitor and R_2, R_3 and R_4 are non-inductive resistors. Balance may be obtained by variation of R_2, R_4 and C. Show that, at balance,

$$L = [R_2R_3C/(1 + \omega^2R_4^2C^2)]$$

and $$R_1 = [R_2R_3R_4\omega^2C^2/(1 + \omega^2R_4^2C^2)],$$

where $\omega = 2\pi \times$ supply frequency.
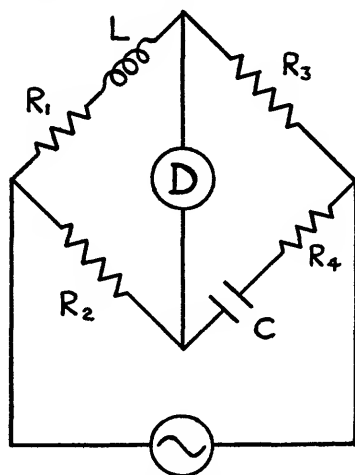
Draw the vector diagram for the network.

318. The connections of Heaviside's bridge, for the measurement of self-inductance, are shown in the diagram. The primary of a mutual inductometer is in the supply circuit, and the secondary, of self-inductance L_2 and resistance R_2, forms one arm of the bridge. The coil under test, of self-inductance L_1 and resistance R_1, is placed in another arm and non-inductive resistors R_3 and R_4 form the remaining arms. Balance may be obtained by variation of the mutual inductance and the resistors R_3 and R_4. Determine the conditions for balance.
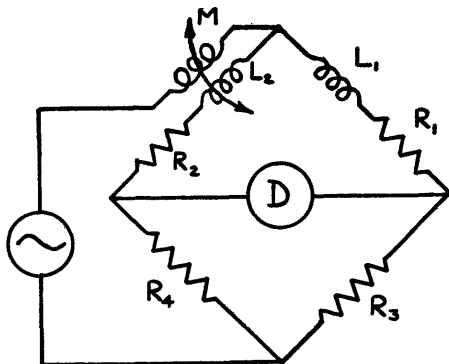
$$[\textit{Ans. } R_2 R_3 = R_1 R_4 \text{ and}$$
$$R_2(L_2 + M) = R_4(L_1 - M)]$$

319. One method of measuring a small capacitance C_x is shown in the diagram. C_1 and C_2 are equal high-quality variable air capacitors. C_3 is a fixed high-quality capacitance of much smaller value than the maximum value of C_2 (about $1/10$ of C_2). The two following balances are obtained:

(1) With switch S open, and with C_2 at its maximum value, C_1 is adjusted for balance.

(2) With switch S closed, and C_1 left unaltered, C_2 is adjusted to C_2' to give a new balance.
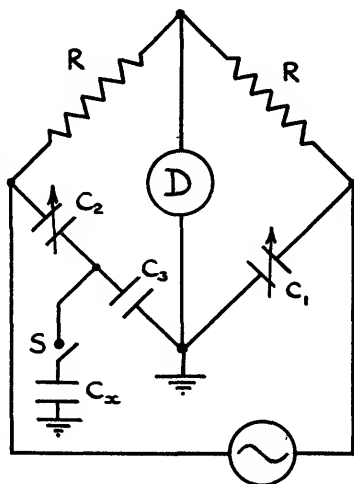
Prove that

$$C_x = C_3{}^2(C_2 - C_2')/(C_2'C_2 + C_2'C_3 - C_2C_3).$$

If $R = 1,000\,\Omega$, C_1 and C_2 are $1,000\,\mu\mu\text{F}$ and $C_3 = 50\,\mu\mu\text{F}$, and assuming the variable capacitors are readable to $\pm\,5\,\mu\mu\text{F}$, with what accuracy could a capacitance of $1\,\mu\mu\text{F}$ be measured?

[*Ans.* $\pm\,3\cdot5\%$]

320. The circuit illustrated is used to determine the amplification factor of a triode. A small alternating voltage is applied from an oscillator and resistors R_1 and R_2 are adjusted until balance is obtained, i.e. until no sound is heard in the earphones. In a certain experiment, when the anode current of the valve was 10 mA and the grid bias was $-3\cdot4$ V, the following values of R_1 and R_2 were recorded at balance:

R_1 (ohms)	49	99	149	249	400	500
R_2 (ohms)	1,000	2,000	3,000	5,000	8,000	10,000

Calculate μ from these test results.
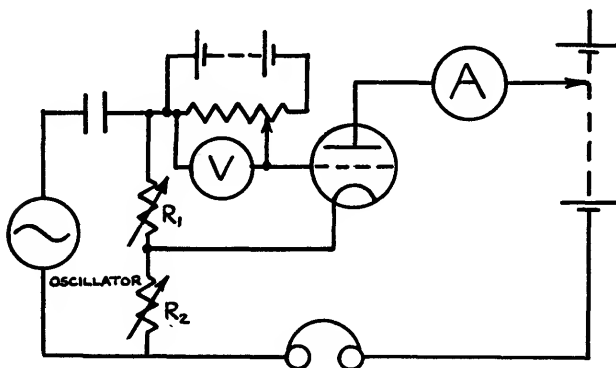
[*Ans.* 20]

321. A triode is incorporated in a bridge network as shown. A small alternating voltage is applied from an oscillator, and resistor R_2 is adjusted for balance. In a particular test $R_1 = R_3$ and, with the anode current of the valve set to 5·4 mÁ, R_2 is 8,000 Ω at balance. What is the value of the anode resistance of the triode?
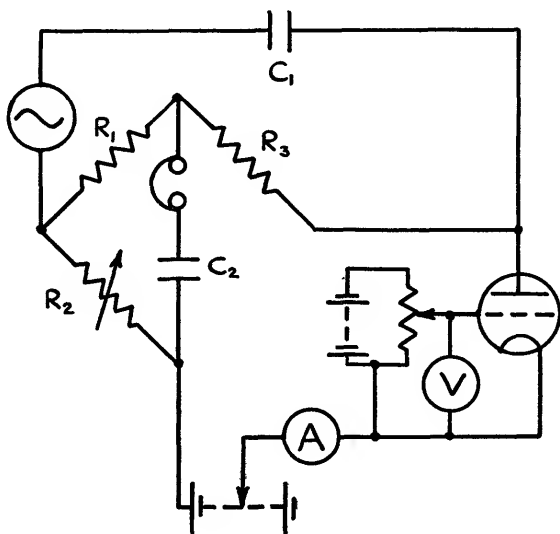
[*Ans.* 8,000 Ω]

322. Show that if a resistor R_3 is connected between the cathode and anode of the valve in the circuit given in Question 320 the mutual conductance of the valve can be measured.

323. The connections for an indirect method of measuring the conversion conductance (g_c) of a hexode valve are shown in the diagram. The change-over switch S allows the phase relationship between the signal and oscillator voltages to be reversed. Assuming that the anode current I_a can be represented by the expression $(a_o + a_1 V_{g1})(b_o + b_1 V_{g3})$, where V_{g1} and V_{g3} are the voltages on grids 1 and 3 respectively, prove that g_c is given by $[(I_{a1} - I_{a2})/2V_s]$. V_s is the maximum value of the signal voltage, I_{a1} is the mean anode current for the in-phase condition of the signal and oscillator voltages and I_{a2} is the mean current for the out-of-phase condition.
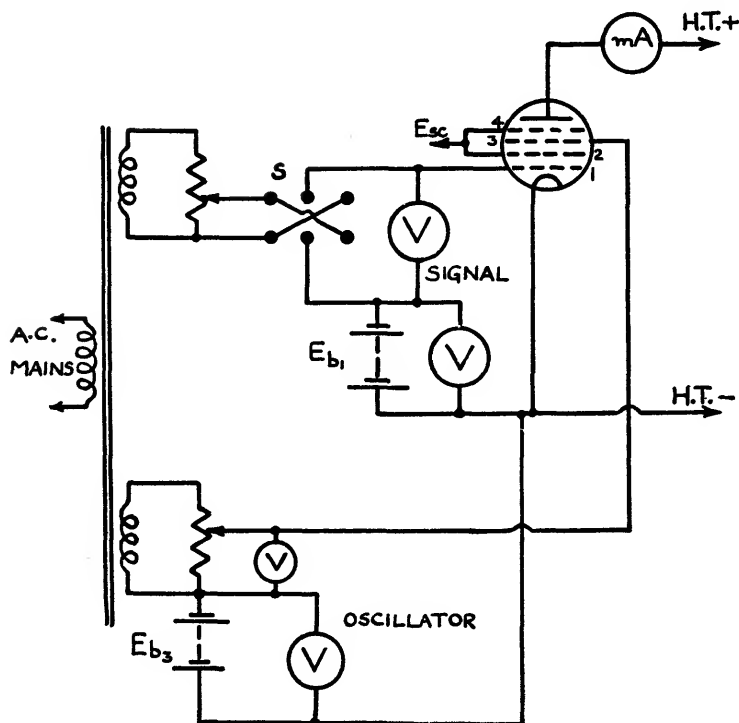
324. The voltage across the horizontal deflector plates of a cathode-ray oscillograph is $V_1 \sin(\omega t + \theta_1)$ and that across the vertical plates is $V_2 \sin(\omega t + \theta_2)$. Prove that the trace on the screen is an ellipse, determine its equation and interpret its meaning.

325. The phase-angle between two sinusoidal voltages can be measured by applying one voltage (maximum value V_1) to the vertical deflector plates of a cathode-ray oscillograph and the other (maximum value V_2) to the horizontal plates. These voltages will produce deflections on the screen of the tube $Y = k_1 V_1$ and $X = k_2 V_2$ where k_1 and k_2 are constants which depend on the sensitivity of the tube. If voltage V_2 leads voltage V_1 by an angle α, an ellipse will result which is bounded by a rectangle with sides $2X$ and $2Y$, as illustrated. Show that the phase-angle can be obtained from the

ellipse in two ways; and derive the expressions $\sin \alpha = BC/AD$ and $\sin \alpha = (2a)(2b)/(2X)(2Y)$.
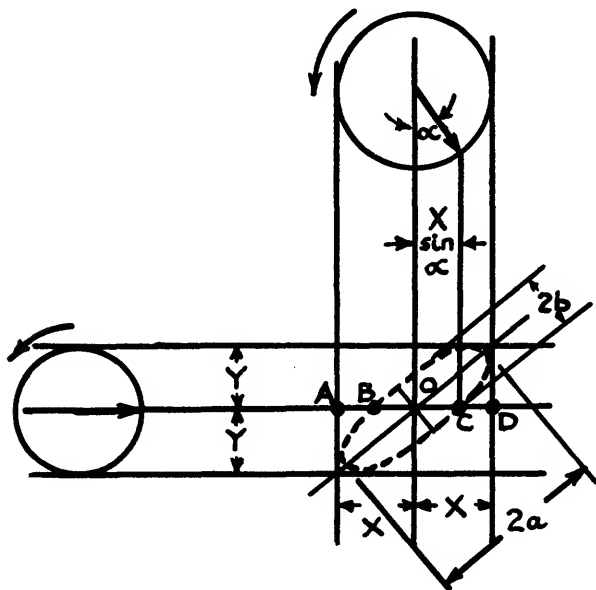
326. In determining phase-angle by the two methods given in the previous problem, an error of ± 0.5 mm may be introduced at either end of each dimension measured. Plot curves showing the resultant phase-angle error, as α varies, for values of AD equal to 40 mm, 70 mm, and 100 mm, suitable for tube screen diameters of about 6 cm, 11 cm, and 21 cm respectively.

327. When harmonics are present in either of the voltage waves of Questions 325 and 326, show that additional errors may be introduced. For both methods plot curves of the error against phase angle when either (a) a 5% positive third harmonic is present in the waveform of V_1 or (b) a 5% positive third harmonic is present in the waveform of V_2.

328. The diagram shows the circuit of a simple electronic phase-meter. Draw the equivalent circuit of the arrangement and explain how the phase angle can be determined.
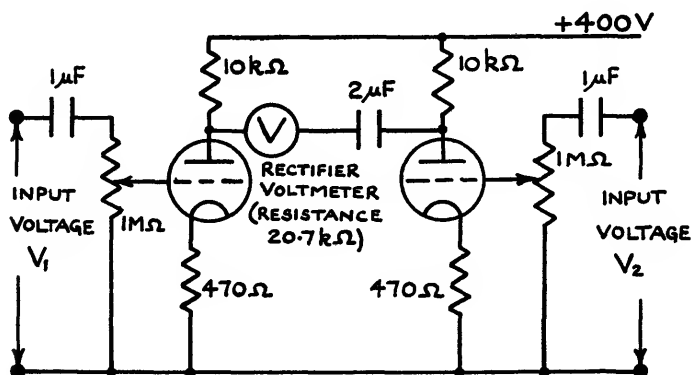
329. Two sinusoidal voltages of equal frequency are simultaneously applied to the two pairs of deflector plates in a cathode-ray tube. The co-ordinates (x, y) of the fluorescent spot may be expressed as $x = \sin (\theta + \phi)$ and $y = \sin \theta$. Plot the figures traced on the screen of the tube for the cases $\phi = 0$, $\phi = 30°$, $\phi = 60°$ and $\phi = 90°$ respectively.

330. Two sinusoidal voltages of unequal frequency are simultaneously applied to the two pairs of deflector plates in a cathode-ray tube. The co-ordinates (x, y) of the fluorescent spot may be expressed as $x = \sin (n\theta + \phi)$ and $y = \sin \theta$. Plot the figures traced on the screen of the tube for the cases $\phi = 0, \phi = 30°, \phi = 60°$ and $\phi = 90°$, respectively, (a) when $n = 2$, and (b) when $n = 3$.

331. Two sinusoidal voltages of unequal frequency are simultaneously applied to the two pairs of deflector plates in a cathode-ray tube. The co-ordinates (x, y) of the fluorescent spot may be expressed as $x = \sin (3\theta + \phi)$ and $y = \sin 2\theta$. Plot the figures traced on the screen of the tube for the cases $\phi = 0$, $\phi = 30°$, $\phi = 60°$, $\phi = 90°$, $\phi = 120°$ and $\phi = 180°$.

332. Two sinusoidal voltages of unequal frequency are simultaneously applied to the two pairs of deflector plates in a cathode-ray tube. The figure traced out on the screen is illustrated at (a). What is the ratio of the frequencies of the two applied voltages?

If the figure traced on the screen is that shown at (b), what is then the frequency ratio?

(a)

(b)
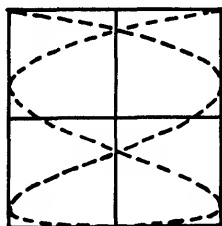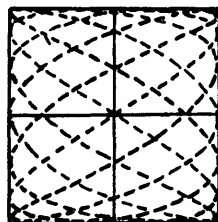
[*Ans.* (a) 5:2; (b) 7:4]

MISCELLANEOUS TOPICS

333. What is the skin depth of current penetration in copper at a frequency of 300 Mc/s?

Repeat the calculation for a frequency of 10,000 Mc/s.

[*Ans.* 3.79×10^{-4} cm; 6.56×10^{-5} cm]

334. What is the skin depth of current penetration in nickel at a frequency of 10,000 Mc/s, if the effective permeability of the metal is 3 at this frequency?

[*Ans.* 8.9×10^{-5} cm]

335. Calculate the inductance of a straight piece of wire 2·54 cm long which has a diameter of 0·254 cm. What is the reactance of this inductance at a frequency of 500 Mc/s?

[*Ans.* 0·014 μH; 44 Ω]

336. The reflected portion of a plane wave, starting in air and incident normally on a space filled with a loss-free dielectric of relative permittivity 4, is to be eliminated by placing a quarter-wavelength plate between the air and the dielectric. Calculate the required relative permittivity of the material of the plate.

[*Ans.* 2]

337. Calculate the bandwidth required for a 405-line television system with 25 complete pictures/sec and an aspect ratio of 5/4, when double side-band transmission is employed. Assume that all the lines are actually in use.

[*Ans.* 5·13 Mc/s]

338. In the double-diode clipping circuit illustrated $e = 200 \sin \omega t$ volts, $E = 2$ V, and the frequency is 100 kc/s. Find the time taken for the output voltage to rise from $-E$ to $+E$.
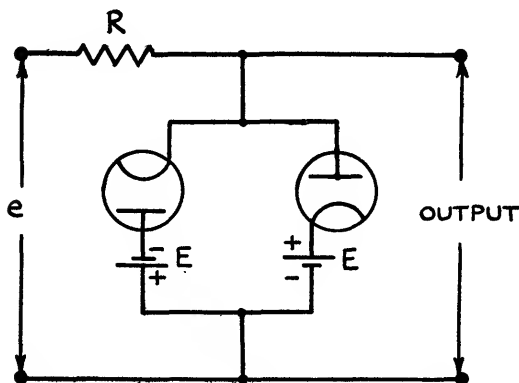
[*Ans.* 0·032 μs]

339. Three conductors 1, 2 and 3 are such that 3 encloses 1 and 2. The coefficients of capacitance are q_{11}, q_{22} and q_{33} respectively, and the coefficients of induction are q_{12}, q_{13} and q_{23}. Determine the capacitance of conductor 3 alone.

[*Ans.* $q_{33} - (q_{11} + 2q_{12} + q_{22})$]

340. Three similar insulated spheres are fixed at the corners of an equilateral triangle. The radii of the conductors are small compared with the sides of the triangle. If the spheres are touched in turn by another small charged sphere prove that the charges they receive are in geometrical progression.

341. A conductor A is enclosed in a second conductor B. Show that A is thereby screened from the effects of a third conductor C external to B.

342. A straight wire of length L is charged with electricity of amount q per unit length. This is placed near an earthed conducting sphere of radius r. The centre of the sphere is at a perpendicular distance s from the wire. The ends of the wire are equidistant from the centre of the sphere. Find the charge on the sphere. Assume that the distribution of charge on the wire is unaffected by induction.

[*Ans.* $- 2qr \sinh^{-1} (L/2s)$]

343. A point charge e is held at a distance f from the centre of an insulated spherical conductor of radius a which carries a charge Q. Prove that the surface density at the point of the sphere most remote from the charge e will be zero if $Q = [- a^2 e(3f + a)/f(f + a)^2]$.

344. A point charge e is placed at a distance f from the centre of an insulated uncharged sphere of radius a. Evaluate the total charge on the smaller part of the sphere cut off by the polar plane of the point.

$$[Ans. \; -e\{1 + a^2/f^2 - \sqrt{1 - a^2/f^2}\}/2]$$

345. The outer of two concentric spherical conductors of radii a and b ($b > a$) is earthed and the inner is charged to potential V. A point charge Q is then placed in the space between the spheres at a distance s from the centre. Prove that the charges on the spheres are in the ratio $[Q/s - Q/b - V]$ to $[Q/a - Q/s + V]$.

346. Laplace's equation can be written in cylindrical coordinates as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Given that V is independent of z and varies with θ as $\cos 4\theta$ determine the dependence of V on r, given that $V \to 0$ as $r \to \infty$.

If the radial electric field strength at a radius r_0 is E_0, what will its value be at a radius $2r_0$ in the same θ-direction?

$$[Ans. \; r^{-4}; \; E_0/32]$$

347. An infinite parallel-plate capacitor is filled with two infinite layers of dielectric each of the same thickness. One of the layers of dielectric has a relative permittivity K and no loss, the other has a relative permittivity also of K and conductivity σ. Show that the composite dielectric appears to have an apparent complex dielectric constant $2K(K - j\sigma/\omega\varepsilon_0)/(2K - j\sigma/\omega\varepsilon_0)$ where ω is the angular frequency and ε_0 is the permittivity of free space.

348. An artificial dielectric consists of a cubic array of metal spheres 2 mm in diameter. The spacing between centres of adjacent spheres is 4 mm. Calculate the dielectric constant using the Clausius–Mossotti formula.   [*Ans.* 1·21]

349. The dielectric constant of polythene is 2·25. Expanded polythene has a density of only 5% of that of polythene. Find its dielectric constant using the Clausius–Mossotti formula. Neglect the polarizability of the gas in the voids.

$$[Ans. \; 1·058]$$

Section 2

SOLUTIONS

SOLUTIONS

1. Resonant frequency $f_r = 1/2\pi\sqrt{LC} = \underline{1{,}126 \text{ kc/s.}}$

$$Q = \omega_r L/R = 2\pi f_r L/R = \underline{177.}$$

Impedance $Z = R + j\left(\omega L - \dfrac{1}{\omega C}\right) = R\left[1 + j\left(\dfrac{\omega L}{R} - \dfrac{1}{\omega C R}\right)\right].$

$\therefore$ the angle ϕ and magnitude $|Z|$ of Z are given by:

$$\tan\phi = \left[\frac{\omega L}{R} - \frac{1}{\omega C R}\right] \text{ and } |Z| = R\sqrt{1 + \tan^2\phi},$$

i.e.
$$\tan\phi = Q\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right).$$

$\therefore$
$$\left(\frac{\omega}{\omega_r}\right)^2 - \frac{\tan\phi}{Q}\left(\frac{\omega}{\omega_r}\right) - 1 = 0$$

and $\dfrac{\omega}{\omega_r} = \dfrac{\tan\phi}{2Q} + \sqrt{1 + \left(\dfrac{\tan\phi}{2Q}\right)^2}$; since at resonance $\omega = \omega_r$ and $\tan\phi = 0$, and only the positive sign has meaning. At the upper half-power frequency $\omega = \omega_1$, $\tan\phi = 1$.

$\therefore$
$$\frac{\omega_1}{\omega_r} = \frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}.$$

At the lower half-power frequency $\omega = \omega_2$, $\tan\phi = -1$.

$\therefore$
$$\frac{\omega_2}{\omega_r} = -\frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}.$$

In this case, $Q = 177$, so $f_1 = \dfrac{\omega_1}{2\pi} = \underline{1{,}129 \text{ kc/s}}.$

and
$$f_2 = \frac{\omega_2}{2\pi} = \underline{1{,}122 \text{ kc/s.}}$$

2. $I_{max} = 344 \text{ mA}$ so $\dfrac{I_{max}}{\sqrt{2}} = 243 \text{ mA.}$

Bandwidth at $I_{max}/\sqrt{2} = 16 \text{ kc/s.}$

$\therefore Q = f_r/(16 \times 10^3) = 884 \times 10^3/(16 \times 10^3) = \underline{55\cdot25,}$

$R = 5/I_{max} = 5 \times 10^3/344 = \underline{14\cdot53\ \Omega,}$

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$$L = RQ/\omega_r = 14 \cdot 53 \times 55 \cdot 25/(2\pi \times 884 \times 10^3) \text{ H} = \underline{144 \cdot 5 \, \mu\text{H}},$$
$$C = 1/4\pi^2 f_r^2 L = 1/\{4\pi^2 \times (884 \times 10^3)^2 \times 144 \cdot 5 \times 10^{-6}\} \text{ F},$$
$$= 224 \, \mu\mu\text{F}.$$

3. Let the resistance, inductance and capacitance be R ohms, L henrys and C farads respectively.

The impedance of the parallel combination

$$Z = \left[\frac{(R + j\omega L)\left(\dfrac{1}{j\omega C}\right)}{R + j\omega L + \dfrac{1}{j\omega C}} \right] = \frac{L}{CR}\left[\frac{1 - jR/\omega L}{1 + j\left(\dfrac{\omega L}{R} - \dfrac{1}{\omega CR}\right)} \right].$$

At what is often taken as resonance, Z is a pure resistance and

$$\left[\frac{-R}{\omega_r L}\right] = \left[\frac{\omega_r L}{R} - \frac{1}{\omega_r CR}\right].$$

$$\therefore \qquad \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 2\pi f_r.$$

Thus, $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{88 \times 10^{-6} \times 375 \times 10^{-12}} - \dfrac{4 \cdot 8^2}{88^2 \times 10^{-12}}}$ c/s

$$= \underline{876 \cdot 4 \text{ kc/s}}.$$

4. $V = I/(1/j\omega L + j\omega C + 1/R).$

V has a maximum value equal to IR when $\omega C = \dfrac{1}{\omega L}$, i.e. when $\omega = 1/\sqrt{LC} = \omega_r$ say.

V drops 3 db from its maximum value when $\omega C - \dfrac{1}{\omega L} = \pm 1/R$

and the corresponding angular frequencies are ω_1 and ω_2,

where $\omega_2/\omega_r - \omega_r/\omega_2 = - 1/\omega_r CR$
and $\omega_1/\omega_r - \omega_r/\omega_1 = + 1/\omega_r CR.$

By comparison with the solution to Problem 1 the Q factor is seen to be $\underline{\omega_r CR = R/\omega_r L}.$

5. Using the solution to Problem 1 it is seen that

$$Q = \omega_r/(\omega_1 - \omega_2).$$

∴ $\qquad Q \simeq \omega_r/2(\omega_1 - \omega_r) \simeq \omega_r/2(\omega_r - \omega_2)$.

In this case $Q \simeq 2\pi \times 10^6/2(2\pi \times 5 \times 10^3) = 100$.

It is easily shown that the parallel resonant impedance $Z = L/CR$, i.e.

$$Z = Q/\omega_r C = 100/(2\pi \times 10^6 \times 200 \times 10^{-12})\,\Omega = \underline{79 \cdot 6\ \text{k}\Omega}.$$

6. The impedance-frequency curve for the circuit will have a maximum value of R at some frequency. The bandwidth is the difference in cycles between the two frequencies at which the impedance is $R/\sqrt{2}$ and can be shown* to be $1/2\pi CR$.

Here bandwidth $= 250 \times 10^3$ c/s, $C = 50 \times 10^{-12}$ F
$$\text{so } R = 12{,}740\ \Omega.$$

7. In reducing circuits such as (a) and (b) to that of (c) the rules that determine the values of Z_p, Z_s and M are:

(1) $Z_p =$ impedance measured between primary terminals of actual circuit when secondary is opened.

(2) $Z_s =$ impedance measured by opening secondary of actual circuit and determining the impedance between these open points when the primary is open-circuited.

(3) M is determined by assuming a current I flows in the primary circuit. The voltage which appears across an open-circuited secondary is then $\pm j\omega MI$.

For circuit (a) applying the above rules:
$$\underline{Z_p = j\omega(L_1 + L_m)},\ \underline{Z_s = j\omega(L_2 + L_m)},\ \underline{j\omega L_m I = j\omega MI}.$$

∴ $\qquad \underline{\text{coefficient of coupling} = L_m/\sqrt{(L_1 + L_m)(L_2 + L_m)}}$.

For circuit (b):
$$\underline{Z_p = (C_1 + C_m)/\omega C_1 C_m},\ \underline{Z_s = (C_2 + C_m)/\omega C_2 C_m},\ \underline{jI/\omega C_m = j\omega MI}.$$

∴ $\qquad \underline{\text{coefficient of coupling} = \sqrt{C_1 C_2/(C_1 + C_m)(C_2 + C_m)}}$.

8. Impedance reflected into the primary circuit from the secondary by mutual coupling $= \omega^2 M^2/Z_s$.

Primary current $I_p = E/(Z_p + \omega^2 M^2/Z_s)$.

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* E.g. See L. B. Arguimbau and R. B. Adler, *Vacuum Tube Circuits and Transistors*, Wiley, 1956, p. 249.

Voltage induced in secondary $= -j\omega M I_p$.

Secondary current $I_s = -j\omega M I_p / Z_s = -j\omega M E / (Z_p Z_s + \omega^2 M^2)$.

$\therefore \quad Z_p = R_p + j(\omega L_p - 1/\omega C_p) = R_p + j\omega L_p(1 - 1/\alpha^2)$.

$Z_s = R_s + j(\omega L_s - 1/\omega C_s) = R_s + j\omega L_s(1 - 1/\alpha^2)$.

$\therefore \quad I_s = -j\omega M E / \{R_p R_s - (1 - 1/\alpha^2)^2 \omega^2 L_p L_s$
$$+ \omega^2 M^2 + j(1 - 1/\alpha^2)(\omega L_p R_s + \omega L_s R_p)\}.$$

Dividing the numerator and denominator by $\omega^2 L_p L_s$ and noting that $Q_p = \omega L_p / R_p$, $Q_s = \omega L_s / R_s$, $M^2 = k^2 L_p L_s$ and $\omega = \alpha \omega_r$,

$$I_s = \frac{-jEk/\alpha\omega_r \sqrt{L_p L_s}\{k^2 + 1/Q_p Q_s - (1 - 1/\alpha^2)^2}{+ j(1 - 1/\alpha^2)(1/Q_p + 1/Q_s)\}}$$

$$I_s = -j\omega M E / [\{R_p + j(\omega L_p - 1/\omega C_p)\}$$
$$\{R_s + j(\omega L_s - 1/\omega C_s)\} + \omega^2 M^2].$$

$\therefore \underline{I_s \text{ reaches a maximum value when the circuits are in resonance}}$
$\underline{\text{and } \omega^2 M^2 = R_p R_s.}$

For maximum I_s, $\omega M = \sqrt{R_p R_s} = \omega \sqrt{L_p L_s} / \sqrt{Q_p Q_s}$.

$\therefore \qquad\qquad \underline{\text{critical value of } k = 1/\sqrt{Q_p Q_s}.}$

9. $k = \underline{\sqrt{300 \times 300}/60 = 0\cdot2.}$

Primary current $I_p = \dfrac{10}{Z + \omega^2 M^2 / Z}$

where

$Z = j(\omega L - 1/\omega C)$
$= j(2 \times 10^6 \times 300 \times 10^{-6} - 10^{12}/2 \times 10^6 \times 10^3) = j100$.

Secondary current $I_s = -j\omega M I_p / Z = \underline{-j0\cdot273 \text{ A}.}$

10. Input impedance $= Z_p + \omega^2 M^2 / Z_s$

where $\qquad\qquad Z_p = (j2\pi \times 10^6 \times 200 \times 10^{-6})\,\Omega$
$\qquad\qquad\qquad Z_s = (100 + j2\pi \times 10^6 \times 20 \times 10^{-6})\,\Omega$

and $\qquad\qquad\qquad M = (0\cdot1\sqrt{200 \times 20 \times 10^{-12}})\text{ H}.$

$\therefore \qquad\qquad\qquad \underline{Z_p = (6\cdot1 + j1{,}249\cdot1)\,\Omega.}$

11. Effective primary impedance (Z_p)

$$= R_1 + j(\omega L_1 - 1/\omega C_1) + \omega^2 M^2 / \{R_2 + j(\omega L_2 - 1/\omega C_2)\}.$$

Substituting the given figures Z_p is found to be $(718 + j0)\,\Omega$,

$\therefore$ Effective resistance $= 718\,\Omega$

and   Effective reactance $= 0$.

Primary current $= 100/718\text{ A} = 0\cdot139\text{ A}$.

Secondary current $= \omega M \times 0\cdot139/R_2 = 1\cdot306\text{ A}$.

12. With the secondary open-circuited the impedance of the primary winding is $j\omega L_1 + \dfrac{1}{j\omega C_1}$ and the resonant frequency f

$= \omega/2\pi = 1/2\pi\sqrt{L_1 C_1}$.

$\therefore$ $C_1 = 1/(2\pi \times 500 \times 10^3)^2 \times 1 \times 10^{-3}\text{ F} = 101\,\mu\mu\text{F}$.

When the secondary is short-circuited the impedance of the primary is $[j\omega L_1 + 1/j\omega C_2 + \omega^2 M^2/j\omega L_2]$, and the

resonant frequency $= \dfrac{1}{2\pi\sqrt{\left(L_1 - \dfrac{M^2}{L_2}\right)C_2}}$,

where C_2 is the new capacitance.

$\therefore$ since the resonant frequencies are the same,

$$(L_1 - M^2/L_2)C_2 = L_1 C_1$$

i.e. $C_2 = \dfrac{L_1 C_1}{L_1 - M^2/L_2} = \dfrac{10^{-3} \times 101 \times 10^{-12}}{10^{-3}(1 - 0\cdot25)}\text{ F} = 135\,\mu\mu\text{F}$.

$\therefore$ The change of capacitance $= (135 - 101)\,\mu\mu\text{F} = 34\,\mu\mu\text{F}$.
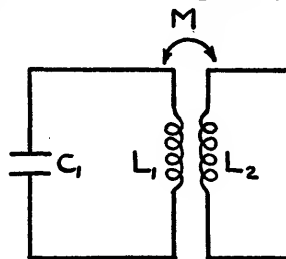
13. The input impedance $= R_1 + j\omega L_1 + 1/j\omega C_1 + \omega^2 M^2/R_2$. When this is purely resistive its value is

$$R_1 + \omega^2 M^2/R_2 = 5 + \frac{(2\pi \times 10^6)^2 \times (10 \times 10^{-6})^2}{20} = 202\cdot4\,\Omega.$$

14. With the currents as shown the equations for the circuit are:

$$e_1 = (R_1 + j\omega L_1)I_1 - j\omega M I_2$$

and $e_2 = (R_2 + j\omega L_2)I_2 - j\omega M I_1$

Now $(R_1 + j\omega L_1) = (60 + j1,885 \times 50 \times 10^{-3})\,\Omega$

$(R_2 + j\omega L_2) = (80 + j1,885 \times 70 \times 10^{-3})\,\Omega$

$-j\omega M = -(j1,885 \times 17\cdot75 \times 10^{-3})\,\Omega$

$$e_1 = \frac{169\cdot7}{\sqrt{2}}\ \underline{/0^\circ}\ \text{V}$$

and $$e_2 = \frac{141\cdot4}{\sqrt{2}}\ \underline{/45^\circ}\ \text{V}.$$

∴ $I_1 = (0\cdot835 - j0\cdot819)\,\text{A} = 1\cdot168\underline{/-45\cdot6^\circ}\,\text{A}$

and $I_2 = (0\cdot874 - j0\cdot212)\,\text{A} = 0\cdot903\underline{/-13\cdot6^\circ}\,\text{A}$.

The vector diagram is as shown below:
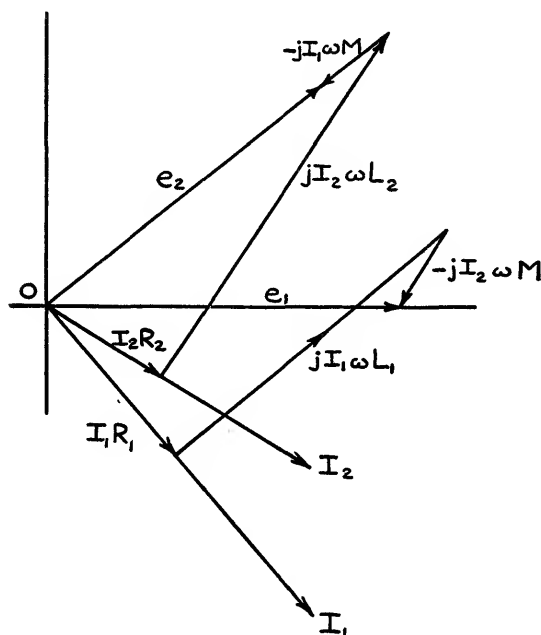
15. The two following equations apply:

$$e = (R_1 + j\omega L_1)I_1 \pm j\omega M I_2$$

and $$e = (R_2 + j\omega L_2)I_2 \pm j\omega M I_1.$$

Writing $Z_1 = R_1 + j\omega L_1$, $Z_2 = R_2 + j\omega L_2$ and $Z_m = \pm j\omega M$,

$$e = Z_1 I_1 + Z_m I_2$$
$$e = Z_m I_1 + Z_2 I_2.$$

$\therefore$ $I_1 = e(Z_2 - Z_m)/(Z_1 Z_2 - Z_m^2)$

and $I_2 = e(Z_1 - Z_m)/(Z_1 Z_2 - Z_m^2).$

$\therefore$ $I = I_1 + I_2 = e(Z_1 + Z_2 - 2Z_m)/(Z_1 Z_2 - Z_m^2).$

Equivalent impedance

$$Z_p = e/I = \underline{(Z_1 Z_2 - Z_m^2)/(Z_1 + Z_2 - 2Z_m).}$$

16. When the coils are in series:

$$L_1 + L_2 + 2M = 2(L + M) = 360 \text{ mH}$$

and $L_1 + L_2 - 2M = 2(L - M) = 40 \text{ mH}.$

$\therefore$ $L = 100 \text{ mH and } M = 80 \text{ mH}.$

Using the result of the previous solution the equivalent inductance L_e of the two coils in parallel is given by:

$$\omega L_e = \{\omega L_1 \omega L_2 - (\pm \omega M)^2\}/\{\omega L_1 + \omega L_2 - (\pm 2\omega M\}$$

i.e. $L_e = (L^2 - M^2)/2(L \pm M) = (100^2 - 80^2)/2(100 \pm 80) \text{ mH}$

$= \underline{10 \text{ or } 90 \text{ mH}.}$

17. Using Thévenin's theorem the circuit to the left of points A and B can be replaced by a single e.m.f. acting in series with a single impedance. The e.m.f. (e) is the voltage between A and B when the network to the right of these points is disconnected. The impedance (Z) is equal to that which would be measured looking to the left at terminals A and B.

If the network is opened at A, B the current I_1 flowing in mesh 1

$= E/(R_1 + j\omega L_1).$

$\therefore$ $e = -j\omega M_1 I_1$

$= -j \times 2 \times 10^6 \times 50 \times 10^{-6} \times 6/(40 + j200)$

$= -2{\cdot}94\underline{/11° \ 19'} \text{ V}.$

Also $Z = j\omega L_2 + \omega^2 M_1^2/(R_1 + j\omega L_1) = (j\omega L_2 + 9{\cdot}6 - j48) \ \Omega.$

The original circuit therefore reduces to the following:
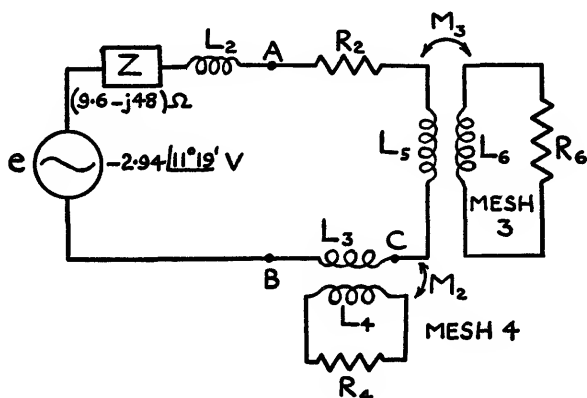
Similarly, mesh 4 can be removed by adding an impedance $\omega^2 M_2{}^2/(R_4 + j\omega L_4) = (15\cdot1 - j68)\,\Omega$ in series with L_3 as shown in the following figure:
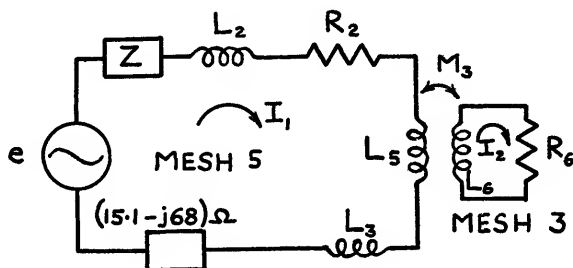
For mesh 5
$$e = I_1\{9\cdot6 - j48 + 15\cdot1 - j68 + R_2 + j\omega(L_2 + L_3 + L_5) + j\omega M_3 I_2.$$
For mesh 3
$$0 = j\omega M_3 I_1 + I_2(R_6 + j\omega L_6).$$
From these two equations I_2 is found to be $0\cdot00369\underline{/-64°\,6'}$ A.

18. The total impedance of the primary loop
$$= \{(3\cdot4 + 5\cdot1) + j\omega(55 + 725) \times 10^{-6} - j/\omega 7\cdot6 \times 10^{-9}\}\,\Omega$$
$$= (8\cdot5 - j174)\,\Omega \text{ since } \omega = 2\pi \times 50 \times 10^3.$$

The total impedance of the secondary loop

$$= \{(0\cdot5 + 120) + j\omega(106 + 450) \times 10^{-6} - j/\omega C\}\,\Omega$$

where

$$1/C = 1/C_2 + 1/C_3 = 10^6[1/0\cdot0421 + 1/0\cdot0076]$$

i.e. total impedance of secondary loop $= (120\cdot5 - j320)\,\Omega$.

The mutual impedance includes the impedance of the common branch and the mutual impedance resulting from M. It is therefore $j(\omega M + 1/\omega C_3)$

$$= j(2\pi \times 50 \times 10^3 \times 268 \times 10^{-6}$$
$$+ \, 1/2\pi \times 50 \times 10^3 \times 0\cdot0076 \times 10^{-6})\,\Omega$$
$$= j503\,\Omega.$$

The apparent impedance which the voltage source sees is

$$\{(8\cdot5 - j174) - (j503)^2/(120\cdot5 - j320)\}\,\Omega$$
$$= (271 + j522)\,\Omega.$$

Primary current $(I_p) = 10/(271 + j522) = 10/589\underline{/62\cdot6°}$
$$= 0\cdot017\underline{/-62\cdot6°}\,\text{A}$$

where e is taken along the real axis.

The current ratio $I_p/I_s = (120\cdot5 - j320)/j503 = 0\cdot679\underline{/20\cdot6°}$.

$\therefore \qquad I_s = 0\cdot017\underline{/-62\cdot6°}/0\cdot679\underline{/20\cdot6°} = 0\cdot0251\underline{/-83\cdot2°}\,\text{A}.$

19. If two coils having inductances L_1 and L_2 respectively and a mutual inductance M are connected together their joint inductances are:

(a) In series aiding $L_1 + L_2 + 2M$.
(b) In series opposing $L_1 + L_2 - 2M$.
(c) In parallel aiding $(L_1L_2 - M^2)/(L_1 + L_2 - 2M)$.
(d) In parallel opposing $(L_1L_2 - M^2)/(L_1 + L_2 + 2M)$.

For (a) here joint inductance is therefore 450 μH.

$\therefore$ frequency range is

$$1/2\pi\sqrt{450 \times 10^{-6} \times 50 \times 10^{-12}}\,\text{c/s}$$

to $\qquad 1/2\pi\sqrt{450 \times 10^{-6} \times 1{,}000 \times 10^{-12}}\,\text{c/s}$

which corresponds to a wavelength range of 283 to 1,265 m.

Similarly, the other ranges are found to be:

(b) 249 to 1,115 m, (c) 122 to 546 m and (d) 107 to 481 m.

20. The impedance Z of the combination

$$= \{(R + j\omega L)/j\omega C\}/(R + j\omega L + 1/j\omega C)$$
$$= (R + j\omega L)(1 - \omega^2 LC - jR\omega C)/\{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2\}.$$

The effective resistance is therefore

$$R_e = R/\{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2\}$$

and the effective inductance is

$$L_e = \{L(1 - \omega^2 LC) - R^2 C\}/\{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2\}.$$

Substituting $L = 5 \times 10^{-3}$ H, $R = 100\,\Omega$, $C = 5 \times 10^{-12}$ F and $\omega = 2\pi \times 500 \times 10^3$, $R_e = 177\,\Omega$ and $L_e = 6\cdot67$ mH.

21. Let the inductance and self-capacitance of the coil be L and C respectively and let the original frequency be f c/s.

Then $f = 1/2\pi\sqrt{L(C + C_1)}$ and $2f = 1/2\pi\sqrt{L(C + C_2)}$

where $C_1 = 250\,\mu\mu\text{F}$ and $C_2 = 55\,\mu\mu\text{F}$.

$\therefore$ $C_1 - 4C_2 = 3C$ so that $C = 10\,\mu\mu\text{F}.$

22. Apparent mutual inductance M_1 is approximately[*]

$$M\{1 + \omega^2(L_1 C_1 + L_2 C_2)\}$$

where $L_1 = 50\,\mu\text{H}$, $L_2 = 200\,\mu\text{H}$, $C_1 = 5\,\mu\mu\text{F}$, $C_2 = 7\,\mu\mu\text{F}$, $\omega = 2\pi \times 2 \times 10^6$ and $M = 0\cdot05\sqrt{50 \times 200}\,\mu\text{H}$.

$\therefore$ $M_1 = 6\cdot3\,\mu\text{H}.$

23. For circuit (a) the impedance

$$Z_a = \left[\frac{1}{j\omega C} + \frac{\dfrac{j\omega L_1}{j\omega C_1}}{\dfrac{1}{j\omega C_1} + j\omega L_1}\right] = \left[\frac{1 - \omega^2 L_1 C_1 - \omega^2 L_1 C}{j\omega C(1 - \omega^2 L_1 C_1)}\right].$$

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[*] This is easily proved from first principles but readers may like to look at the following: F. K. Harris, *Electrical Measurements*, Wiley, 1952, pp. 671 and 672; S. Butterworth, 'Capacity and Eddy Current Effects in Inductometers,' *Proc. Phys. Soc.*, **33**, p. 312, 1921; L. Hartshorn, 'The Properties of Mutual Inductance Standards at Telephonic Frequencies,' *Proc. Phys. Soc.*, **38**, p. 302, 1926.

For circuit (b) the impedance

$$Z_b = \left[\frac{\dfrac{1}{j\omega C'} + \left(j\omega L_2 + \dfrac{1}{j\omega C_2} \right)}{\dfrac{1}{j\omega C'} + \dfrac{1}{j\omega C_2} + j\omega L_2} \right] = \left[\frac{1 - \omega^2 L_2 C_2}{j\omega C_2(1 + C'/C_2 - \omega^2 L_2 C')} \right].$$

If $Z_a = Z_b$,

$$C_2(1 - \omega^2 L_1 C_1 - \omega^2 L_1 C)\left(1 + \frac{C'}{C_2} - \omega^2 L_2 C' \right)$$
$$= C(1 - \omega^2 L_2 C_2)(1 - \omega^2 L_1 C_1).$$

Equating the ω^4 terms gives

$$C' = CC_1/(C + C_1) \quad . \qquad . \qquad . \qquad . \quad (1)$$

Equating the ω^2 terms gives

$$L_2 = \frac{L_1}{C_2}\left[\frac{C_1 C_2 + C_1 C' + CC_2 + CC' - CC_1}{C - C'} \right].$$

Substituting for C' from (1) gives

$$L_2 = L_1(C_1 + C)^2/C^2 \quad . \qquad . \qquad . \qquad . \quad (2)$$

Equating the terms which do not contain ω gives

$$C_2 + C' = C$$

Substituting for C' from (1) gives

$$C_2 = C^2/(C + C_1) \quad . \qquad . \qquad . \qquad . \quad (3)$$

24. Impedance Z

$$= \{(R + j\omega L)(R + 1/j\omega C)\}/\{(R + j\omega L) + (R + 1/j\omega C)\}$$
$$= \{R^2 + L/C + jR(\omega L - 1/\omega C)\}/\{2R + j(\omega L - 1/\omega C)\}.$$

If $L/C = R^2$ then $Z = R$.

25.

$$r + j\omega l = Rj\omega L/(R + j\omega L) = (j\omega L R^2 + \omega^2 L^2 R)/(R^2 + \omega^2 L^2).$$

Equating real and imaginary parts,

$$r = \omega^2 L^2 R/(R^2 + \omega^2 L^2) \text{ and } l = LR^2/(R^2 + \omega^2 L^2).$$

$$\therefore \qquad R = r + (\omega^2 l^2/r) \text{ and } L = l + (r^2/\omega^2 l).$$

26. Let the impedance of the source be $Z_s = R_s + jX_s$, the load impedance $Z_L = R_L + jX_L$ and the voltage of the source V. Then the load current $I_L = V/[(R_s + R_L) + j(X_s + X_L)]$.

$\therefore$ the power in the load $(W) = R_L \cdot V^2/[(R_s + R_L)^2 + (X_s + X_L)^2]$.

If X_L is variable, W is a maximum when $X_L = -X_s$ and W is then $R_L V^2/(R_s + R_L)^2$. This is a maximum when $R_L = R_s$,

i.e.   W is a maximum when $\underline{Z_L = R_L + jX_L = R_s - jX_s.}$

Let the loudspeaker impedance be $(R + jX)\ \Omega$.

Then the total load on the generator

$$= \frac{(R + jX)(-j5)}{R + jX - j5}.$$

Conjugate of source impedance $= (3 - j4)\ \Omega$.

$\therefore$ $\dfrac{(R + jX)(-j5)}{R + jX - j5} = 3 - j4.$

Cross-multiplying and equating real and imaginary parts gives:

$$X - 3R = -20 \quad \text{and} \quad 3X + R = 15$$

$\therefore$ $\underline{R = 7 \cdot 5\ \Omega \quad \text{and} \quad X = 2 \cdot 5\ \Omega.}$

27. For the series circuit

$$\tan \phi = 1/\omega C \rho,$$

where ϕ is the phase angle between current and voltage.

Power factor, $\cos \phi = \rho/\sqrt{\rho^2 + 1/\omega^2 C^2} \simeq \omega C \rho$.

For the parallel circuit

$$\tan \phi = \omega Cr \text{ and } \cos \phi = 1/\sqrt{1 + C^2\omega^2 r^2} \simeq 1/\omega Cr$$

$\therefore$ $1/\omega Cr = \omega C\rho \text{ or } \omega^2 C^2 \rho r = 1.$

Here, $\cos \phi = 0 \cdot 001$, $\omega = 2\pi f$, $C\rho = 25 \times 10^{-10}$ so

$$\underline{f = 63 \cdot 7 \text{ kc/s.}}$$

28. The vector diagram for the network is as shown. I is the current through R and C.
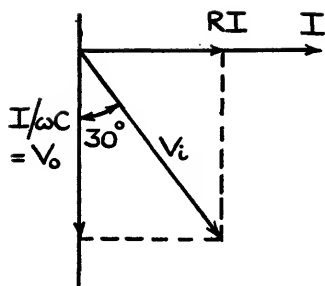
$$(R^2 + 1/C^2\omega^2)^{1/2} = 5{,}000 \qquad \cdot \quad \cdot \quad \cdot \quad (1)$$
$$R\omega C = \tan 30° = 1/\sqrt{3} \qquad \cdot \quad \cdot \quad \cdot \quad (2)$$

$\therefore \qquad R = 2{,}500\,\Omega$ and $C = 0{\cdot}037\,\mu\text{F}$ and V_o lags behind V_i.

29. *Mesh analysis.*

The equations for the three loops are:

$$E = (600 + j600 + 400)I_1 - j600I_2 - 400I_3. \qquad \cdot \quad (1)$$
$$0 = -j600I_1 + (900 + j600 - j600)I_2 - (-j600)I_3 \quad \cdot \quad (2)$$
$$0 = -400I_1 - (-j600)I_2 + (600 + 400 - j600)I_3 \quad \cdot \quad (3)$$

Currents I_1, I_2 and I_3 can be found from equations (1), (2) and (3) using Cramer's Rule. They are:

$$I_1 = E(83{\cdot}3 - j35{\cdot}7)10^{-5},$$
$$I_2 = E(23{\cdot}8 + j23{\cdot}8)10^{-5},$$

and $\qquad I_3 = E(47{\cdot}6)10^{-5}.$

$\therefore \qquad V_1 = E - 600I_1 = \underline{E(0{\cdot}499 + j0{\cdot}214)},$

$\qquad V_2 = 400(I_1 - I_3) = \underline{E(0{\cdot}143 - j0{\cdot}143)},$

and $\qquad V_3 = 600I_3 = \underline{E(0{\cdot}284)}.$

Nodal analysis

The various admittances in the network are:

$$Y_g = 1/R_g = 1{\cdot}67 \times 10^{-3}\,\text{mho},$$
$$Y_1 = 1/R_1 = 1{\cdot}1 \times 10^{-3}\,\text{mho},$$
$$Y_2 = 1/R_2 = 2{\cdot}5 \times 10^{-3}\,\text{mho},$$
$$Y_3 = 1/X_3 = -j1{\cdot}67 \times 10^{-3}\,\text{mho},$$
$$Y_4 = 1/X_4 = j1{\cdot}67 \times 10^{-3}\,\text{mho},$$
$$Y_l = 1/R_l = 1{\cdot}67 \times 10^{-3}\,\text{mho}.$$

The nodal equations are:

$$E/600 = V_1(1{\cdot}67 \times 10^{-3} + 1{\cdot}1 \times 10^{-3} - j1{\cdot}67 \times 10^{-3})$$
$$- V_2(-j1{\cdot}67 \times 10^{-3}) - V_3(1{\cdot}1 \times 10^{-3}) \qquad \cdot \quad \cdot \quad (4)$$

$$0 = - V_1(-j1\cdot67 \times 10^{-3})$$
$$+ V_2(2\cdot5 \times 10^{-3} - j1\cdot67 \times 10^{-3} + j1\cdot67 \times 10^{-3})$$
$$- V_3(j1\cdot67 \times 10^{-3}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$0 = - V_1(1\cdot1 \times 10^{-3}) - V_2(j1\cdot67 \times 10^{-3})$$
$$+ V_3(1\cdot67 \times 10^{-3} + 1\cdot1 \times 10^{-3} + j1\cdot67 \times 10^{-3}) \quad . \quad . \quad (6)$$

Node voltages V_1, V_2 and V_3 can be found directly from equations (4), (5) and (6). They are:

$$\underline{V_1 = E(0\cdot499 + j0\cdot214),}$$
$$\underline{V_2 = E(0\cdot143 - j0\cdot143),}$$
$$\text{and } \underline{V_3 = E(0\cdot284)}$$

30. The nodal equations are obtained by simply applying Kirchhoff's first law at nodes 1 and 2. Thus:

$$\underline{-E_1 Y_1 + V_1(Y_1 + Y_3 + Y_4 + Y_5) - V_2(Y_4 + Y_5) = 0}$$
$$\text{and } \underline{-E_2 Y_2 + V_2(Y_2 + Y_4 + Y_5 + Y_6) - V_1(Y_4 + Y_5) = 0}$$

31. The following equation holds for the circuit:

$$L\frac{di}{dt} + Ri + \frac{q}{C} = V$$

where i is the current, V is the applied voltage and q is the charge on the capacitor.

Differentiating $\dfrac{d^2i}{dt^2} + \dfrac{R}{L}\dfrac{di}{dt} + \dfrac{i}{LC} = 0$.

The solution of this equation depends on the relative magnitudes of the constants R, L and C. In this case $\dfrac{R^2}{4L^2} < \dfrac{1}{LC}$ and the solution is:

$$i = Ae^{-\frac{Rt}{2L}} \cos\left[\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\, t + B\right]$$

where A and B are constants.

Initially when $t = 0$, $i = 0$ and $q = 0$.

$$\therefore \qquad A = - V/L \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ and } B = \pi/2.$$

$$\therefore \qquad i = \frac{V}{L\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}}\, e^{-\frac{Rt}{2L}} \sin\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\, t \text{ amperes.}$$

Substituting the given circuit values in this expression gives:
$$i = 40e^{-50t} \sin 1{,}000t \text{ amperes,}$$
i.e. the current is oscillatory, of gradually decreasing amplitude and of frequency $1{,}000/2\pi$ c/s $= 159$ c/s.
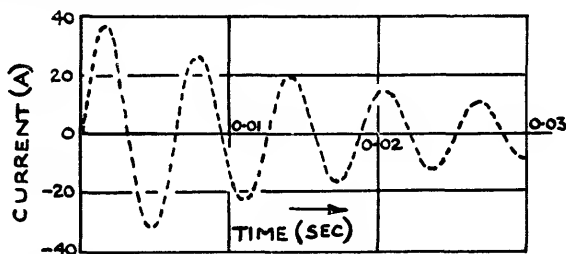
The current/time curve is plotted from the above expression.

When $R = 10\,\Omega$, $\dfrac{R^2}{4L^2} = \dfrac{1}{LC}$ and the solution of the above differential equation is:
$$i = (At + B)e^{-\frac{Rt}{2L}}$$
where A and B are constants.

When $t = 0$, $i = 0$ and $q = 0$.

$\therefore$ $\qquad\qquad\qquad\qquad A = V/L$ and $B = 0$.

$\therefore$ $\qquad\qquad\qquad\qquad i = \dfrac{V}{L}\,t\,e^{-\frac{Rt}{2L}}$ amperes.

Substituting the given circuit values in this expression gives:
$$i = 40{,}000t\,e^{-1000t} \text{ amperes.}$$

The current/time curve is plotted from this expression.
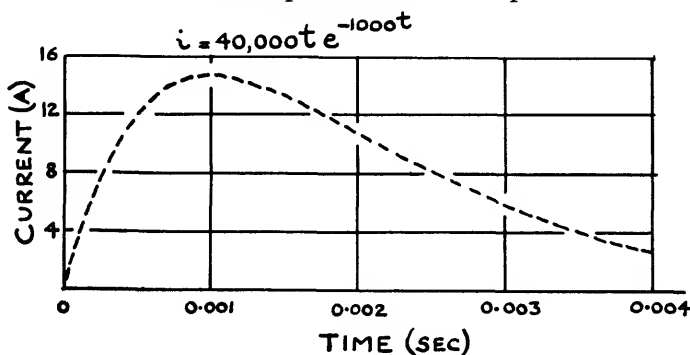
When $R = 20\,\Omega$, $\dfrac{R^2}{4L^2} > \dfrac{1}{LC}$ and the solution of the differential equation is $i = A_1 e^{m_1 t} + B_1 e^{m_2 t}$, where A_1 and B_1 are constants and m_1 and m_2 are the two roots of $m^2 + \dfrac{R}{L}\,m + \dfrac{1}{LC} = 0$.

With the same initial conditions as before

$$A_1 = -\,B_1 = V/L(m_1 - m_2)$$

and

$$i = \frac{V}{L(m_1 - m_2)}\,(e^{m_1 t} - e^{m_2 t}) \text{ amperes.}$$

Using the given circuit values, $m_1 = -\,3{,}732$ and $m_2 = -\,268$.

$\therefore \qquad\qquad i = 11\!\cdot\!54\,(e^{-268t} - e^{-3732t}) \text{ amperes.}$

The current/time curve is plotted from this expression.
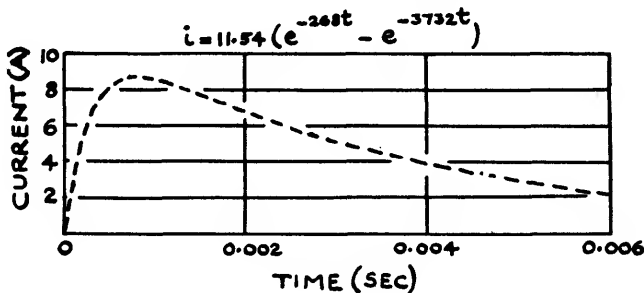
32. At any time t after closing the switch the current in the circuit (i) is given by:

$$L\frac{di}{dt} + Ri = 200 \sin 628t = V \sin \omega t.$$

The solution of this equation is:

$$i = A\,e^{-\frac{Rt}{L}} + V \sin (\omega t - \theta)/\sqrt{R^2 + \omega^2 L^2}$$

where A is a constant and $\tan \theta = \omega L/R$.

With the given initial conditions

$$A = V \sin \theta/\sqrt{R^2 + \omega^2 L^2}.$$

$\therefore$ the transient current $i_1 = A\,e^{-\frac{Rt}{L}} = V \sin \theta\,e^{-\frac{Rt}{L}}/\sqrt{R^2 + \omega^2 L^2}$

and the steady cyclic current $i_2 = V \sin (\omega t - \theta)/\sqrt{R^2 + \omega^2 L^2}$.

Substituting the given circuit values in these expressions:

$$i_1 = 15 \cdot 52\, e^{-100t} \text{ amperes}$$

and $\qquad i_2 = 15 \cdot 71 \sin(628t - 81°) \text{ amperes.}$

Thus i_1, i_2 and i can be plotted against t as shown. It is seen that after a time corresponding to about three complete cycles of the supply voltage i_1 is small.
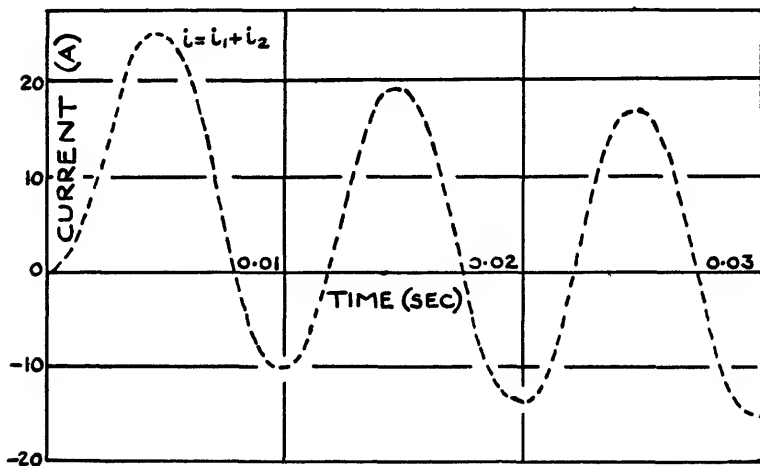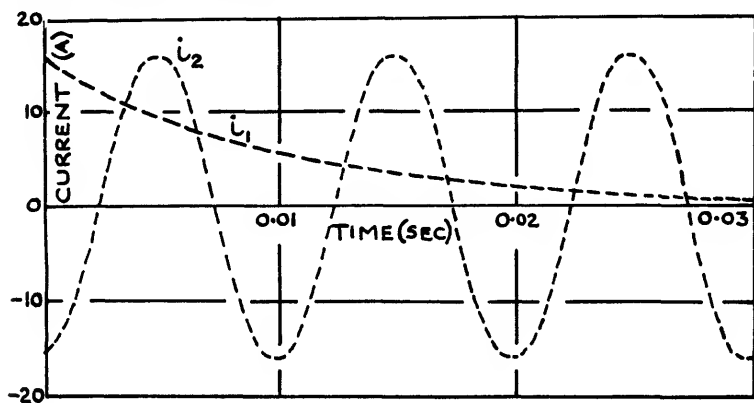
When $R = 0$, $\theta = 90°$, and with the same initial conditions

$$i = V(1 - \cos \omega t)/\omega L,$$

i.e. i never becomes negative; its minimum value is 0. It is also seen that the voltage and current values are zero at the same instant.

33. The following equation holds for the circuit:

$$L\frac{di}{dt} + Ri + \frac{q}{c} = V\cos(\omega t - \theta)$$

where i is the current and q is the charge on the capacitor.

$$\therefore \quad \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC}$$

$$= \frac{V\omega}{L}\cos(\omega t - \theta + \pi/2).$$
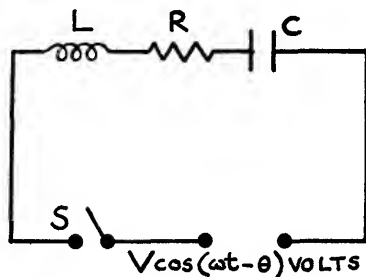
In this case $\dfrac{R^2}{4L^2} < \dfrac{1}{LC}$ and the solution is:

$$i = A\,e^{-\frac{Rt}{2L}}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \alpha\right)$$

$$+ \frac{\dfrac{V\omega}{L}\cos\left(\omega t - \theta + \dfrac{\pi}{2} - \beta\right)}{\sqrt{\left(\dfrac{1}{LC} - \omega^2\right)^2 + R^2\omega^2/L^2}}$$

where $\tan\beta = \dfrac{R\omega/L}{(1/LC - \omega^2)}$ and A, α and θ are constants determined by the initial conditions that when $t = 0$, $i = 0$, $q = 0$ and the applied voltage is zero.

Using the given circuit values it is found that

$$i = 41{\cdot}5\,e^{-50t}\cos(1{,}000t + 173{\cdot}5°) + 41{\cdot}3\cos(628t - 6°)\text{ amperes}$$
$$= i_1 + i_2.$$

The first term is the transient and it has a frequency of 159 c/s. It is seen from the graph of this current against time that about 0·05 sec after closing S its amplitude is less than 10% of its maximum amplitude of 41·3 A.

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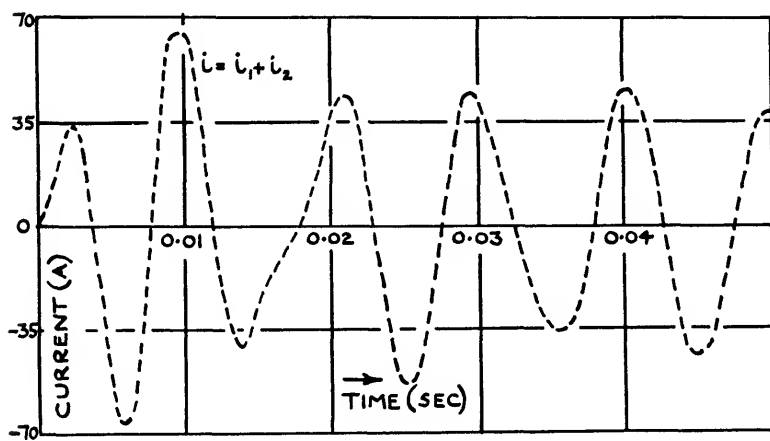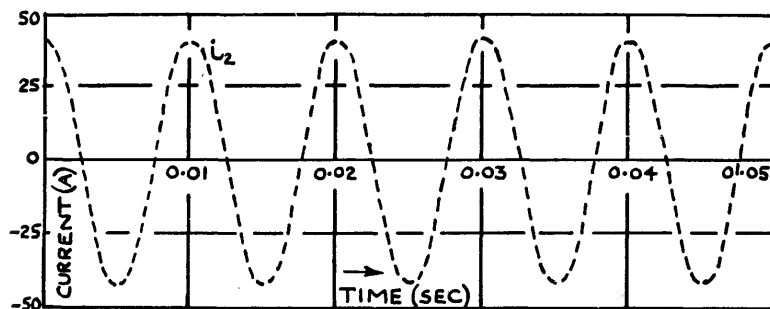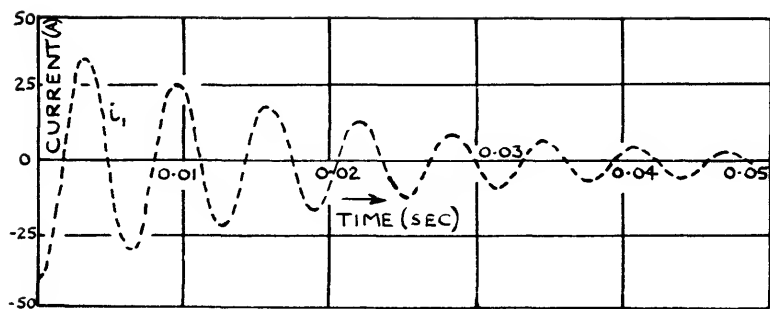
34. In this case $\dfrac{R^2}{4L^2} < \dfrac{1}{LC}$ and the natural frequency of the circuit is the same as the frequency of the applied voltage.

Proceeding in the same way as in the previous solution it is found that

$$i = -800\, e^{-25t} \sin 1{,}000t + 800 \sin 1{,}000t \text{ amperes}$$
$$= i_1 + i_2.$$

The i/t, i_1/t and i_2/t curves are plotted from this expression.
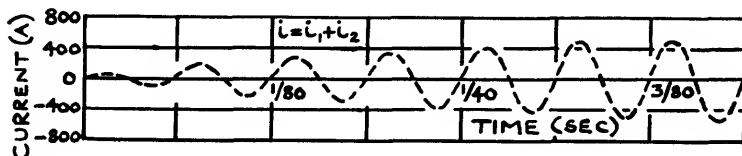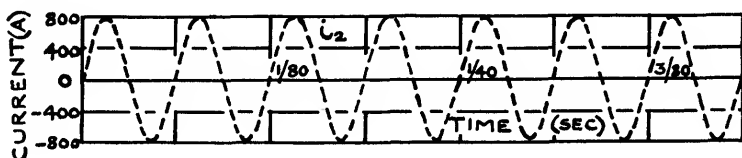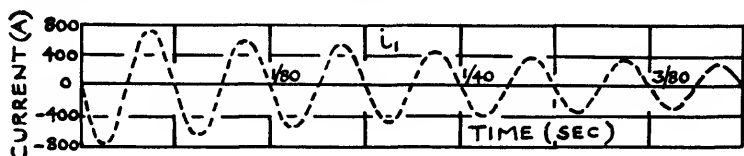
35. In this case $\dfrac{R^2}{4L^2} > \dfrac{1}{LC}$ and the solution of the differential equation of Problem 33 is now of the form:

$$i = A\, e^{m_1 t} + B\, e^{m_2 t}$$
$$+ \frac{V\omega}{L} \cos\left(\omega t - \theta + \frac{\pi}{2} - \beta\right) \Big/ \sqrt{\left(R\omega/L\right)^2 + \left(\frac{1}{LC} - \omega^2\right)^2}.$$

Substituting the given circuit values in this expression gives—

$$i = A\, e^{-1151t} + B\, e^{-868t} + 11{\cdot}4 \cos(314t - \theta + \pi/2 - 35{\cdot}3°)$$

where A, B and θ are constants depending on the initial conditions.
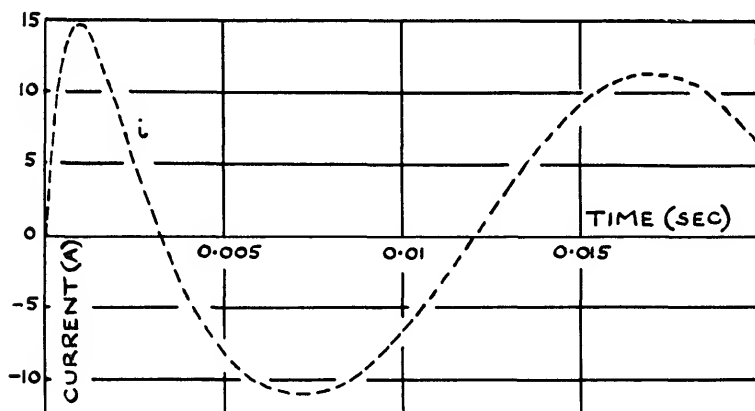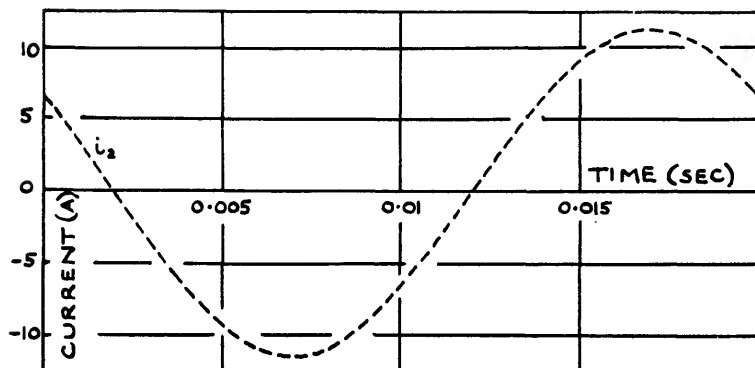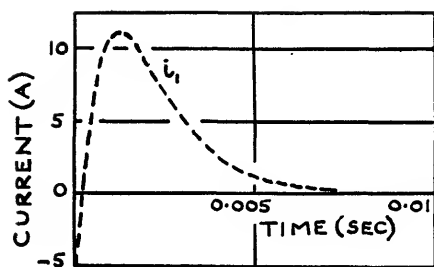
If when $t = 0$, $i = 0$, $q = 0$ and the applied voltage is at its maximum positive value, i.e. $\theta = 0$, it is found that

$$i = -131 \cdot 6\, e^{-1151t} + 125\, e^{-868t} + 11 \cdot 4 \cos (314t + 54 \cdot 7°) \text{ amperes.}$$

The transient current $i_1 = 131 \cdot 6\, e^{-1151t} + 125\, e^{-868t}$ amperes, the permanent current $i_2 = 11 \cdot 4 \cos (314t + 54 \cdot 7°)$ amperes and i can therefore be plotted against t as illustrated.

36. The solution to this problem has been given elsewhere.*

37. The solution to this problem has been given elsewhere.†

38. The solution to this problem has been given elsewhere.‡

39. The solution to this problem has been given elsewhere.§

40. Suppose $y(t)$ is a known function of t for values of $t > 0$. Then the Laplace transform $\bar{y}(p)$ of $y(t)$ is defined as

$$\bar{y}(p) = \int_0^\infty e^{-pt} y(t)\, dt \qquad . \qquad . \qquad . \quad (1)$$

where p is a number sufficiently large to make the integral convergent.

If a is any number, real or complex, then $\bar{y}(p + a)$ is the Laplace transform of $e^{-at} y(t)$.

Using (1) it is found that if $y(t) = t^{n-1}/(n-1)!$, then $\bar{y}(p) = 1/p^n$. Thus, the transform of $e^{-at} t^{n-1}/(n-1)!$ is $1/(p + a)^n$. Similarly, using (1) if $y(t) = \sin at$, then $\bar{y}(p) = a/(p^2 + a^2)$.

Thus, the transform of $e^{-bt} \sin at = a/\{(p + b)^2 + a^2\}$.
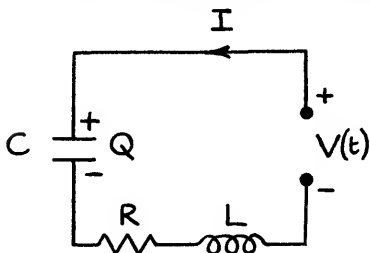
For the circuit illustrated,

$$I = dQ/dt . \qquad . \quad (2)$$

Also,

$$L\, dI/dt + RI + Q/C = V \quad (3)$$

The problem is to solve (2) and (3) with given initial values, $I = I_0$, $Q = Q_0$ at $t = 0$ . . (4)

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* F. A. Benson and D. Harrison, *Electric Circuit Theory*, Arnold, 2nd Edition, 1963, pp. 240–1.

† *Ibid.*, pp. 230–2.   ‡ *Ibid.*, pp. 243–4.   § *Ibid.*, pp. 247–9.

Forming the subsidiary equations for (2), (3) and (4) in the usual way:

$$(Lp + R)\bar{I} + \bar{Q}/C = LI_0 + \bar{V} \quad . \qquad . \qquad . \quad (5)$$

$$\bar{Q} = \bar{I}/p + Q_0/p \quad . \qquad . \qquad . \qquad . \quad (6)$$

From (5) and (6)

$$(Lp + R + 1/Cp)\bar{I} = \bar{V} + LI_0 - Q_0/Cp \quad . \qquad . \quad (7)$$

If a constant voltage E is applied at $t = 0$ and $I_0 = Q_0 = 0$, the subsidiary equation (7) becomes:

$$(Lp + R + 1/Cp)\bar{I} = E/p \quad . \qquad . \qquad . \quad (8)$$

Thus, $\qquad\qquad \bar{I} = E/L\{(p + \alpha)^2 + \beta^2\} \quad . \qquad . \qquad . \quad (9)$

where $\qquad\qquad \alpha = R/2L \quad$ and $\quad \beta^2 = 1/LC - R^2/4L^2$

The solution of (9) is $I = Ete^{-\alpha t}/L$, when $R = 2\sqrt{L/C}$. Similarly, the transient response of the circuit can be investigated using (7) no matter what the form of the applied voltage.

41. (*a*) The subsidiary equations are:

$$(p^2 + 2)\bar{x} - p\bar{y} = \frac{1}{p} + px_0$$

$$p\bar{x} + (p^2 + 2)\bar{y} = x_0$$

Solving:

$$\bar{x} = \frac{p^4x_0 + p^2(3x_0 + 1) + 2}{p(p^2 + 1)(p^2 + 4)} = \frac{1}{2p} + \frac{p(2x_0 - 1)}{3(p^2 + 1)} + \frac{p(2x_0 - 1)}{6(p^2 + 4)}$$

$$\bar{y} = \frac{2x_0 - 1}{(p^2 + 1)(p^2 + 4)} = \frac{(2x_0 - 1)}{3}\left(\frac{1}{p^2 + 1} - \frac{1}{p^2 + 4}\right)$$

$\therefore \qquad\qquad x = \tfrac{1}{2} + \tfrac{1}{3}(2x_0 - 1)\cos t + \tfrac{1}{6}(2x_0 - 1)\cos 2t$

and $\qquad\qquad y = \tfrac{1}{3}(2x_0 - 1)\sin t - \tfrac{1}{6}(2x_0 - 1)\sin 2t$

(*b*) The subsidiary equations are:

$$Z_1\bar{I}_1 = \bar{V} - \bar{V}_g$$

$$Z_4(\bar{I}_1 + \bar{I}_2 - \bar{I}_a) = \bar{V}_g$$

$$Z_3(\bar{I}_2 - \bar{I}_a) = \bar{V}_a - \bar{V}_g$$

and $\qquad\qquad Z_2\bar{I}_2 = -\bar{V}_a$

For the valve:

$$r_a\bar{I}_a = \bar{V}_a + \mu\bar{V}_g$$

Solving these equations for $\bar{V}_a$ in terms of $\bar{V}$ gives

$$\bar{V}_a = \frac{\bar{V}Z_4Z_2(r_a - \mu Z_3)}{Z_4Z_1Z_2(1 + \mu) + (Z_4 + Z_1)\{Z_2Z_3 + r_a(Z_2 + Z_3)\} + r_aZ_4Z_1}$$

(c) The subsidiary equations are:

$$L_1 p\bar{\imath}_1 + M p\bar{\imath}_2 = \bar{v}$$

$$M p\bar{\imath}_1 + \left(L_2 p + R_2 + \frac{1}{C_2 p}\right)\bar{\imath}_2 = 0$$

$\therefore \qquad \bar{\imath}_2 = \dfrac{-\, M p\bar{v}}{(L_1L_2 - M^2)p^2 + R_2L_1p + L_1/C_2}$

e.g. if $v = $ a constant V, $\bar{v} = V/p$

Then, $\qquad \bar{\imath}_2 = \dfrac{-\, MV}{(L_1L_2 - M^2)p^2 + R_2L_1p + L_1/C_2}$

$\therefore \qquad i_2 = \dfrac{-\, MV}{\beta(L_1L_2 - M^2)}\, e^{-\alpha t}\sin\beta t$

where $\quad \alpha = \dfrac{R_2L_1}{2(L_1L_2 - M^2)} \quad$ and $\quad \beta^2 = \dfrac{L_1}{C_2(L_1L_2 - M^2)} - \alpha^2$

42. (a) Logarithmic decrement $\delta = \pi R/\omega L = R/2fL$.

The frequency f corresponding to a wavelength of 300 m is 1 Mc/s.

$\therefore \qquad \delta = 10/(2 \times 10^6 \times 150 \times 10^{-6}) = \underline{0{\cdot}033}.$

(b) Let the number of oscillations be N.

Then the amplitude of the Nth oscillation, $I_N = I_1\, e^{-(N-1)\delta}$, where δ is the logarithmic decrement.

$\therefore \qquad 100 = e^{(N-1)0{\cdot}1} \text{ and } \underline{N = 47}.$

43. The natural frequency of free oscillations is

$$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \underline{478 \text{ kc/s.}}$$

To make the discharge non-oscillatory R must be at least equal to $2\sqrt{\dfrac{L}{C}}$, i.e. $\underline{12{\cdot}1\ \Omega}.$

44. For any waveform which is cyclic, repeating itself at intervals of 2π,

$$f(\theta) = A + a_1 \sin \theta + \ldots + a_n \sin n\theta + \ldots$$
$$+ b_1 \cos \theta + \ldots + b_n \cos n\theta + \ldots$$

where
$$A = \frac{1}{2\pi} \int_0^{2\pi} f(\theta)\, d\theta$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \sin n\theta\, d\theta$$

and
$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(\theta) \cos n\theta\, d\theta.$$

In the case of a half-wave rectifier $f(\theta) = E \sin \theta$ from 0 to π and $f(\theta) = 0$ from π to 2π.
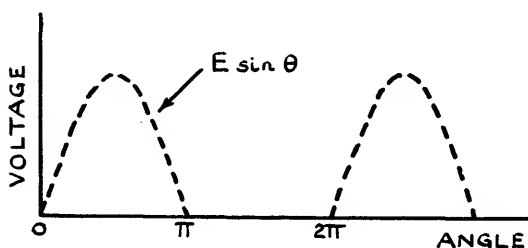
$$\therefore \qquad A = \frac{1}{2\pi} \int_0^{\pi} E \sin \theta\, d\theta = E/\pi.$$

$a_n = \dfrac{1}{\pi} \displaystyle\int_0^{\pi} E \sin \theta \sin n\theta\, d\theta$ which is zero except for $n = 1$ in which case $a_1 = E/2$.

$$b_n = \frac{1}{\pi} \int_0^{\pi} E \sin \theta \cos n\theta\, d\theta.$$

When n is odd, $b_n = 0$; when n is even $b_n = -2E/\pi(n^2 - 1)$.
The Fourier expansion is therefore

$$E \left[\frac{1}{\pi} + \frac{1}{2} \sin \theta - \frac{2}{\pi} \sum_{n=2,4,6} \frac{\cos n\theta}{(n^2 - 1)} \right].$$

The full-wave circuit consists essentially of two half-wave circuits, one circuit operates during one half-cycle and the second operates during the next half-cycle. Also analysing each half-wave separately using the above result it is seen that for the full-wave circuit the component fundamentals cancel out, the negative even cosine harmonics are coincident, and are therefore present with twice the amplitude, and it is evident that the value of the constant term is twice the value for the previous case.

The Fourier expansion for the full-wave case is therefore

$$E\left[\frac{2}{\pi} - \frac{4}{\pi} \sum_{n=2,4,6\ldots} \frac{\cos n\theta}{(n^2 - 1)}\right].$$

The r.m.s. value $= E\sqrt{[4/\pi^2 + 16/2\pi^2\{(1/3)^2 + (1/15)^2 + \ldots\}]}$
$= E/\sqrt{2}.$

45. The method of solution is the same as that already given for Problem 44 and is therefore not given again here.

46. The method of solution for the first part of the problem is the same as that already given for Problem 44 and is therefore not given again here.

The solution to the second part of the problem can be found elsewhere.*

47. The method of solution is the same as that already given for Problem 44 and is therefore not given again here.

48. The method of solution is the same as that already given for Problem 44 and is therefore not given again here.

49. The solution to this problem has been given elsewhere.†

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* W. H. Middendorf, *Analysis of Electric Circuits*, Wiley, 1956, Chapter 17.
† F. A. Benson and D. Harrison, *Electric Circuit Theory*, Arnold, 2nd Edition, 1963, pp. 183 and 184.

50. The formula for $f(t)$ is:

$$f(t) = \int_{-\infty}^{\infty} g(\omega)e^{j\omega t}d\omega$$

The relationship between the transform pairs and the Fourier series is discussed in many textbooks*

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

$$= \frac{1}{2\pi} \int_{0}^{\infty} e^{-(\alpha+j\omega)t}dt$$

$$= \frac{1}{2\pi} \left[\frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)} \right]_0^{\infty}$$

i.e.

$$g(\omega) = 1/2\pi(\alpha + j\omega)$$

For amplifier of complex gain $G(\omega)$ spectrum is $G(\omega)\,g(\omega)$ and output is:

$$F(t) = \mathscr{L}_{\alpha \to 0} \int_{-\infty}^{+\infty} G(\omega)g(\omega)e^{j\omega t}d\omega$$

or

$$F(t) = \mathscr{L}_{\alpha \to 0} \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{G(\omega)}{(\alpha + j\omega)} e^{j\omega t}d\omega$$

51. (a) The permissible energy levels W_n may be expressed as†:

$$W_n = -(13{\cdot}6\, Z^2/n^2)\text{eV}$$

where Z is the atomic number of the atom. For a hydrogen atom $Z = 1$.

The lowest energy state ($n = 1$) for hydrogen is therefore

$$-13{\cdot}6 \text{ eV}$$

When $n = 2$, $W_2 = -(13{\cdot}6/4)$ eV $= -3{\cdot}4$ eV

When $n = 3$, $W_3 = -(13{\cdot}6/9)$ eV $= -1{\cdot}51$ eV

When $n = 4$, $W_4 = -(13{\cdot}6/16)$ eV $= -0{\cdot}85$ eV

* See, for example, R. E. Scott, *Linear Circuits* (complete), Addison Wesley, 1960, Chapter 20.

† See, for example, D. J. Harris and P. N. Robson, *Vacuum and Solid State Electronics*, Pergamon, 1963, p. 61.

(b) Energy released $= (13 \cdot 6 - 3 \cdot 4)$ eV $= 10 \cdot 2$ eV

Frequency of radiation (f) is given by:

$$hf = (10 \cdot 2 \times 1 \cdot 602 \times 10^{-19}) \text{ J}$$

$$\therefore \qquad f = \frac{10 \cdot 2 \times 1 \cdot 602 \times 10^{-19} \text{ c/s}}{6 \cdot 624 \times 10^{-34}}$$

$$= \underline{2 \cdot 465 \times 10^{15} \text{ c/s}}$$

52. The conductivity σ of a semiconductor is given by*:

$$\sigma = e(p\mu_p + n\mu_n)$$

where $\qquad n =$ electron density

$\qquad\qquad p =$ hole density

$\qquad\qquad \mu_n =$ electron mobility

$\qquad\qquad \mu_p =$ hole mobility.

For an intrinsic semiconductor $n = p = n_i$, where n_i is the density of holes and electrons/cm³. Therefore the intrinsic conductivity σ_i is given by:

$$\sigma_i = e\, n_i(\mu_p + \mu_n) \text{ mhos/cm.}$$

For germanium

$$\sigma_i = 1 \cdot 602 \times 10^{-19} \times 2 \cdot 5 \times 10^{13}(3600 + 1700)$$

$$= \underline{0 \cdot 0212 \text{ mho/cm}}$$

The resistivity

$$\rho_i = 1/\sigma_i = 1/0 \cdot 0212 = \underline{47 \cdot 2 \text{ ohm-cm}}$$

For silicon

$$\sigma_i = 1 \cdot 602 \times 10^{-19} \times 1 \cdot 6 \times 10^{10}(1500 + 500)$$

$$= \underline{5 \cdot 12 \times 10^{-6} \text{ mho/cm}}$$

$$\rho_i = 1/(5 \cdot 12 \times 10^{-6}) = \underline{195,300 \text{ ohm-cm}}$$

53. Neglecting the effect of minority carriers the conductivity σ_n of n-type material is given by:

$$\sigma_n \simeq N_D e\, \mu_n$$

where N_D is the density of donor atoms.

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* See, for example, D. J. Harris and P. N. Robson, *Vacuum and Solid State Electronics*, Pergamon, 1963, Chapter 4.

Here $N_D = 4{\cdot}4 \times 10^{22}/10^6 = 4{\cdot}4 \times 10^{16}/cm^3$

∴ $\sigma_n \simeq 4{\cdot}4 \times 10^{16} \times 1{\cdot}602 \times 10^{-19} \times 3600$

               $\simeq 25{\cdot}37 \text{ mho/cm}$

∴ resistivity $\rho_n \simeq 1/25{\cdot}37$

               $\simeq \underline{0{\cdot}039 \text{ ohm-cm.}}$

54. Electron density $= 4{\cdot}4 \times 10^{16}/cm^3$

  Hole density $= (2{\cdot}5 \times 10^{13})^2/4{\cdot}4 \times 10^{16}$

                    $= \underline{1{\cdot}41 \times 10^{10}/cm^3.}$

55. The drift velocity v_D (cm/sec) is proportional to the electric field E (volts/cm)*

i.e. $v_D \, \alpha \, E$

or $v_D = \mu E$

where μ is the carrier mobility in $cm^2/$(volt-sec).

For germanium

  Drift velocity of holes $= 1{,}700 \times 100 = \underline{17 \times 10^4 \text{ cm/sec.}}$

  Drift velocity of electrons $= 3{,}600 \times 100 = \underline{36 \times 10^4 \text{ cm/sec.}}$

For silicon

  Drift velocity of holes $= 500 \times 100 = \underline{5 \times 10^4 \text{ cm/sec.}}$

  Drift velocity of electrons $= 1{,}500 \times 100 = \underline{15 \times 10^4 \text{ cm/sec.}}$

56. The Einstein relation between mobility μ and diffusion constant D is:

$$D = kT\mu/e$$

where k is Boltzmann's constant and T is the absolute temperature.

  (*a*) For holes,

  $D_p = 1{\cdot}38 \times 10^{-23} \times 300 \times 1{,}700/1{\cdot}602 \times 10^{-19}$

       $= \underline{44 \text{ cm/sec.}}$

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* See D. J. Harris and P. N. Robson, *Vacuum and Solid State Electronics*, Pergamon, 1963, p. 79.

For electrons,

$$D_n = 1\cdot38 \times 10^{-23} \times 300 \times 3{,}600/1\cdot602 \times 10^{-19}$$
$$= \underline{93\cdot1 \text{ cm/sec.}}$$

(b) For holes,

$$D_p = 1\cdot38 \times 10^{-23} \times 300 \times 500/1\cdot602 \times 10^{-19}$$
$$= \underline{12\cdot9 \text{ cm/sec.}}$$

For electrons,

$$D_n = 1\cdot38 \times 10^{-23} \times 300 \times 1{,}500/1\cdot602 \times 10^{-19}$$
$$= \underline{38\cdot8 \text{ cm/sec.}}$$

57. The diffusion lengths for holes (L_p) and electrons (L_n) may be expressed as:*

$$L_p = \sqrt{D_p\, t_p}$$
$$L_n = \sqrt{D_n\, t_n}$$

where D_p = diffusion constant for holes

D_n = diffusion constant for electrons

t_p = average lifetime of a hole

t_n = average lifetime of an electron.

$\therefore \qquad t_p = L_p{}^2/D_p = (0\cdot1)^2/44 \quad = 2\cdot27 \times 10^{-4} \text{ sec}$
$$= \underline{227\ \mu\text{s}}$$

and $\qquad t_n = L_n{}^2/D_n = (0\cdot1)^2/93\cdot1 = 1\cdot07 \times 10^{-4} \text{ sec}$
$$= \underline{107\ \mu\text{s.}}$$

58. The solution to this problem can be found elsewhere.†

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* See, for example, D. J. Harris and P. N. Robson, *Vacuum and Solid State Electronics*, Pergamon, 1963, p. 235.

† See, for example, M. V. Joyce and K. K. Clarke, *Transistor Circuit Analysis*, Addison–Wesley, 1961, p. 9, or D. J. Harris and P. N. Robson, *Vacuum and Solid State Electronics*, Pergamon, 1963, pp. 107 and 243.

59. The solution to the first part of the problem can be found elsewhere.*

$$J = J_s\{\exp(eV/kT) - 1\}$$

$\therefore \qquad \exp(eV/kT) - 1 = 5/25 \times 10^{-6} = 2 \times 10^5$

$\therefore \qquad (eV/kT) = \log_e(2 \times 10^5) = 12{\cdot}21$

$\therefore \qquad V = \dfrac{12{\cdot}21 \times 1{\cdot}38 \times 10^{-23} \times 300}{1{\cdot}602 \times 10^{-19}} = \underline{0{\cdot}316 \text{ volt}}$

60. The derivation of the formula can be found elsewhere.† The formula is:

Hall coefficient $\underline{R = (p\mu_p{}^2 - n\mu_n{}^2)/e(p\mu_p + n\mu_n)^2}$

(a) For intrinsic material, $p = n = n_i$

$\therefore \qquad R = \dfrac{n_i(\mu_p{}^2 - \mu_n{}^2)}{en_i{}^2(\mu_p + \mu_n)^2}$

i.e. $\qquad \underline{R = (\mu_p - \mu_n)/en_i(\mu_p + \mu_n)}$

(b) For the highly-doped n-type material, $n \gg p$

$\therefore \qquad R = \dfrac{1}{e}\left(\dfrac{-n\mu_n{}^2}{n^2\mu_n{}^2}\right) = \underline{-1/ne}$

61. Let the voltage across the diode be V_a and the current through the diode I_a mA,

$\therefore \qquad V_a = 200 - 20\left(\dfrac{V_a}{60} + I_a\right)$, i.e. $V_a = 150 - 15I_a$

which is the equation of the load line.

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* See, for example, D. J. Harris and P. N. Robson, *Vacuum and Solid State Electronics*, Pergamon, 1963, pp. 116–118.

† See, for example, W. Shockley, *Electrons and Holes in Semiconductors*, D. van Nostrand, 1956, p. 217.
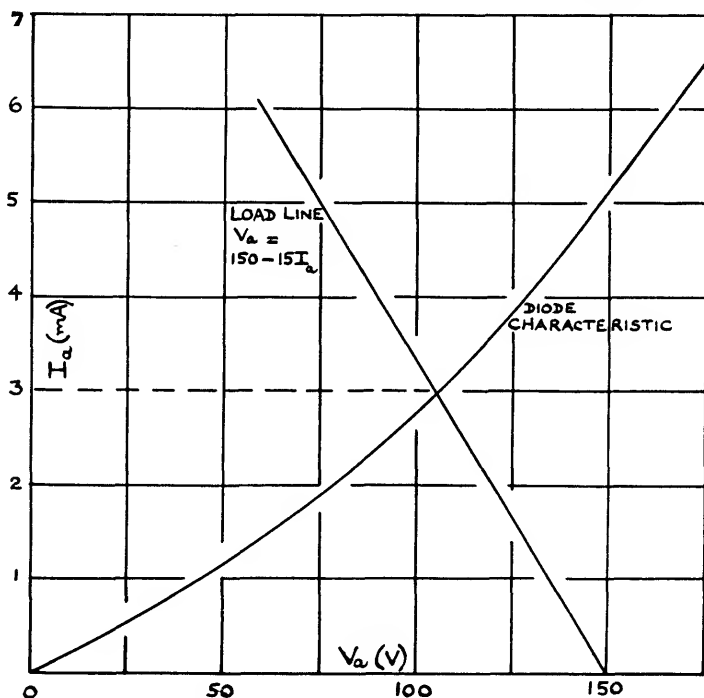
At the point of intersection of the I_a/V_a curve and the load line $I_a = 3$ mA.

62. The static characteristic has been plotted from the given figures.

$$V_i = V_a + R_l I_a \qquad . \qquad . \qquad . \qquad . \qquad (1)$$

where $\qquad\qquad\qquad R_l = 2{,}500\ \Omega.$

If (1) is plotted on the same sheet as the static curve a straight line (the load line) results.

A typical load line (for $V_i = 40$ V) is shown. The point of intersection of the load line with the static curve, P, indicates the current flowing. Thus the dynamic curve can be plotted because the current is that corresponding to P when the input voltage is 40 V and so the first point on the dynamic curve is P'.
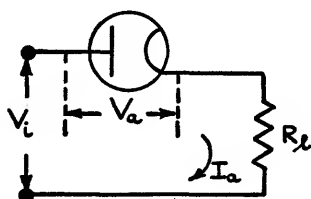
Hence, when the supply voltage is 50 V the load current is 14·5 mA.
The voltage across the load $= 2,500 \times (14·5 \times 10^{-3}) = \overline{36·25 \text{ V.}}$

From the static curve and the 50-V load line the voltage across
the diode is 13·75 V. The voltage across the load is therefore
$(50 - 13·75) \text{ V} = \overline{36·25 \text{ V}}$ which agrees with the result already
obtained.
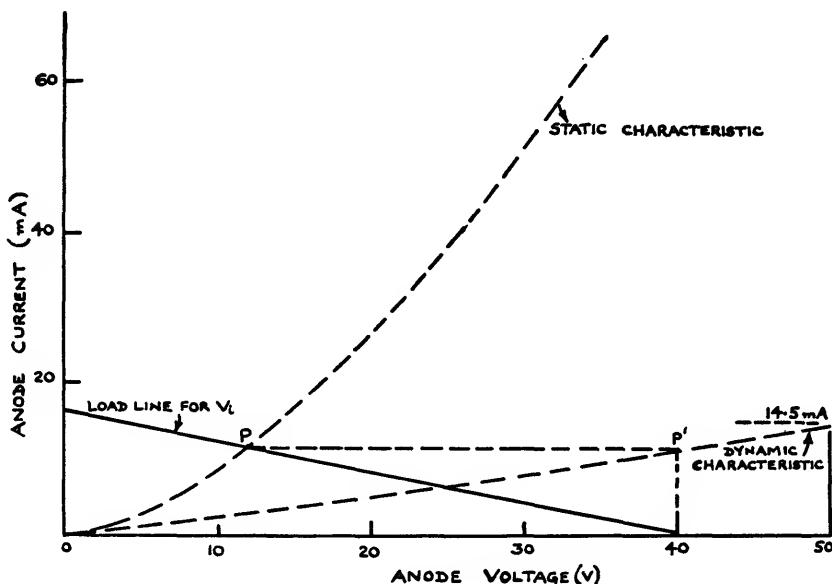
63. If $I_a = KV_a{}^n$

$$\log_{10} I_a = \log_{10} K + n \log_{10} V_a.$$

Thus, if $\log_{10} I_a$ is plotted against $\log_{10} V_a$ a straight line should
result with a slope n and an intercept on the $\log_{10} I_a$ axis of $\log_{10} K$.

In the present case $\log_{10} I_a$ plotted against $\log_{10} V_a$ does give a straight line (the plot is not shown) and the slope $n \simeq 1 \cdot 14$. K is found to be $\simeq 1$.

64. The control characteristic is shown below.
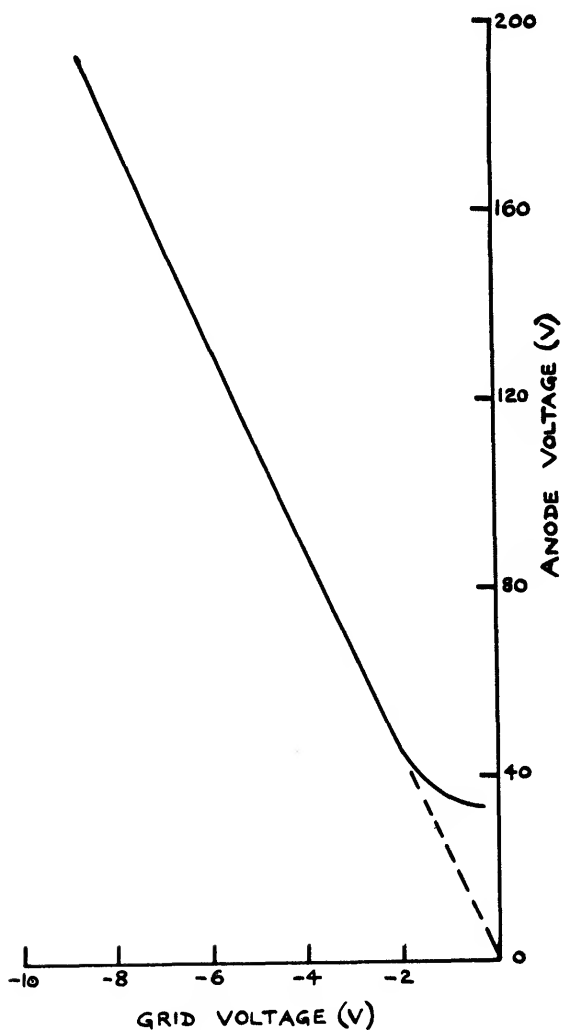
Control ratio $= \dfrac{\text{Anode Voltage}}{\text{Grid Voltage}}$ at striking which is about $21 \cdot 8$.

65. The control characteristic can be plotted since it passes through the point where the anode voltage is 200 V and the grid voltage is − 8 V and it has a slope of 35. If this is done it is found that when the grid voltage is − 20 V the anode voltage is 620 V and when the anode voltage is 340 V the grid voltage is − 12 V.

66. The control characteristics are as shown. From these, when $V_a = 400$ V, the change in critical grid voltage required is seen to be about 2·8 V.
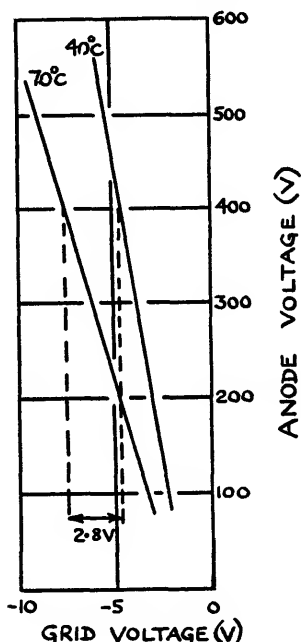
67. When the temperature is 40°C and $V_g = -4$ V, $V_a = 320$ V. But $V_a = 350 \sin \theta$, therefore $\theta = \sin^{-1}(320/350) = 66° \, 5'$.

When the temperature is 70°C and $V_g = -4$ V, $V_a = 150$ V and
$$\theta = \sin^{-1}(150/350) = 25° \, 22'.$$

68. The effective mutual conductance is the sum of the individual mutual conductances since the anode currents add directly, i.e. $(2 + 5 + 3)$ mA/V = 10 mA/V.

The equivalent anode resistance is obtained by adding the individual anode resistances as one adds resistances in parallel, i.e.

$$1/(1/5{,}000 + 1/4{,}000 + 1/10{,}000)\,\Omega = \underline{1{,}818\,\Omega}.$$

The equivalent amplification factor

$$= 1{,}818 \times 10 \times 10^{-3} = \underline{18{\cdot}18}.$$

69. Thermionic emission of electrons is in accordance with the expression:

$$I = AT^2\, e^{-b/T}\ \text{amps/cm}^2,$$

where T is the absolute temperature.

For the thoriated-tungsten filament:

$$85 \times 10^{-3} = 3 \times (1{,}900)^2\, e^{-30{,}500/1{,}900} \times \text{area} \qquad . \quad (1)$$

For the pure-tungsten filament:

$$i \times 10^{-3} = 60{\cdot}2 \times (2{,}500)^2\, e^{-52{,}400/2{,}500} \times \text{area} \qquad . \quad (2)$$

Dividing (2) by (1):

$$\frac{i}{85} = \frac{60{\cdot}2 \times (2{,}500)^2\, e^{-52{,}400/2{,}500}}{3 \times (1{,}900)^2\, e^{-30{,}500/1{,}900}}\ \text{and}\ \underline{i = 21{\cdot}8\ \text{mA}.}$$

70. The formula of the previous solution is used.

$$\frac{10{,}000}{1} = \frac{A(2{,}100)^2\, e^{-b/2{,}100}}{A(1{,}600)^2\, e^{-b/1{,}600}} \qquad . \qquad . \qquad . \quad (1)$$

Also, $b = 11{,}600\,\phi$ where ϕ is the work function . . (2)

From (1) and (2), $\underline{\phi = 5\ \text{V}.}$

71. The Child–Langmuir equation for plane-parallel electrodes gives* $J = 2{\cdot}34 \times 10^{-6}\, V_a^{3/2}/d^2$, where J is the current density in amperes/m², V_a is the anode voltage in volts and d is the anode-cathode distance.

Here $V_a = 200$ V, $d = 0{\cdot}2$ cm so $\underline{J = 165\ \text{mA/sq. cm}.}$

72. The current is given by the following expression:†

$$I = 1{\cdot}47 \times 10^{-5}\, V_a^{3/2}\, l/r_a\beta^2\ \text{amperes}$$

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* This equation is proved in many textbooks. For example, see S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, p. 12, or P. Parker, *Electronics*, Arnold, 1950, pp. 93–6; or K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, pp. 170–1.

† For example, see S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, p. 12, or P. Parker, *Electronics*, Arnold, 1950, pp. 98–9, or K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, p. 176.

where V_a is the anode voltage, l is the active length of the valve, r_a is the anode radius and β^2 is a quantity that is determined from the ratio of anode radius to cathode radius (r_f).*

For the first valve $r_a = 2$ mm, $l = 2$ cm, $r_f = 0.05$ mm and $V_a = 25$ V. Thus $r_a/r_f = 40$ and $\beta^2 = 1.0946$,

$\therefore \quad I = 17$ mA.

For the second valve $r_f = 0.75$ mm, $r_a/r_f = 2.67$ and $\beta^2 \simeq 0.45$,

$\therefore \quad I = 41$ mA.

73. The solution to this problem can be found in many textbooks.†

74. Power radiated $= W'ld = 263.0 \times 2 \times 0.025 = 13.17$ W.

Resistance $= R'l/d^2 = 98.66 \times 10^{-6} \times 2/(0.025)^2 = 0.3155\ \Omega$.

Filament current $= I_f' \times d^{3/2} = 1{,}632 \times (0.025)^{3/2} = 6.45$ A.

Voltage drop $= V_f' \times l/d^{1/2} = 161.1 \times 10^{-3} \times 2/(0.025)^{1/2}$
$$= 2.04 \text{ V.}$$

Emission current $= I_e'ld = 2.25 \times 2 \times 0.025 = 0.1125$ A.

Life for 10% reduction in mass
$$= \left(\frac{\text{volume} \times \text{density}}{10 \times 3{,}600 \times M}\right) \text{hours}$$

where $\qquad M = M'ld = 2.76 \times 10^{-8} \times 0.025 \times 2.$

$\therefore$ life is 376 hr.

$$\beta = \alpha - \frac{2}{5}\alpha^2 + \frac{11}{120}\alpha^3 - \frac{47}{3{,}300}\alpha^4 + \cdot\cdot$$

where $\alpha = \log_e (r_a/r_f)$.

Values of β^2 corresponding to various values of the ratio (r_a/r_f) have been plotted in Parker's book, Fig. 82, and tabulated in Appendix 11 of that book.

† E.g. see K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, pp. 181–2; or P. Parker, *Electronics*, Arnold, 1950, pp. 99–100.

75. The energy diagram is obtained from the voltage diagram by multiplying each ordinate on (a) by $-e$.

From the diagram, $x/4 = 2/8$ so $\underline{x = 1 \text{ cm}}$.
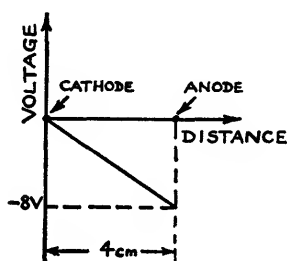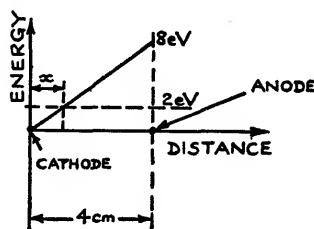
(a.)   (b)

76. The current I under the condition of an accelerating field of E volts/m at the cathode surface is* $I_1 \, e^{+0.44E^{1/2}/T}$, where I_1 is the zero-field thermionic current and T is the absolute temperature of the cathode.

$\therefore \quad \log_{10}(I/I_1) = 0.4343 \times 0.44 \times (10^6)^{1/2}/2{,}600 = 0.07345.$

thus $I/I_1 = 1.184$, which shows that the Schottky theory predicts an increase of $\underline{18.4\%}$ over the zero-field emission current.

77. (a) The amplification factor $\mu = -2\pi a_2/a_g \log_e(2 \sin \pi r_w/a_g)$ where a_2 is the grid-anode spacing, a_g is the grid-wire spacing and r_ω is the grid-wire radius.†

Since a_g is large compared with r_ω, $\mu \simeq 2\pi a_2/a_g \log_e(a_g/2\pi r_\omega)$.

Now $a_2 = 0.19$ cm, $a_g = 0.127$ cm and $r_\omega = 0.0064$ cm.

$\therefore \quad \underline{\mu \simeq 8.}$

(b) The amplification factor $\mu \simeq 2\pi r_g \log_e(r_a/r_g)/a_g \log_e(a_g/2\pi r_w)$ where r_a is the anode radius, r_g is the radius of the grid-wire circle, r_ω is the radius of the grid wire and a_g is the linear distance between the grid-wire centres at radius r_g.‡

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* For the proof of this expression see J. Millman and S. Seely, *Electronics*, McGraw-Hill, 1951, Section 5–19, pp. 151–6.

† For the proof of this see, for example, K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, pp. 125–8.

‡ *ibid*, pp. 135–7.

If $N = 1/a_g$, $\mu \simeq 2\pi N r_g \log_e (r_a/r_g)/\log_e (1/2\pi N r_\omega)$.

Here, $\mu = 20$, $r_a = 1\cdot05$ cm, $r_g = 0\cdot5$ cm and $r_\omega = 0\cdot04$ cm, so $N \simeq 3$.

Total number of grid wires $= 2\pi N r_g \simeq 10$.

(c) The expressions are:

(i) *For plane-electrode triode,*

$$\mu = \{2\pi a_2/a_g - \log_e \cosh (2\pi r_\omega/a_g)\}/\{\log_e (\coth 2\pi r_\omega/a_g)\}$$

where a_2 is the grid-anode distance, a_g is the grid-wire spacing and r_ω is the grid-wire radius.

(ii) *For cylindrical triode,*

$$\mu = \{2\pi N r_g \log_e (r_a/r_g) - \log_e (\cosh 2\pi N r_\omega)\}/\log_e \coth (2\pi N r_\omega)$$

where the symbols have the same meaning as in the solution to the previous problem.

The derivations of the expressions can be found elsewhere.*

78.
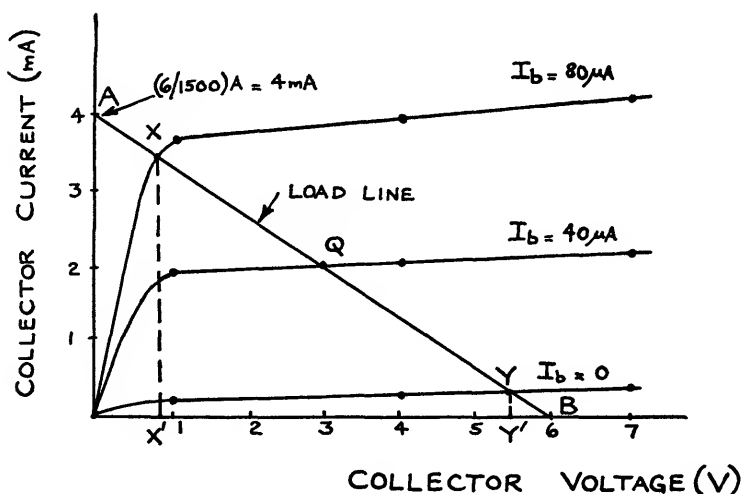
The load line passes through the points A (0, 4 mA) and B (6 V, 0). The quiescent working point is at Q. When an input signal of 40 μA

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* See F. B. Vodges and F. R. Elder, 'Formulas for the Amplification Constant for Three-element Tubes,' *Phys. Rev.*, **24**, pp. 683-9, 1924. W. G. Dow, *Fundamentals of Engineering Electronics*, Wiley, 2nd Edition, 1952, Chapter 4. K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, pp. 142–52.

peak current is applied, the peak-to-peak input signal will be 80 μA; the base current will vary between 0 and 80 μA. The extremes of the working range are given by points X and Y.

Peak-to-peak collector-emitter voltage excursion is $X'Y' \simeq \underline{4 \cdot 7 \text{ V.}}$

At Q, collector current $\simeq 2$ mA.

Power supplied by battery $= (2 \times 6) \text{ mW} = 12 \text{ mW}$.

Power dissipated as heat in 1,500-Ω load $= (2^2 \times 10^{-6} \times 1500) \text{W}$ $= 6 \text{ mW}$.

$\therefore$ power dissipated in the transistor itself $= (12 - 6) \text{ mW} = \underline{6 \text{ mW.}}$

79. Consider first the I_c/V_{cb} characteristics.

With the collector-base voltage constant at $- 4$ V a change in I_e from 1 mA to 5 mA gives a change in collector current from $- 1 \cdot 03$ mA to $- 4 \cdot 95$ mA.

Thus, $\alpha = - \left\{ - \dfrac{(4 \cdot 95 - 1 \cdot 03)}{(5 - 1)} \right\} = \underline{0 \cdot 98}$

Consider now the I_c/V_{ce} curves and a constant value of collector-emitter voltage of $- 4$ V. A change of I_b from $- 20 \ \mu$A to $- 80 \ \mu$A gives a change of collector current from $- 1 \cdot 1$ mA to $- 4 \cdot 5$ mA.

Thus, $\alpha' = \dfrac{- (4 \cdot 5 - 1 \cdot 1) \cdot 10^{-3}}{- (80 - 20) \cdot 10^{-6}} \simeq \underline{57}$

$$\alpha' = (\partial i_c / \partial i_b)$$

But $\partial i_b = - (\partial i_e + \partial i_c)$

so $\alpha' = - \partial i_c / (\partial i_e + \partial i_c)$

$$= - (\partial i_c / \partial i_e) / (\partial i_e / \partial i_e + \partial i_c / \partial i_e)$$

i.e. $\alpha' = \alpha / (1 - \alpha)$

From this expression for α' it is seen that:

$$\alpha = \alpha' / (1 + \alpha')$$

This equation can also be obtained directly from the definition of α, substituting $- (\partial i_b + \partial i_c)$ for ∂i_e and dividing each term in the numerator and denominator by ∂i_b.

80. Current gain $\alpha' = \delta I_c / \delta I_b$

When $V_c = -5$ V and $I_b = -70$ μA, $I_c = 2\cdot46$ mA

When $V_c = -5$ V and $I_b = -50$ μA, $I_c = 1\cdot72$ mA

$\therefore$ $\quad$ gain $= \dfrac{(2\cdot46 - 1\cdot72)\,10^{-3}}{(70 - 50)\,10^{-6}} = \underline{37}$

The load line is as shown. It passes through the points $V_c = -9$, $I_c = 0$ and $V_c = 0$, $I_c = \dfrac{9 \times 10^3}{1800}$ mA (i.e. $I_c = 5$ mA)

For $V_c = -4$ V, operating point is Q where $I_b \simeq \underline{-82\ \mu\text{A}}$.
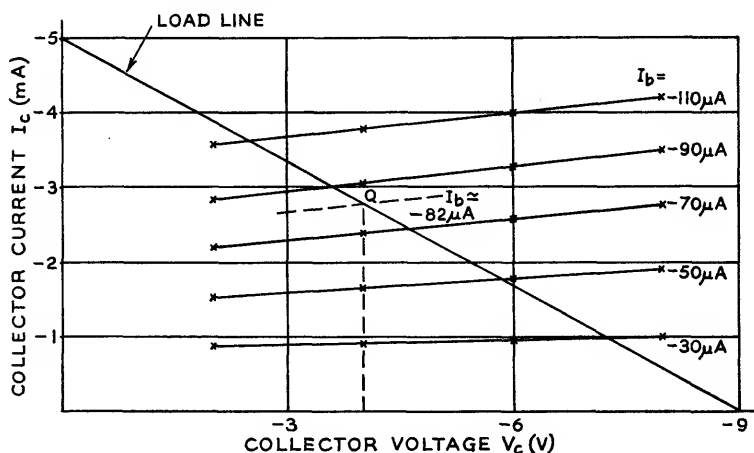
81. The equivalent circuit is as shown.
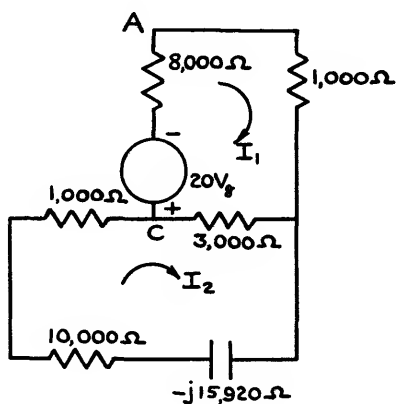
Reactance of capacitance

$$= \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 2{,}000 \times 0{\cdot}005} = 15{,}920\,\Omega.$$

Let currents I_1 and I_2 in milliamps circulate as shown.

For the I_2 mesh:

$$(10 + 1 + 3 - j15{\cdot}92)I_2 - 3I_1 = 0 \qquad . \qquad . \quad (1)$$

For the I_1 mesh:

$$(8 + 1 + 3)I_1 - 3I_2 + 20V_g = 0 \ . \qquad . \qquad . \quad (2)$$

Also, $\qquad\qquad\qquad V_g = 1 + I_2 \qquad . \qquad . \qquad . \qquad . \quad (3)$

From (1), (2) and (3) $I_2 = (-\,0{\cdot}1556 - j0{\cdot}1357)$ mA.

The capacitor blocks the d.c. and the meter reads the product of its resistance and the a.c. through it, i.e. I_2.

$\therefore \qquad$ meter reads $10[0{\cdot}1556^2 + 0{\cdot}1357^2]^{1/2} = \underline{2{\cdot}06\text{ V}}.$

82. The equivalent circuit is shown.

Let currents I_1 and I_2 in milliamps circulate as shown.

For the I_1 mesh:

$$(5 + 20)I_1 - 20I_2 + 20V_g = 0 \ . \qquad . \qquad . \quad (1)$$

For the I_2 mesh:

$$(20 + 5)I_2 - 20I_1 = e = 0{\cdot}2 \qquad . \qquad . \qquad . \quad (2)$$

Also, $\qquad\qquad\qquad V_g = 10(I_1 - I_2) \qquad . \qquad . \qquad . \qquad . \quad (3)$
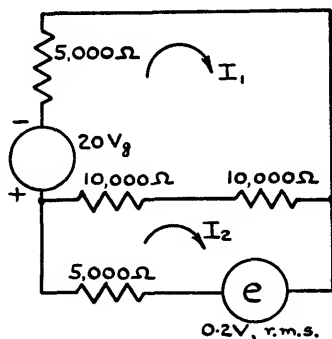
From (1), (2) and (3): $\underline{I_1 = 35{\cdot}9\ \mu\text{A}}.$

83. The equivalent circuit is as shown. Let the currents I_1, I_2 and I_3 circulate as shown.
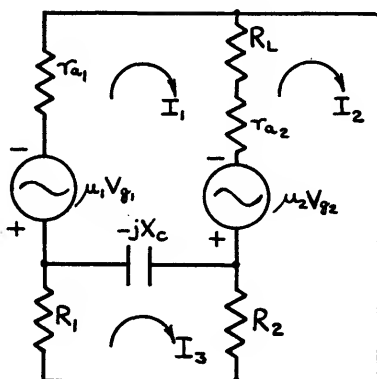
If
$$e_1 = 1 + j0$$
$$e_2 = 2 (\cos 30° + j \sin 30°)$$
$$= 1\cdot73 + j1.$$

For the I_1 mesh:

$$(r_{a_1} + R_L + r_{a_2} - jX_c)I_1$$
$$- (R_L + r_{a_2})I_2 - (-jX_c)I_3 + \mu_1 V_{g_1} - \mu_2 V_{g_2} = 0 \quad (1)$$

For the I_2 mesh:

$$(R_L + r_{a_2} + R_2)I_2 - (R_L + r_{a_2})I_1 - R_2 I_3 + \mu_2 V_{g_2} = 0 \qquad . \quad (2)$$

For the I_3 mesh—

$$(R_1 - jX_c + R_2)I_3 - (-jX_c)I_1 - R_2 I_2 = 0 \ . \qquad . \quad (3)$$

Also, $V_{g_1} = e_1 + R_1 I_3 = 1 + R_1 I_3 \ . \qquad . \qquad . \qquad . \quad (4)$

and $\quad V_{g_2} = e_2 + R_2(I_2 - I_3) = 1\cdot73 + j1 + R_2(I_2 - I_3) \quad . \quad (5)$

$\therefore \ I_1$, I_2 and I_3 can be found.

84. For a triode, a change δI_a in the anode current I_a can be written

$$\left(\frac{\partial I_a}{\partial V_a}\right)_{V_g \text{ const.}} \delta V_a + \left(\frac{\partial I_a}{\partial V_g}\right)_{V_a \text{ const.}} \partial V_g = \frac{1}{r_a} \cdot \delta V_a + g_m \cdot \delta V_g.$$

$\therefore \qquad\qquad g_m \cdot \delta V_g = \delta I_a - \delta V_a / r_a.$
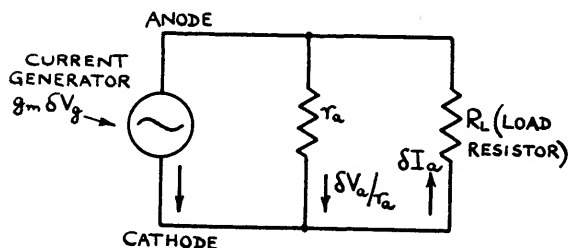
The current-source equivalent circuit shown follows from this expression.

85. The equivalent circuit of the arrangement is as shown.
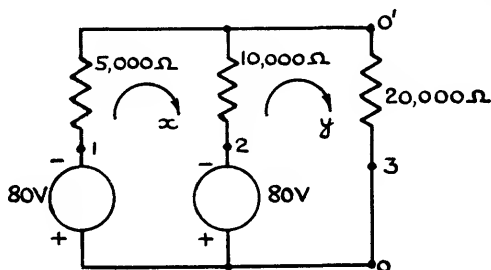
Millman's Theorem* states that

$$V_{00}' = \frac{V_{01} Y_1 + V_{02} Y_2 + V_{03} Y_3}{Y_1 + Y_2 + Y_3}$$

where V_{00}' is the voltage drop from 0 to 0'

V_{01} ,,   ,,   ,,   ,, 0 to 1, etc.

In this case $V_{01} = 80$, $V_{02} = 80$, $V_{03} = 0$, $Y_1 = 1/5,000$, $Y_2 = 1/10,000$, $Y_3 = 1/20,000$.

∴ $V_{00}' = 68.6$ V.

Let the currents in the two meshes be x and y mA.

For the x mesh: $5x + 10(x - y) - 80 + 80 = 0$.

,,   ,, y ,, $10(y - x) + 20y + 80 = 0$.

From these equations $x = -2.3$ mA, $y = -3.4$ mA.

The valve currents are $-x = 2.3$ mA and $x - y = 1.1$ mA.

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* See J. Millman, 'A Useful Network Theorem,' *Proc. I.R.E.*, **28**, pp. 413–17, 1940, and F. A. Benson, *Electrical Engineering Problems with Solutions*, Spon, 1954, pp. 104–9.

86. (*a*) The equivalent circuit is as shown.
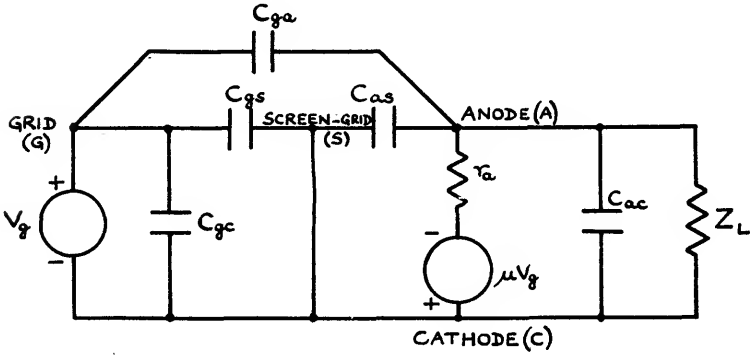
Neglecting C_{ga}, this simplifies to the following:
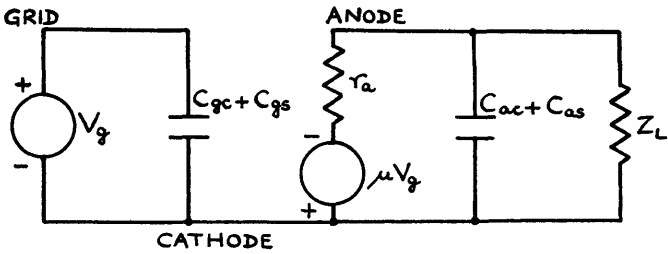
(*b*) The equivalent circuit is as shown.
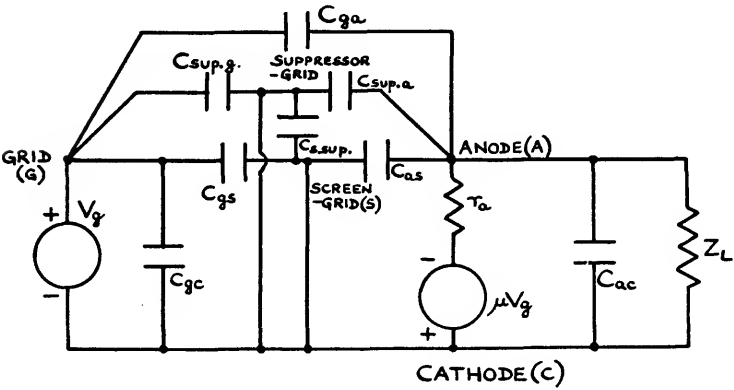
Neglecting C_{ga} and $C_{\text{sup.g.}}$, this simplifies to the following:
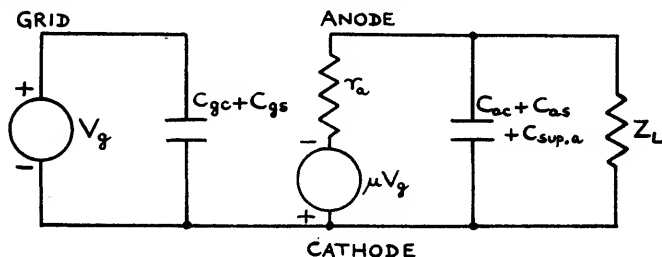
87. Consider the common-base transistor connection. The emitter and collector voltages, V_e and V_c, measured with respect to the base, are functions of the independent variables I_e and I_c, the emitter and collector currents,

i.e. $\qquad\qquad V_c = f_1(I_e, I_c)$ . . . . (1)

and $\qquad\qquad V_c = f_2(I_e, I_c)$ . . . . (2)

For small-signal variations the voltage variations are given by:

$$\delta V_e = \left(\frac{\partial V_e}{\partial I_e}\right)_{I_c} \delta I_e + \left(\frac{\partial V_e}{\partial I_c}\right)_{I_e} \delta I_c \qquad . \qquad . \text{ (3)}$$

and $\qquad \delta V_c = \left(\frac{\partial V_c}{\partial I_e}\right)_{I_c} \delta I_e + \left(\frac{\partial V_c}{\partial I_c}\right)_{I_e} \delta I_c \qquad . \qquad . \text{ (4)}$

If δV_e, δV_c, δI_e and δI_c are written as v_e, v_c, i_e and i_c respectively, these equations may be written as:

$$v_e = r_{11} i_e + r_{12} i_c \qquad . \qquad . \qquad . \qquad . \text{ (5)}$$
$$v_c = r_{21} i_e + r_{22} i_c \qquad . \qquad . \qquad . \qquad . \text{ (6)}$$

where the coefficients r_{11}, r_{12}, r_{21} and r_{22} are defined as:

$$r_{11} = \left(\frac{\partial V_e}{\partial I_e}\right)_{I_c} \qquad . \qquad . \qquad . \qquad . \text{ (7)}$$

$$r_{12} = \left(\frac{\partial V_e}{\partial I_c}\right)_{I_e} \qquad . \qquad . \qquad . \qquad . \text{ (8)}$$

$$r_{21} = \left(\frac{\partial V_c}{\partial I_e}\right)_{I_c} \qquad . \qquad . \qquad . \qquad . \text{ (9)}$$

and $\qquad\qquad r_{22} = \left(\frac{\partial V_c}{\partial I_c}\right)_{I_e} \qquad . \qquad . \qquad . \text{ (10)}$

It is possible to draw several equivalent circuits which satisfy equations (5) and (6). These four-terminal networks are active, not passive, so four independent parameters are needed to specify their performances. In some equivalent circuits the four parameters used are r_e, r_b, r_c and r_m (or α). By comparing the mesh equations for the various networks it is easily shown that*:

$$r_{11} = r_e + r_b \qquad . \qquad . \qquad . \qquad . \quad (11)$$

$$r_{12} = r_b \qquad . \qquad . \qquad . \qquad . \quad (12)$$

$$r_{21} = r_b + r_m \qquad . \qquad . \qquad . \quad (13)$$

$$r_{22} = r_b + r_c \qquad . \qquad . \qquad . \quad (14)$$

and
$$\alpha = r_{21}/r_{22} \qquad . \qquad . \qquad . \quad (15)$$

Equations (5) and (6) can be re-arranged to give the voltage v_e and current i_c in terms of i_e and v_c. The h parameters (or hybrid parameters) are then defined by these equations as follows:

$$v_e = h_{11}i_e + h_{12}v_c \qquad . \qquad . \qquad . \quad (16)$$

$$i_c = h_{21}i_e + h_{22}v_c \qquad . \qquad . \qquad . \quad (17)$$

Similar parameters may be defined for common-emitter and common-collector arrangements; these are frequently distinguished by single and double dashes respectively (e.g. h_{11}' and h_{11}''). The relationships between the h and r parameters can easily be determined as follows:

From equation (5), $r_{11} = v_e/i_e$ with $i_c = 0$. Under this condition:

$$v_e = h_{11}i_e + h_{12}v_c$$

and
$$0 = h_{21}i_e + h_{22}v_c$$

Thus,
$$r_{11} = v_e/i_e = (h_{11}h_{22} - h_{12}h_{21})/h_{22} \qquad . \qquad . \quad (18)$$

Similarly,
$$r_{21} = v_c/i_e \text{ when } i_c = 0$$

Then,
$$0 = h_{21}i_e + h_{22}v_c$$

$\therefore$
$$r_{21} = - h_{21}/h_{22} \qquad . \qquad . \qquad . \quad (19)$$

Also,
$$r_{12} = v_e/i_c \text{ with } i_e = 0$$

* See, for example, S. Seely, *Electronic Engineering*, McGraw-Hill, 1956, Chapter 16 or L. M. Krugman, *Fundamentals of Transistors*, Rider and Chapman and Hall, 2nd Edition, 1959.

Then, $$v_e = h_{12}v_c$$

and $$i_c = h_{22}v_c$$

so $$r_{12} = h_{12}/h_{22} \qquad . \qquad . \qquad . \qquad . \ (20)$$

Finally, $$r_{22} = v_c/i_c \text{ with } i_e = 0.$$

Then, $$i_c = h_{22}v_c$$

i.e. $$r_{22} = 1/h_{22} \qquad . \qquad . \qquad . \qquad . \ (21)$$

Also, $$\alpha = r_{21}/r_{22} = -h_{21} \qquad . \qquad . \qquad . \ (22)$$

It follows that:

$$r_e = r_{11} - r_{12} = h_{11} - h_{12}(1 + h_{21})/h_{22} \qquad . \qquad . \ (23)$$

$$r_b = r_{12} = h_{12}/h_{22} \qquad . \qquad . \qquad . \qquad . \ (24)$$

$$r_c = r_{22} - r_{12} = (1 - h_{12})/h_{22} \simeq 1/h_{22} \qquad . \qquad . \ (25)$$

and $$r_m = r_{21} - r_{12} = -(h_{21} + h_{12})/h_{22} \qquad . \qquad . \ (26)$$

In the example given:

$$r_{11} = (35 \times 1 \times 10^{-6} + 7 \times 10^{-4} \times 0.976)/(1 \times 10^{-6}) = \underline{718.2 \ \Omega}$$

$$r_{12} = (7 \times 10^{-4})/(1 \times 10^{-6}) = \underline{700 \ \Omega}$$

$$r_{21} = \{0.976/(1 \times 10^{-6})\}\Omega = \underline{976 \ k\Omega}$$

$$r_{22} = 1/(1 \times 10^{-6})\Omega = \underline{1 \ M\Omega}$$

$$\alpha = \underline{-0.976}$$

$$r_e = (718.2 - 700) = \underline{18.2 \ \Omega}$$

$$r_b = \underline{700 \ \Omega}$$

$$r_c = r_{22} \simeq \underline{1 \ M\Omega}$$

$$r_m = (976,000 - 700)\Omega = \underline{975.3 \ k\Omega}$$

88. The solution to this problem can be found elsewhere.*

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* See, for example, S. Seely, *Electronic Engineering*, McGraw-Hill, 1956, Chapter 16.
See also, L. M. Krugman, *Fundamentals of Transistors*, Rider and Chapman & Hall,
2nd Edition, 1959.

89. The solution to this problem can be found elsewhere.*

90. Consider the two arrangements shown at (a) and (b).
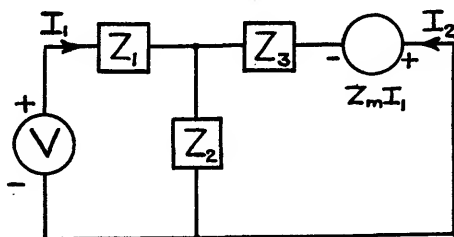
(a)
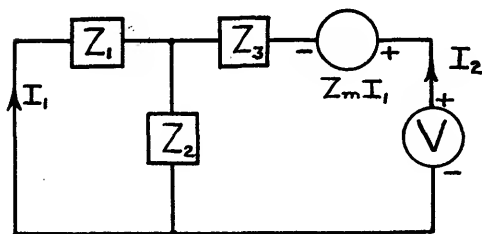
(b)

<table>
<tr><td align="center">For circuit (a)</td><td align="center">For circuit (b)</td></tr>
</table>

$(Z_1 + Z_2)I_1 + Z_2I_2 = V,$	$(Z_1 + Z_2)I_1 + Z_2I_2 = 0,$
$(Z_2 + Z_m)I_1 + (Z_2 + Z_3)I_2 = 0$	$(Z_2 + Z_m)I_1 + (Z_2 + Z_3)I_2 = V$

$$\therefore I_2 = \frac{\begin{vmatrix} (Z_1 + Z_2) & V \\ (Z_2 + Z_m) & 0 \end{vmatrix}}{\begin{vmatrix} (Z_1 + Z_2) & Z_2 \\ (Z_2 + Z_m) & (Z_2 + Z_3) \end{vmatrix}}. \quad \therefore I_1 = \frac{\begin{vmatrix} 0 & Z_2 \\ V & (Z_2 + Z_3) \end{vmatrix}}{\begin{vmatrix} (Z_1 + Z_2) & Z_2 \\ (Z_2 + Z_m) & (Z_2 + Z_3) \end{vmatrix}}.$$

In general, $I_1 \neq I_2$, so the original circuit does not satisfy the reciprocity condition. For the reciprocity condition to be satisfied Z_m must be zero.

* See, for example, S. Seely, *Electronic Engineering*, McGraw-Hill, 1956, Chapter 16. See also, L. M. Krugman, *Fundamentals of Transistors*, Rider and Chapman & Hall, 2nd Edition, 1959 and J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957, Chapter 12.

91. It is found* that α varies with frequency according to the following expressions:

$$\alpha = \alpha_0 \left\{ \frac{1}{1 + j(f/f_\alpha)} \right\}$$

where α_0 is the low-frequency value of α and f_α, called the alpha cut-off frequency, is that frequency where $\alpha = \alpha_0/\sqrt{2}$.

Thus, if $\alpha_0 = 0\cdot96$, $f_\alpha = 5$ Mc/s and $f = 10$ Mc/s,

$$\alpha = 0\cdot96/\sqrt{1 + (10/5)^2}$$
$$= \underline{0\cdot43}$$

If $\alpha_0 = 0\cdot96$, $\alpha = 0\cdot6$ and $f = 5$ Mc/s,

$$f = f\sqrt{(\alpha_0/\alpha)^2 - 1}$$
$$= (5\sqrt{(0\cdot96/0\cdot6)^2 - 1}) \text{ Mc/s}$$
$$= \underline{6\cdot25 \text{ Mc/s.}}$$

92. $\alpha' = \alpha/(1 - \alpha)$

$$= \frac{\alpha_0/\{1 + j(f/f_\alpha)\}}{1 - \alpha_0/\{1 + j(f/f_\alpha)\}} = \underline{\frac{\alpha_0}{(1 - \alpha_0) + j(f/f_\alpha)}}$$

The cut-off frequency is defined as that for which the gain falls to $1/\sqrt{2}$ of its original value. This occurs for the common-emitter circuit when the frequency is f_α' such that $f_\alpha'/f_\alpha = 1 - \alpha_0$

i.e. $$\underline{f_\alpha' = f_\alpha(1 - \alpha_0)}$$

93–98. The solutions to these problems can be found elsewhere.†

99. (*a*) Mean load current

$$(I_{\text{d.c.}}) = \frac{1}{2\pi} \int_0^\pi \frac{300\sqrt{2} \sin\theta}{(150 + 1{,}000)} \, d\theta = \underline{117 \text{ mA.}}$$

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* R. L. Pritchard, *Frequency Variations of Current-Amplication Factor for Junction Transistors*, Proc.I.R.E., **40**, p. 1476, 1952.
and
   D. E. Thomas, *Transistor Amplifier Cut-off Frequency*, ibid., **40**, p. 1481, 1952.
   † S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, Chapter 8.

(b) Alternating load current

$$(I_{r.m.s.}) = \left[\frac{1}{2\pi}\int_0^\pi \left(\frac{300\sqrt{2}\sin\theta}{1,150}\right)^2 d\theta\right]^{1/2} = \underline{184\ mA.}$$

(c) D.C. power supplied to the load $= (I_{d.c.})^2 \times 1,000 = \underline{13\cdot8\ W.}$

(d) Power supplied to anode circuit $= (I_{r.m.s.})^2 \times 1,150 = \underline{39\cdot1\ W.}$

(e) Rectification efficiency

$$= \frac{d.c.\ output\ power}{power\ supplied\ to\ anode\ circuit} \times 100\% = \underline{35\cdot3\%.}$$

(f) Ripple factor $= [(I_{r.m.s.}/I_{d.c.})^2 - 1]^{1/2} = \underline{1\cdot21.}$

100. (a) The d.c. load voltage

$$(E_{d.c.}) = \frac{1}{2\pi}\int_{\theta_1}^{\theta_2}(300\sqrt{2}\sin\theta - 10)\,d\theta$$

where θ_1 and θ_2 are the angles at the striking and extinction points. Since $300\sqrt{2} \gg 10$, θ_1 may be taken as zero and θ_2 as π.

$$\therefore \qquad\qquad E_{d.c.} = \underline{130\ V.}$$

(b) D.C. power supplied to load $= (E_{d.c.})^2/1,000 = \underline{16\cdot9\ W.}$

(c) Input power to circuit

$$= \frac{1}{2\pi}\int_0^\pi 300\sqrt{2}\sin\theta\left(\frac{300\sqrt{2}\sin\theta - 10}{1,000}\right)d\theta = \underline{43\cdot7\ W.}$$

(d) Rectification efficiency $= \dfrac{16\cdot9}{43\cdot7} \times 100\% = \underline{38\cdot7\%.}$

(e) It can be shown* that the ripple factor is

$$approximately\ 1\cdot21\left[1 + 0\cdot5 \times \frac{10}{300\sqrt{2}}\right] = \underline{1\cdot225.}$$

101. Mean load voltage $E_{d.c.} = $ Mean load current $I_{d.c.} \times R_l$, where R_l is the resistance of the load and $I_{d.c.} = \dfrac{230\sqrt{2}}{\pi(500 + R_l)}$.

$$\therefore \qquad\qquad E_{d.c.} = 230\sqrt{2}/\pi - 500 I_{d.c.}$$

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* J. Millman and S. Seely, *Electronics*, McGraw-Hill, 2nd Edition, 1951, p. 350.

$E_{\text{d.c.}}$ changes from 103·5 V when $I_{\text{d.c.}} = 0$ to 63·5 V when $I_{\text{d.c.}} = 80$ mA, i.e. regulation is $(103·5 - 63·5)$ V $= \underline{40 \text{ V}}$.

$$\text{Efficiency} = \frac{(I_{\text{d.c.}})^2 R_l}{(I_{\text{r.m.s.}})^2(R_l + 500)} \times 100\% = \frac{40·6}{1 + \dfrac{500}{R_l}} \%.$$

∴ efficiency decreases from 40·6% when $I_{\text{d.c.}} = 0$ to 24·9% when $I_{\text{d.c.}} = 80$ mA.

Maximum output power is obtained when $R_l = 500 \,\Omega$ and the efficiency is then 20·3%.

∴ the current at which maximum power is obtained is given by $20·3 = 40·6[1 - 500 I_{\text{d.c.}}/103·5]$.

∴ $$I_{\text{d.c.}} = \underline{103·5 \text{ mA}}.$$

102. (a) Mean load current $(I_{\text{d.c.}}) = \dfrac{2}{\pi}\left(\dfrac{300\sqrt{2}}{500 + 2{,}000}\right) = \underline{108 \text{ mA}}.$

(b) Alternating load current $(I_{\text{r.m.s}}) = \dfrac{1}{\sqrt{2}}\left(\dfrac{300\sqrt{2}}{500 + 2{,}000}\right)$
$$= \underline{120 \text{ mA}}.$$

(c) D.C. output power $= (I_{\text{d.c.}})^2 \times 2{,}000 = \underline{23·3 \text{ W}}.$

(d) Input power $= (I_{\text{r.m.s.}})^2(500 + 2{,}000) = \underline{36 \text{ W}}.$

(e) Rectification efficiency $= \dfrac{23·3}{36} \times 100\% = \underline{64·8\%}.$

(f) Ripple factor $= [(I_{\text{r.m.s.}}/I_{\text{d.c.}})^2 - 1]^{1/2} = \underline{0·482}.$

(g) D.C. output voltage $E_{\text{d.c.}} = \dfrac{2}{\pi} \times 300\sqrt{2} - I_{\text{d.c.}}\,500.$

∴ $E_{\text{d.c.}}$ changes from 270 V when $I_{\text{d.c.}} = 0$ to 216 V when $I_{\text{d.c.}} = 108$ mA, i.e. regulation is 54 V.

103. R.m.s. current $= 5 \text{ A} = \left[\dfrac{1}{2\pi} \displaystyle\int_0^\pi I_m{}^2 \sin^2 \theta \, d\theta \right]^{1/2} = I_m/2$

where I_m is the maximum value of the current

$\therefore$ $\qquad\qquad\qquad\qquad\qquad I_m = 10 \text{ A.}$

Moving-coil ammeter reads $\dfrac{1}{2\pi} \displaystyle\int_0^\pi I_m \sin \theta \, d\theta = \dfrac{I_m}{\pi} = \underline{3 \cdot 18 \text{ A.}}$

For full-wave rectification:

A.C. ammeter reads r.m.s. value $= I_m/\sqrt{2} = \underline{7 \cdot 07 \text{ A.}}$

Moving-coil ammeter reads mean value $= \dfrac{2}{\pi} I_m = \underline{6 \cdot 37 \text{ A.}}$

104.
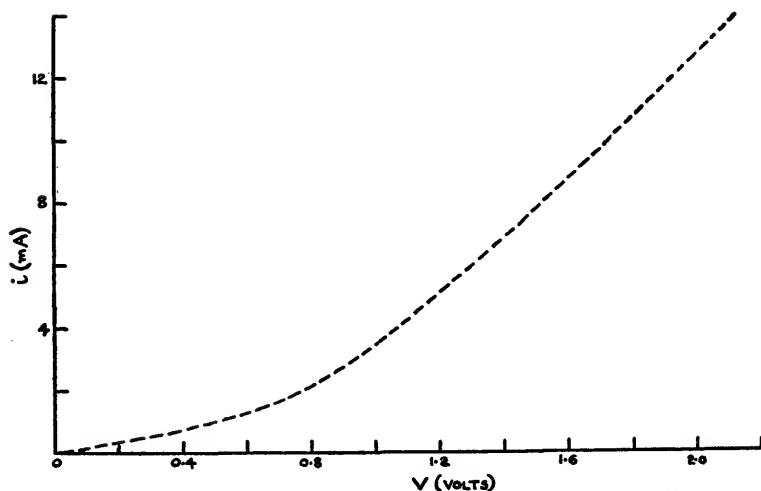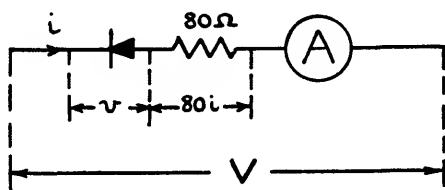
The following table can be drawn up using the rectifier characteristic:

Current i (mA) . .	2	4	6	8	10	12	14
80i volts . . .	0·16	0·32	0·48	0·64	0·80	0·96	1·12
v volts . . .	0·60	0·76	0·82	0·88	0·93	0·98	1·03
$V = (v + 80i)$ volts .	0·76	1·08	1·30	1·52	1·73	1·94	2·15

The i/V characteristic is drawn. The current wave corresponding to the positive half-cycle of voltage can then be obtained as shown. The current during the negative half-cycle is so small that it can be neglected. The moving-coil ammeter reads the mean current taken over the whole cycle. This is found to be 3·32 mA.
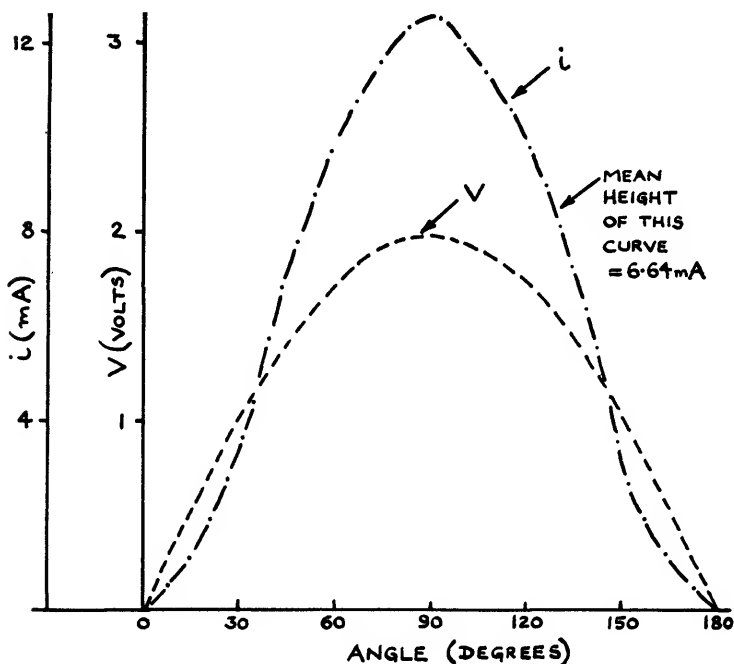
105. Current flows in each cycle for an angle θ, where $\cos \theta/2 = V_R/V$. The current through R is V_R/R which must equal the mean current through the rectifier,

$$\therefore \qquad V_R/R = \frac{1}{2\pi} \int_{-\theta/2}^{\theta/2} \frac{V(\cos \phi - \cos \theta/2)\, d\phi}{10}.$$

But θ is given as $2\pi/6$, so $\underline{R = 585\ \Omega}$.
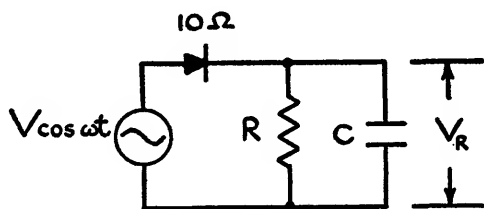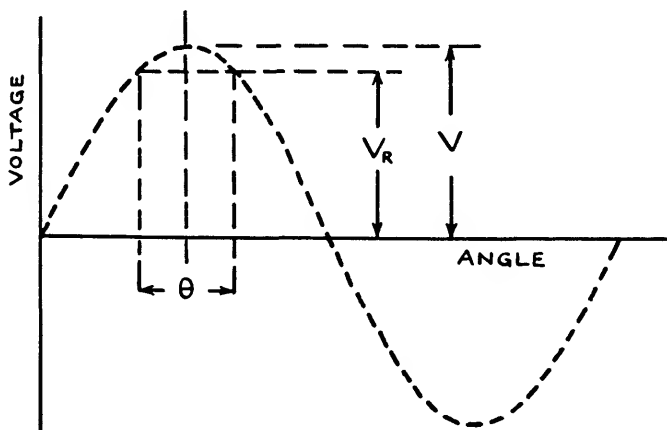
Component of fundamental frequency in the a.c. supply is

$$I_1 = \frac{1}{10\pi} \int_{-\theta/2}^{\theta/2} V(\cos \phi - \cos \theta/2) \cos \phi\, d\phi = 0{\cdot}0287 \times V/10$$

A.C. power input $= \frac{1}{2}VI_1 = \frac{1}{2}V^2 \times 0{\cdot}0287/10$

D.C. power output $= (V_R)^2/R = (V \cos \theta/2)^2/R = V^2 \left(\dfrac{\sqrt{3}}{2}\right)^2 \Big/ 585.$

$\therefore$ Efficiency of rectification

$$= \frac{\text{d.c. power output}}{\text{a.c. power input}} \times 100\% = \underline{89{\cdot}4\%}.$$

106. It is evident from the diagram that, since $200\sqrt{2} \gg 10$, conduction may be assumed to continue until the end of each positive half-cycle.
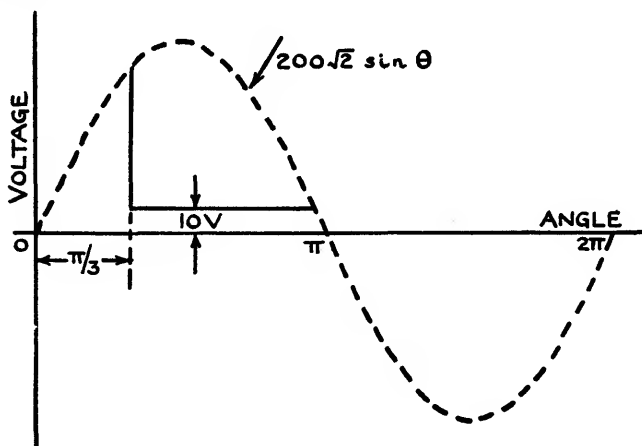
(a) R.m.s. load current

$$= \sqrt{\frac{1}{2\pi} \int_{\pi/3}^{\pi} \left(\frac{200\sqrt{2}\sin\theta - 10}{200}\right)^2 d\theta} = \underline{0.63 \text{ A.}}$$

(b) R.m.s. value of voltage across valve

$$= \sqrt{\frac{1}{2\pi} \left[\int_0^{\pi/3} (200\sqrt{2}\sin\theta)^2 \, d\theta + \int_{\pi/3}^{\pi} 10^2 \, d\theta + \int_{\pi}^{2\pi} (200\sqrt{2}\sin\theta)^2 \, d\theta \right]}$$

$$= \underline{155 \text{ V.}}$$

(c) Power

$$= \frac{1}{2\pi} \int_{\pi/3}^{\pi} \left(\frac{200\sqrt{2}\sin\theta - 10}{200}\right) (200\sqrt{2}\sin\theta) \, d\theta = \underline{77 \text{ W.}}$$

107. The filter is shown in the diagram. Any losses in the valves, transformer and choke will be neglected.
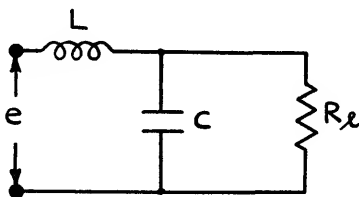
Suppose e is given by the first two terms of the Fourier-series representation of the rectifier output voltage, i.e. $\dfrac{2E_m}{\pi}\left[1 - \dfrac{2}{3}\cos 2\omega t\right]$ where E_m is the maximum value of the transformer voltage to the centre-tap.

R.m.s. value of $e(e_{\text{r.m.s.}}) = \sqrt{2}\,\dfrac{E_m}{\pi}\cdot\dfrac{2}{3}$.

$\therefore$ a.c. through the circuit is approximately $\sqrt{2}\,\dfrac{E_m}{\pi}\cdot\dfrac{2}{3}\cdot\dfrac{1}{X_L} = I_{\text{r.m.s.}}$ where $X_L = 2\pi(2f)L$ and f is the supply frequency.

$\therefore$ ripple voltage across load is approximately $I_{\text{r.m.s.}}X_C = E_{\text{r.m.s.}}$, where $X_C = 1/2\pi(2f)C$.

Ripple factor $= E_{\text{r.m.s.}}/(2E_m/\pi) = \sqrt{2}X_c/3X_L = \sqrt{2}/3(4\pi f)^2 LC = 10/300$ in this case.

$\therefore$ when $f = 50$ c/s, $LC = 35\cdot86 \times 10^{-6}$, and when $f = 60$ c/s, $LC = 24\cdot87 \times 10^{-6}$.

If the rectifier is to pass current throughout the whole cycle the peak current delivered must be less than the direct current, i.e. $4E_m/3\pi X_L \leqslant 2E_m/\pi R_l$.

The limiting condition for this is when $L = R_l/6\pi f$.

In this case $R_l = 300/0\cdot12 = 2{,}500\ \Omega$.

$\therefore$ when (a) $f = 50$ c/s, $L = 2\cdot65$H, and when (b), $f = 60$ c/s, $L = 2\cdot21$H.

The above expressions give the minimum values of L and LC that may be used to obtain the required results. Since the minimum value of $L = 2\cdot65$ H for case (a) and $2\cdot21$ H for case (b) choose a 10-H choke, in both instances. This is a readily available item and its size must be such as to carry the necessary current.

If $L = 10$ H and $f = 50$ c/s, $C = 35\cdot86 \times 10^{-6}/10 = 3\cdot586\ \mu\text{F}$.
If $L = 10$ H and $f = 60$ c/s, $C = 24\cdot87 \times 10^{-6}/10 = 2\cdot487\ \mu\text{F}$.

$\therefore$ in both cases choose a 4-μF capacitor which is also readily available.

108. (*a*) *Simple inductor filter.*

$$\text{Load current} = \frac{2E_m}{\pi R_l} - \frac{4E_m}{3\pi} \frac{\cos(2\omega t - \phi)}{\sqrt{R_l^2 + 4\omega^2 L^2}}$$

where the symbols E_m, R_l and L have the same meanings as in the previous solution, and $\tan \phi = 2\omega L / R_l$.

$$\therefore \qquad \text{the ripple factor} = \frac{\dfrac{4E_m}{3\pi\sqrt{2}} \Big/ \sqrt{R_l^2 + 4\omega^2 L^2}}{2E_m/\pi R_l}.$$

If $\omega = 100\pi$, $L = 20$ H and $R_l = 2{,}000\ \Omega$.

$$\therefore \qquad \underline{\text{the ripple factor} = 0.074.}$$

If $\omega = 120\pi$, the ripple factor $= 0.062$.

(*b*) *Simple inductor filter.*

When $L = 40$ H, the ripple factor is 0·037 for $f = 50$ c/s and 0·031 for $f = 60$ c/s.

(*c*) *Simple capacitor filter.*
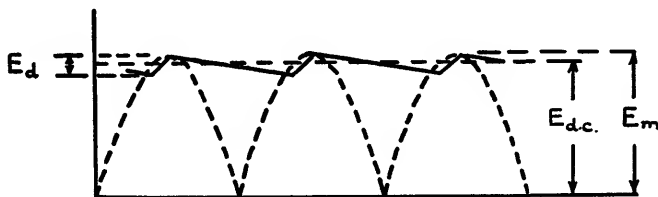
The diagram shows the voltage curves.

The r.m.s. value of the ripple voltage $e_{\text{r.m.s.}} = E_d/2\sqrt{3}$.

Assume the capacitor discharge continues for the full half-cycle at a constant rate equal to the average value of the load current $I_{\text{d.c.}}$*

In the time for half a cycle $(1/2f)$ the capacitor will lose an amount of charge $I_{\text{d.c.}}/2f$ coulombs.

$$\therefore \qquad\qquad E_d = I_{\text{d.c.}}/2fC.$$

The ripple factor $= e_{\text{r.m.s.}}/E_{\text{d.c.}} = \dfrac{I_{\text{d.c.}}/(2fC . 2\sqrt{3})}{I_{\text{d.c.}}R_l}$

---

* This gives a ripple factor for a given C which is too large. See J. Millman and S. Seely, *Electronics*, McGraw-Hill, 2nd Edition, 1951, p. 393.

In this case $C = 16\,\mu\mathrm{F}$ and $R_l = 2{,}000\,\Omega$.

$\therefore$ when $f = 50$ c/s, the ripple factor $= 0\cdot090$, and when $f = 60$ c/s, the ripple factor $= 0\cdot075$.

(d) *Simple capacitor filter.*

When $C = 32\,\mu\mathrm{F}$ the ripple factor is $0\cdot045$ for $f = 50$ c/s, and $0\cdot0375$ for $f = 60$ c/s.

(e) *Single L-type filter.*

In the previous solution it has been shown that the ripple factor for a single L-type filter is $\sqrt{2}/3(4\pi f)^2 LC$.

Substituting the values of f, L and C it is found that the ripple factor is $0\cdot0037$ for $f = 50$ c/s, and $0\cdot0025$ for $f = 60$ c/s.

(f) *Single L-type filter.*

When $L = 40$ H and $C = 32\,\mu\mathrm{F}$ the ripple factor is $0\cdot0009$ for $f = 50$ c/s, and $0\cdot0006$ for $f = 60$ c/s.

(g) *Double L-type filter.*

The reactances of the chokes are much larger than the reactances of the capacitors. Assume reactance of C small compared with R_l.
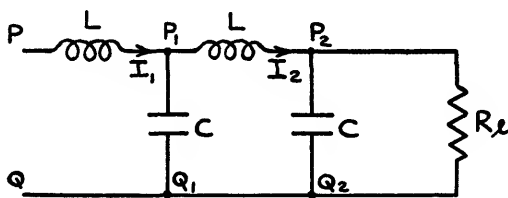
$\therefore$ impedance between P_2 and Q_2 is approximately $X_C = \dfrac{1}{2\pi(2f)C}$

   ,,   ,, P_1 ,, Q_1 ,,   ,, X_C

   ,,   ,, P ,, Q ,,   ,, $X_L = 2\pi(2f)L$

Alternating current I_1 is approximately $\dfrac{\sqrt{2}}{3} \cdot \dfrac{2E_m}{\pi} \cdot \dfrac{1}{X_L}$.

Alternating voltage across P_1Q_1 is $I_1 X_C$.

Also $\qquad I_2 = I_1 X_C / X_L.$

$\therefore$ Alternating voltage across $P_2 Q_2 = I_2 X_C = \dfrac{\sqrt{2}}{3} \cdot \dfrac{2E_m}{\pi} \left(\dfrac{X_C}{X_L}\right)^2$

$\therefore$ $\qquad\qquad$ ripple factor $= \dfrac{\sqrt{2}}{3} \left(\dfrac{X_C}{X_L}\right)^2.$

When $L = 20$ H and $C = 16\,\mu$F the ripple factor is found to be $2\cdot95 \times 10^{-5}$ for $f = 50$ c/s, and $1\cdot42 \times 10^{-5}$ for $f = 60$ c/s.

109. An upper limit to the ripple can be found by assuming that cut-out takes place for the entire half-cycle.* The triangular ripple waveform shown in the solution to Question 108 becomes a triangular wave with vertical sides.
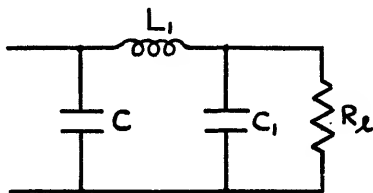
The Fourier analysis of such a waveform gives

$$E_{\text{d.c.}} - \frac{E_d}{\pi} \left(\sin 2\omega t - \frac{\sin 4\omega t}{2} + \frac{\sin 6\omega t}{3} - \cdots \right)$$

where $E_d = I_{\text{d.c.}} / 2fC$ as in Question 108. Harmonics greater than the second will be neglected. R.m.s. second-harmonic voltage $E_2 = I_{\text{d.c.}} / 2\pi f C \sqrt{2}$ and this is impressed on an L-section filter.

The output ripple is therefore approximately $E_2 \cdot X_{C_1} / X_{L_1}$

where $\qquad X_{C_1} = 1/2\pi(2f)C_1$ and $X_{L_1} = 2\pi(2f)L_1.$

Ripple factor $= E_2 \cdot X_{C_1} / X_{L_1} \cdot E_{\text{d.c.}} = \sqrt{2}/L_1 C_1 C R_l (2\pi \cdot 2f)^3.$

If C and C_1 are in microfarads and $f = 50$ c/s, ripple factor $= 5{,}700/CC_1 L_1 R_l.$

In this case $R_l = 250 \times 1{,}000/50 = 5{,}000\,\Omega$ and the ripple factor $= 0\cdot01/100.$

$\therefore$ $\qquad\qquad$ if $C_1 = C,$ $C^2 L_1 = 11{,}400.$

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* J. Millman and S. Seely, *Electronics*, McGraw-Hill, 2nd Edition, 1951, p. 402.

A value for L_1 is usually chosen to be that of a readily available item.

For example if $L_1 = 20$ H, $C = 23{\cdot}9\,\mu$F.

Alternatively, if $L_1 = 40$ H, $C = 16{\cdot}9\,\mu$F.

The capacitors chosen for these two values of L_1 would need to be not less than the corresponding figures quoted.

Having chosen a suitable choke its d.c. resistance will be known and therefore the d.c. voltage drop across it can be calculated. This gives the voltage drop across the first capacitor from which the peak transformer voltage to the centre-tap can be evaluated.

If C and C_1 are in microfarads and $f = 60$ c/s, ripple factor $= 3{,}300/CC_1L_1R_l$. In this case, if $C_1 = C$, $C^2L_1 = 6{,}600$.

If now $L_1 = 20$ H, $C = 18{\cdot}2\,\mu$F. Alternatively, if $L_1 = 40$ H, $C = 12{\cdot}8\,\mu$F.

110. A π-section filter with a resistor replacing the inductor may be analysed as in the previous solution.

$\therefore$ the ripple factor is now

$$\sqrt{2}X_C \, . \, X_{C_1}/R_l \, . \, R \text{ instead of } \sqrt{2}X_C \, . \, X_{C_1}/R_l \, . \, X_{L_1}$$

i.e. for the same ripple factor $R = X_{L_1} = 4\pi fL_1$
$$= 12{,}568\,\Omega \text{ for } 50 \text{ c/s, and}$$

15,082 Ω for 60 c/s.

When output current $= 100$ mA, power dissipated is

(a) $(0{\cdot}1)^2 \times 12{,}568$ W $= \underline{125{\cdot}7 \text{ W.}}$

(b) $(0{\cdot}1)^2 \times 15{,}082$ W $= \underline{150{\cdot}8 \text{ W.}}$

When output current $= 10$ mA, power dissipated is (a) $\underline{1{\cdot}257 \text{ W,}}$ (b) $1{\cdot}508$ W.

111. The solution to this problem can be found elsewhere.*

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* See F. A. Benson, 'Voltage Stabilizers,' *Electronic Engineering Monograph,* 1950, pp. 49–50. See also 'Stabilovolts,' *Marconi Publication* S.P.10.

112. The solution to this problem can be found elsewhere.*

113. Let resistance of series resistor be R ohms.
Current through $R = (20 + 30)$ mA $= 50$ mA.
Voltage across $R = (400 - 200)$ V $= 200$ V.

$\therefore$ $$R = 200/50 \times 10^{-3} = 4\,\text{k}\Omega.$$

Since the load current $= 20$ mA, and the tube current may vary from 10 to 50 mA, the current through R varies from 30 mA to 70 mA.

$\therefore$ the voltage across R varies from 120 V to 280 V,

i.e. the input voltage varies from 320 V to 480 V.

Load current can vary from zero to 40 mA (when tube current is at its minimum value of 10 mA),

i.e. load resistance varies from $5\,\text{k}\Omega$ to ∞, since voltage across load is 200 V.

114. The solution to this problem has been given elsewhere by the author.†

115. The solution to this problem has been given elsewhere by the author.‡

116. The solution to this problem has been given elsewhere by the author.‡

117. The solution to this problem has been given elsewhere by the author.‡

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* See F. A. Benson, 'Voltage Stabilizers,' *Electronic Engineering Monograph*, 1950, pp. 30–2. See also E. W. Titterton, 'Some Characteristics of Glow-discharge Voltage Regulator Tubes,' *J. Sci. Instrum.*, **26**, p. 33, 1949.

† F. A. Benson, 'Voltage Stabilizers,' *Electronic Engineering Monograph*, 1950, Chapter 4.

‡ F. A. Benson and G. V. G. Lusher, 'Voltage Stabilizers for Microwave Oscillators,' *Electronic Engineering*, **26**, p. 106, 1954.

118. The voltage-current curve is as shown.
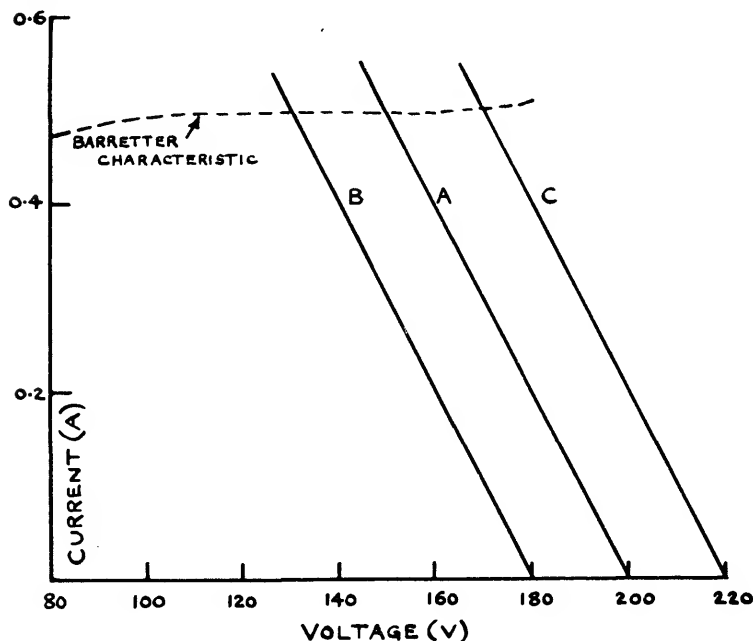
If v is the voltage across the barretter and I the current through it, then for the 200-V input

$$200 = v + 100I.$$

This is the equation of straight line A which cuts the barretter characteristic at 0·5 A.

For 180-V input line B is obtained which cuts the barretter characteristic at 0·5 A.

For 220-V input line C is obtained which cuts the barretter characteristic at 0·504 A.

Current variation if input voltage changes by $\pm 10\%$ is 0·004A.

119. Supply voltage $= \{250 + (10 \times 10^3 \times 9 \times 10^{-3})\}$ V $= 340$ V.
Resistance of load $= (430 - 250)/9 \times 10^{-3} \,\Omega = 20$ kΩ.

120. Resistance $= 8/(9 \times 10^{-3}) = 889\ \Omega$.

The capacitor should have low reactance compared with $889\ \Omega$. The greater the capacitance the more effective is the capacitor in taking most of the alternating component of the anode current.

Suppose the reactance of the capacitor is chosen to be 1/10 of the resistance. Then at 1,000 c/s, $C = 10^6/2\pi \times 1,000 \times 88 \cdot 9\ \mu F$,

i.e. $C = 1 \cdot 79\ \mu F$, say $2\ \mu F$.

At 100 c/s, $C = 17 \cdot 9\ \mu F$, say $20\ \mu F$.

121. Power input $= 1/600$ W.

Power output $= 10I^2$, where I is the load current.

$\therefore$ $10 \log_{10}(10I^2 \times 600) = 60$, so $I = 12 \cdot 9$ A

60 db $= 60 \times 0 \cdot 1151$ nepers $= 6 \cdot 9$ nepers.

122. The equivalent circuit is as shown.
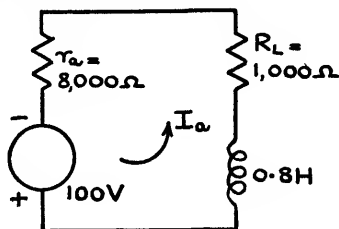
$$I_a = \frac{100}{8,000 + 1,000 + j(2\pi \times 300 \times 0 \cdot 8)}$$

$$= (10 \cdot 81 - j1 \cdot 81)\ \text{mA}.$$

The output voltage

$$V_o = -(10 \cdot 81 - j1 \cdot 81)(1,000 + j1,508)10^{-3}\ \text{V}$$

$$= -(13 \cdot 54 + j14 \cdot 49) = 19 \cdot 61\underline{/-133°}.$$

The gain $A = \dfrac{19 \cdot 61\underline{/-133°}}{5} = 3 \cdot 92\underline{/-133°}.$
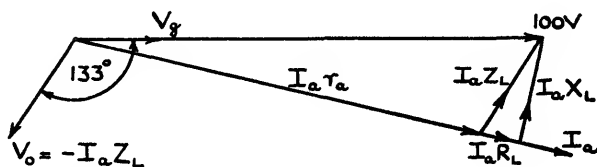
The vector diagram is therefore as illustrated.

When the frequency is 2,000 c/s:

$$I_a = \frac{100}{8,000 + 1,000 + j(2\pi \times 2,000 \times 0{\cdot}8)} = (4{\cdot}94 - j5{\cdot}52) \text{ mA.}$$

The output voltage

$$V_o = -(4{\cdot}94 - j5{\cdot}52)(1,000 + j10,060) \times 10^{-3} \text{ V}$$
$$= 74{\cdot}87\underline{/-143{\cdot}8°}.$$

∴   the gain $A = 14{\cdot}97\underline{/-143{\cdot}8°}.$

The gain at 2,000 c/s is greater than the gain at 300 c/s, i.e. frequency distortion is present. The results also show that phase-shift distortion exists in the amplifier.

123. The solution to this problem can be found elsewhere.*

124. The equivalent circuit of the arrangement is as shown.
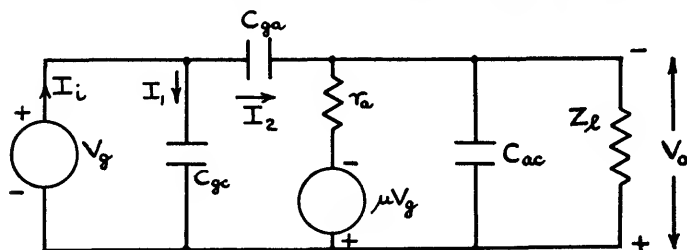
Using the Millman Theorem, $V_o = \dfrac{\mu V_g Y_a - V_g Y_3}{Y_a + Y_l + Y_2 + Y_3}$

where $Y_a = 1/r_a$, $Y_2 = j\omega C_{ac}$, $Y_3 = j\omega C_{ga}$ and $Y_l = 1/Z_l$.

∴   the gain $= -\dfrac{V_o}{V_g} = \dfrac{Y_3 - g_m}{Y_a + Y_l + Y_2 + Y_3} = A$ say.

In this case, since $\omega = 2\pi \times 10,000$, $Y_2 = j2{\cdot}26 \times 10^{-7}$ mho, and $Y_3 = j1{\cdot}88 \times 10^{-7}$ mho. Also $Y_l = 1{\cdot}11 \times 10^{-5}$ mho, $Y_a = 2{\cdot}5 \times 10^{-5}$ mho and $g_m = 1{\cdot}5 \times 10^{-3}$ mho.

∴ $\text{gain} = \dfrac{-1{\cdot}5 \times 10^{-3} + j1{\cdot}88 \times 10^{-7}}{3{\cdot}61 \times 10^{-5} + j4{\cdot}14 \times 10^{-7}}.$

Thus the j terms which come from Y_2 and Y_3 are negligible. Neglecting these j terms the gain is $-41{\cdot}6$.

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* See, for example, J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957.

Since A is real the input impedance consists of a capacitance of value $C_i = C_{gc} + (1 + A)C_{ga}$

$$= (3\cdot 0 + 42\cdot 6 \times 3\cdot 0)\,\mu\mu\text{F} = 130\cdot 8\,\mu\mu\text{F}.$$

For a two-stage amplifier the input impedance of the second stage acts as a shunt for the load of the first stage. Thus C_i, along with the C_{ac} of the first tube, shunts the load. It should also be remembered that every $1\,\mu\mu\text{F}$ of stray capacitance between the leads to the anode and grid of the second stage adds effectively $42\cdot 6\,\mu\mu\text{F}$ across the load resistor of the first stage. It is reasonable to assume therefore that the 90,000-Ω load of the first stage is shunted by a capacitance of $200\,\mu\mu\text{F}$ (a conservative figure).

$$\therefore \qquad Y_l = 1\cdot 11 \times 10^{-5} + j1\cdot 26 \times 10^{-5}\ \text{mho}.$$

$$\therefore \quad \text{the gain} = \frac{Y_3 - g_m}{Y_a + Y_l + Y_2 + Y_3} \simeq \frac{-g_m}{Y_a + Y_l + Y_2 + Y_3}$$

$$= -36\cdot 79 + j13\cdot 24 = 39\cdot 1\underline{/160\cdot 2^\circ}.$$

125. As in the previous solution the gain

$$= \frac{Y_3 - g_m}{Y_a + Y_l + Y_2 + Y_3}.$$

In this case, since $\omega = 2\pi \times 10{,}000$,

$Y_2 = j\omega C_{ac} = j2\cdot 26 \times 10^{-7}$ mho, $Y_3 = j\omega C_{ga} = j2\cdot 13 \times 10^{-7}$ mho, $Y_a = 1/r_a = 1\cdot 3 \times 10^{-4}$ mho, $g_m = 26 \times 10^{-4}$ mho,

$$Y_l = \frac{1}{R_l + j\omega L} = \frac{1}{2{,}500 + j1{,}257} = (3\cdot 193 - j1\cdot 605)10^{-4}\ \text{mho}.$$

$$\therefore \quad \text{the gain} = -5\cdot 13 - j1\cdot 83 = 5\cdot 4\underline{/199\cdot 6^\circ} = 5\cdot 4\underline{/-160\cdot 4^\circ}.$$

Referring to the diagram of the previous solution,

$$I_i = I_1 + I_2 = V_g\{Y_1 + (1 - A)Y_3\}$$

where $\qquad Y_1 = j\omega C_{gc}$

i.e. the input admittance

$$= Y_i = I_i/V_g = Y_1 + (1 - A)Y_3.$$

In this case, $Y_1 = j2\cdot 13 \times 10^{-7}$ mho.

$$\therefore \qquad Y_i = (-3\cdot 9 + j15\cdot 2)10^{-7}\ \text{mho}.$$

If the input circuit is supposed to consist of a resistor R and a capacitor C in parallel,

$$R = \frac{1}{-3\cdot 9 \times 10^{-7}}\,\Omega = -2\cdot 564\ \text{M}\Omega$$

and $\qquad C = \dfrac{15 \cdot 2 \times 10^{-7}}{2\pi \times 10^{4}} \text{ F} = 24 \cdot 2 \ \mu\mu\text{F}.$

126. The equivalent circuit of the arrangement is shown.

$$V_g = v_i - i_a Z_c \qquad . \qquad . \qquad (1)$$

From the equivalent circuit:

$$i_a = \mu V_g / (R_l + r_a + Z_c) \qquad . \qquad (2)$$

From (1) and (2):

$$i_a = \mu v_i / \{R_l + r_a + Z_c(1 + \mu)\} \qquad . \qquad (3)$$

Now $\qquad\qquad v_o = -R_l i_a \qquad . \qquad . \qquad (4)$

From (3) and (4),

$$v_o / v_i = -\mu R_l / \{r_a + R_l + Z_c(1 + \mu)\} \quad (5)$$

But $R_l = 100{,}000 \ \Omega$, $Z_c = \dfrac{2{,}000/j\omega C}{2{,}000 + \dfrac{1}{j\omega C}}$,

$r_a = 50{,}000 \ \Omega$ and $\mu = 80$

$$\therefore \qquad v_o / v_i = -8{,}000 \Big/ \left\{ 150 + \dfrac{162}{1 + 2{,}000 j\omega C} \right\}.$$

$\therefore$ v_o / v_i is a maximum when $\omega \to \infty$ and is 53·3, and v_o / v_i is a minimum when $\omega \to 0$ and is 25·6.

v_o / v_i is equal to 0·707 of its maximum value when the frequency $(\omega / 2\pi) = 121 \cdot 2$ c/s.

127. The equivalent circuit of one stage is as shown. Applying the Millman Theorem between points O and B:

$$V_{oB} = \dfrac{V_{oA} Y_C}{Y_C + Y_{Rg} + Y_{Cg}}$$
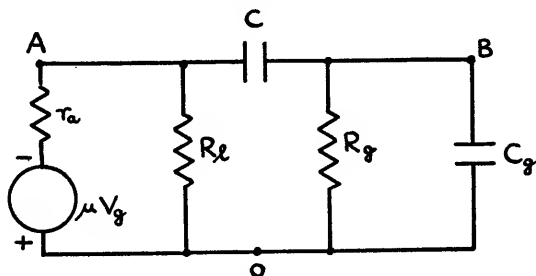
where $Y_C = j\omega C$, $Y_{R_g} = 1/R_g$ and $Y_{C_g} = j\omega C_g$.

Applying the Millman Theorem between points O and A:

$$V_{oA} = \frac{\mu V_g Y_a + V_{oB} Y_C}{Y_a + Y_l + Y_C}, \text{ where } Y_a = 1/r_a \text{ and } Y_l = 1/R_l.$$

$\therefore$ gain $A = - V_{oB}/V_g$

$$= \frac{- \mu Y_a Y_C}{(Y_C + Y_{R_g} + Y_{C_g})(Y_a + Y_l) + Y_C(Y_{R_g} + Y_{C_g})}.$$

At intermediate frequencies where Y_C is large and Y_{C_g} is small,

$$A = A_0 = - \mu Y_a/(Y_a + Y_l + Y_{R_g}).$$

At low frequencies the effect of C_g is negligible and

$$A = A_1 = - \mu Y_a Y_C/\{Y_C(Y_a + Y_l + Y_{R_g}) + Y_{R_g}(Y_a + Y_l)\}.$$

$\therefore \quad \dfrac{A_1}{A_0} = \dfrac{1}{1 - jf_1/f} \text{ where } f_1 = \dfrac{Y_{R_g}(Y_a + Y_l)}{2\pi C(Y_a + Y_l + Y_{R_g})}.$

If the load is a pure resistance f_1 is real and $\dfrac{A_1}{A_0} = \dfrac{1}{\sqrt{1 + (f_1/f)^2}}$,

i.e. f_1 represents that frequency at which the gain falls to $1/\sqrt{2}$ of its intermediate-frequency value.

At high frequencies Y_{R_g} and Y_{C_g} can be neglected in comparison with Y_C and $A = A_2 = - \mu Y_a/(Y_a + Y_l + Y_{R_g} + Y_{C_g})$.

$\therefore \quad \dfrac{A_2}{A_0} = \dfrac{1}{1 + jf/f_2}, \text{ where } f_2 = (Y_a + Y_l + Y_{R_g})/2\pi C_g.$

If the load is a pure resistance f_2 is real and $\dfrac{A_2}{A_0} = \dfrac{1}{\sqrt{1 + (f/f_2)^2}}$,

i.e. f_2 represents that frequency (at the high-frequency end) where the gain falls to $1/\sqrt{2}$ of its intermediate-frequency value.

In this case,

$$Y_a = 1 \cdot 3 \times 10^{-4} \text{ mho}, \quad Y_l = 0 \cdot 2 \times 10^{-4} \text{ mho},$$

and $\qquad\qquad Y_{R_g} = 0 \cdot 02 \times 10^{-4} \text{ mho}.$

$\therefore \qquad A_0 = - 17 \cdot 1, f_1 = 31 \text{ c/s and } f_2 = 121,000 \text{ c/s}.$

$A_0 = - 17 \cdot 1$ so when $A = 14$

the minimum gain ratio $\dfrac{A}{A_0} = \dfrac{14}{17 \cdot 1} = 0 \cdot 8187.$

If f' is the low frequency where the gain drops to 14 and f'' is the high frequency where the gain drops to 14,

$$\frac{1}{\sqrt{1 + (f_1/f')^2}} = 0.8187 = \frac{1}{\sqrt{1 + (f''/f_2)^2}}.$$

$$\therefore \qquad f' = 44 \text{ c/s and } f'' = 84{,}960 \text{ c/s}.$$

128. Gain per stage $= \sqrt{6000} = 78$.

Using the symbols introduced in the solution to Problem 127:

$$\left|\frac{A_2}{A_0}\right| = \left[\frac{1}{1 + (f/f_2)^2}\right]^{n/2}$$

where n is the number of stages (in this case 2)

$$\therefore \qquad 0.95 = \left[\frac{1}{1 + (f/f_2)^2}\right]^{2/2}$$

i.e. $\qquad\qquad\qquad f/f_2 = 0.223$

Since $\qquad\qquad f = 100 \text{ kc/s}, f_2 = 450 \text{ kc/s}.$

Now $\qquad f_2 = 1/2\pi C_g R_l$, where $C_g = 20.5 \ \mu\mu\text{F}$ (given)

$$\therefore \quad R_l = 1/(2\pi \times 20.5 \times 10^{-12} \times 450 \times 10^3) = \underline{17{,}200 \ \Omega}$$

The gain per stage at mid-frequency is

$$- g_m R_l = - 5.2 \times 10^{-3} \times 17{,}200 = - 89.4$$

The overall gain $= 89.4^2 = \underline{7{,}992.}$

129. The equivalent circuit of one stage of the amplifier is shown.
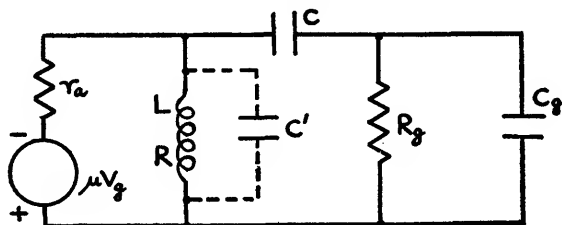
L is the inductance and R is the resistance of the inductor. C' is the distributed capacitance of the winding. The other symbols have the same meaning as on the equivalent circuit given in the

solution to Question 127. It should be observed, in fact, that the two equivalent circuits differ only in the anode-circuit impedances. Thus, the analysis given in the solution to Question 127 is valid here provided Y_l is now written as $j\omega C' + 1/(R + j\omega L)$. The frequency-response characteristic of this amplifier may therefore be examined in the same way as that for the resistor-capacitor arrangement given already.

130. It is shown in Solution 127 that:

$$A_0 = -\mu\, Y_a/(Y_a + Y_l + Y_{Rg})$$

With pentodes it is possible to assume that $r_a \gg R_l$. Further, it may be assumed that $R_g \gg R_l$ because R_l must be made small to raise f_2, whereas R_g must be large to lower f_1. With these assumptions $A_0 = -g_m R_l$ and the high-frequency gain becomes:

$$A_2 = -g_m R_l/(1 + j\omega C_g R_l)$$

The gain ratio for a pentode is then

$$\left|\frac{A_2}{A_0}\right| = \left[\frac{1}{1 + (f/f_2)^2}\right]^{\frac{1}{2}}$$

where $f_2 = 1/2\pi C_g R_l =$ bandwidth in c/s since f_1 will be low.

Figure of merit (gain × bandwidth) $= g_m R_l/2\pi C_g R_l = g_m/2\pi C_g$.

This is constant for a given type of valve. If $C_g = C_{ac} + C_{gc}$ (the irreducible minimum value of C_g for pentodes) $= 9\cdot2\ \mu\mu\text{F}$ in this case and $g_m = 5\cdot7\ \text{mA/V}$:

Figure of merit $= 5\cdot7 \times 10^{-3}/(2\pi \times 9\cdot2 \times 10^{-12})$
$$= \underline{98\cdot6}.$$

131. If an amplifier is made up of n cascaded RC-coupled stages, not necessarily identical, the overall high-frequency gain ratio is:

$$\left|\frac{A_2}{A_0}\right| = \left[\frac{1}{1 + (f/f_2')^2}\right]^{\frac{1}{2}} \left[\frac{1}{1 + (f/f_2'')^2}\right]^{\frac{1}{2}} \cdots$$

If the stages are identical

$$\left|\frac{A_2}{A_0}\right| = \left[\frac{1}{1 + (f/f_2)^2}\right]^{n/2}$$

If f_s designates the 3 dB point for the overall amplifier of n stages:

$$1/\sqrt{2} = \left[\frac{1}{1 + (f_s/f_2)^2}\right]^{n/2}$$

Similarly, for the three non-identical stages

$$1/\sqrt{2} = \left[\frac{1}{1 + (f/f_2')^2}\right]^{\frac{1}{2}} \left[\frac{1}{1 + (f/f_2'')^2}\right]^{\frac{1}{2}} \left[\frac{1}{1 + (f/f_2''')^2}\right]^{\frac{1}{2}}$$

$$\therefore \quad 1/2 = \left[\frac{1}{1 + (f/250)^2}\right] \left[\frac{1}{1 + (f/350)^2}\right] \left[\frac{1}{1 + (f/550)^2}\right]$$

This expression gives f at the high-frequency end in kc/s.

A similar calculation gives the low-frequency limit of the overall bandwidth:

$$\left|\frac{A_1}{A_0}\right| = 1\sqrt{2} = \left[\frac{1}{1 + (100/f)^2}\right]^{\frac{1}{2}} \left[\frac{1}{1 + (80/f)^2}\right]^{\frac{1}{2}} \left[\frac{1}{1 + (50/f)^2}\right]^{\frac{1}{2}}$$

132. The solution to this problem can be found elsewhere.[*]

133. The solution to this problem can be found elsewhere.[†]

134. The solution to this problem can be found elsewhere.[‡]

135. Consider first the low-frequency equivalent circuit (a) of the amplifier. The secondary of the transformer feeds the grid circuit of the next valve which is assumed to have infinite impedance.

$$\therefore \qquad I_p = \mu V_g/(r_a + R_p + j\omega L_p) \qquad . \qquad . \qquad . \quad (1)$$

Voltage across primary $= j\omega L_p I_p$ $\qquad . \qquad . \qquad . \qquad .$ (2)

$$= \mu V_g/\{1 - j(r_a + R_p)/\omega L_p\} \quad . \quad (3)$$

[*] See J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957, p. 107.
[†] *ibid.*, 108.
[‡] See, for example, V. C. Rideout, *Active Networks*, Constable, 1954, pp. 224–7; or S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, pp. 319–21.

(a)
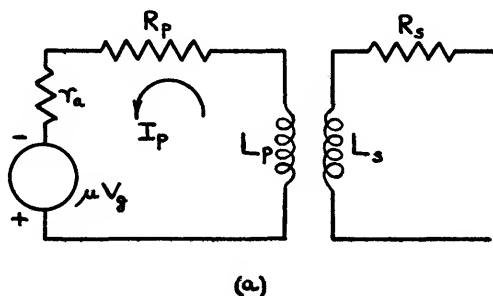
Voltage across secondary is n times as large as this.

$$\therefore \qquad \text{gain} = \pm\, n\mu/\{1 - j(r_a + R_p)/\omega L_p\} \qquad . \qquad . \quad (4)$$

The magnitude of the gain $(A) = n\mu/\sqrt{1 + \{(r_a + R_p)/\omega L_p\}^2}$ (5)

When $\omega L_p \gg (r_a + R_p)$, $A = n\mu = 30$ in this case. The gain drops off at low frequencies because ωL_p is not large compared with $(r_a + R_p)$. When $\omega L_p = (r_a + R_p)$ the gain is only 70·7% of its value $n\mu$ approached at higher frequencies. Thus the gain drops to 70·7% of 30, i.e. 21·2 when $f = (8{,}000 + 3{,}500)/(2\pi \times 70) = 26$ c/s.

Now consider circuit (b) which is the high-frequency equivalent circuit.

(b)
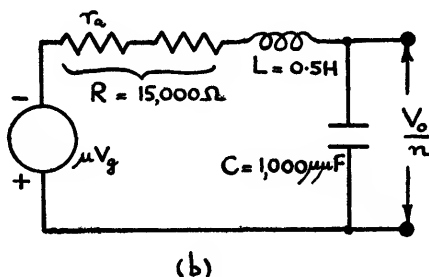
An analysis of this circuit shows the gain to be

$$\frac{\pm\, jn\mu \left(\dfrac{1}{\omega C}\right)}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)} \cdot \qquad . \qquad . \qquad . \quad (6)$$

The magnitude of the gain $A = \dfrac{n\mu\left(\dfrac{1}{\omega C}\right)}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$.   . (7)

At low frequencies, ωL is small and $1/\omega C$ is large, therefore the gain approaches $n\mu$ as already stated. At very high frequencies $1/\omega C$ is small and ωL is large and the gain falls rapidly to zero. The gain passes through a maximum between these two extremes which is found by putting $dA/d\omega = 0$ to occur when

$$1/\omega C = \{2(\omega L)^2 + R^2\}/2\omega L.$$

Since ωL is usually much greater than R, the maximum occurs when $\omega L = 1/\omega C$. This is the condition for series resonance.

In this case the frequency for maximum gain

$$= \frac{1}{2\pi\sqrt{0{\cdot}5 \times 1{,}000 \times 10^{-12}}} = 7{,}117 \text{ c/s.}$$

An analysis of the circuit shows that the corresponding gain is

$$\frac{n\mu}{R}\sqrt{\frac{L}{C}} = \frac{30}{15{,}000}\sqrt{\frac{0{\cdot}5}{1{,}000 \times 10^{-12}}} = 44{\cdot}7.$$

A gain-frequency curve can be plotted by finding other values of gain at the low-frequency end using equation (5) and at the high-frequency end using equation (7). Both expressions give a gain of $n\mu = 30$ for the mid-band frequencies. A sketch of the resulting response curve is shown in diagram (c).
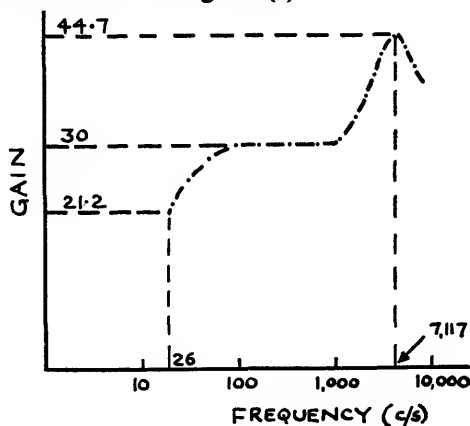
(c)

136. The equivalent resistance of r_a and R in parallel, in the equivalent circuit shown, is $R' = (8 \times 50)/(8 + 50) = 200/29$ kΩ.
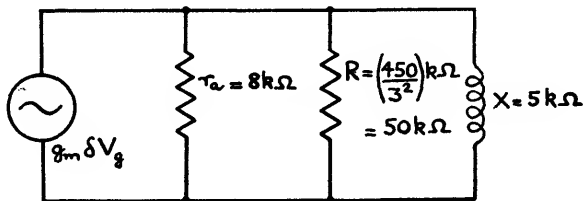
The equivalent impedance of X and R' is $Z' = \dfrac{200 \times 5/29}{\sqrt{5^2 + (200/29)^2}}$ kΩ.

Stage gain $= g_m Z' \times 3$ and $g_m = 2$ mA/V.

$\therefore$ stage gain $= 24\cdot3$.

137. If I_1 and I_2 are the peak values of the fundamental and second-harmonic currents,

$$I_1 + 2I_2 = 150 - 80 = 70 \text{ mA}$$
$$I_1 - 2I_2 = 80 - 20 = 60 \text{ mA}.$$

$\therefore$ $\qquad\qquad\qquad I_1 = 65 \text{ mA}$

and $\qquad\qquad\qquad I_2 = 2\cdot5 \text{ mA}.$

The mean anode current with the signal is $80 + I_2 = 82\cdot5$ mA. Power delivered to load

$$= \tfrac{1}{2}(I_1^2 + I_2^2)\,2{,}000 \text{ W}$$
$$= 4\cdot23 \text{ W}.$$

D.C. power supplied $= 300 \times 82\cdot5/1{,}000 = 24\cdot75$ W.

$\therefore$ $\qquad\qquad$ Efficiency $= 4\cdot23/24\cdot75 = 0\cdot17.$

Percentage second-harmonic current $= (2\cdot5/65)100 = 3\cdot85\%.$

138. Let the input voltage be sinusoidal and of the form $v_g = V_g \cos \omega t$. The anode current i_a is of the form

$$I_a + B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t \qquad . \qquad . \quad (1)$$

B_0, B_1, etc., can be found from the characteristic curves of the valve.

From the figure when:

$$\omega t = 0, \; i_a = I_{max}$$
$$\omega t = \pi/2, \; i_a = I_a$$
$$\omega t = \pi, \; i_a = I_{min}.$$

Substituting in (1):

$$I_{max} = I_a + B_0 + B_1 + B_2 \qquad \cdot \qquad \cdot \qquad \cdot \quad (2)$$

$$I_a = I_a + B_0 - B_2 \qquad \cdot \qquad \cdot \qquad \cdot \quad (3)$$

$$I_{min} = I_a + B_0 - B_1 + B_2 \qquad \cdot \qquad \cdot \qquad \cdot \quad (4)$$

From (3), $B_0 = B_2$ $\cdot$ $\cdot$ $\cdot$ $\cdot$ $\cdot$ (5)

From (2) and (4), $B_1 = (I_{max} - I_{min})/2$ $\cdot$ $\cdot$ $\cdot$ (6)

From (2) and (6), $B_2 = (I_{max} + I_{min} - 2I_a)/4$ $\cdot$ $\cdot$ (7)
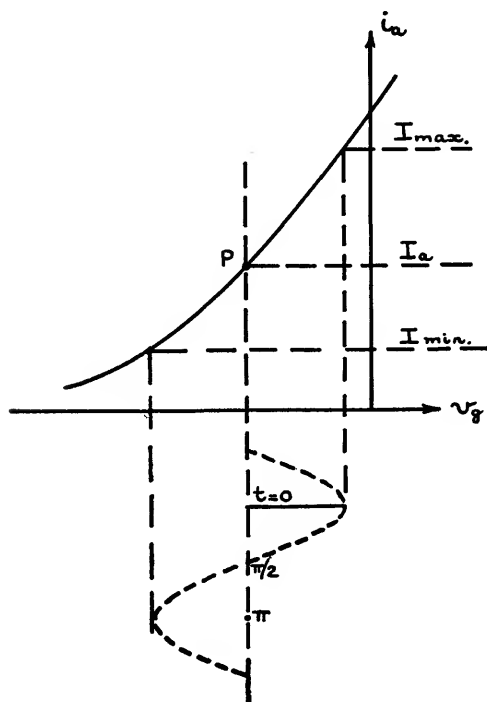
In this case maximum current corresponds to a grid voltage of $(-8+6)V = -2$ V and minimum current corresponds to a grid voltage of $(-8-6)V = -14$ V.

From the characteristic curves and the load line,

$$I_{max} = 21{\cdot}6 \text{ mA}, \ I_a = 13{\cdot}2 \text{ mA and } I_{min} = 6{\cdot}3 \text{ mA}.$$

$\therefore \qquad B_2 = B_0 = (21{\cdot}6 + 6{\cdot}3 - 26{\cdot}4)/4 = 0{\cdot}375 \text{ mA}.$

Total steady current $= (13{\cdot}2 + 0{\cdot}375) \text{ mA} = \underline{13{\cdot}575 \text{ mA.}}$

Peak fundamental current is

$$B_1 = (21{\cdot}6 - 6{\cdot}3)/2 \text{ mA} = 7{\cdot}65 \text{ mA.}$$

Peak fundamental output voltage

$$= \frac{7{\cdot}65}{1,000} \times 8,000 = 61{\cdot}2 \text{ V.}$$

Fundamental gain

$$= 61{\cdot}2/6 = \underline{10{\cdot}2.}$$

Percentage of second-harmonic distortion

$$= 100B_2/B_1 = 100 \times 0{\cdot}375/7{\cdot}65$$
$$= \underline{4{\cdot}9\%.}$$

139. Let I_{max}, I', I_a, I'' and I_{min} correspond respectively to the grid voltages when $\omega t = 0$, $\pi/3$, $\pi/2$, $2\pi/3$ and π. Then proceeding in the same manner as in the previous solution it is found that:

$$B_0 = (I_{max} + 2I' + 2I'' + I_{min})/6 - I_a$$
$$B_1 = (I_{max} + I' - I'' - I_{min})/3$$
$$B_2 = (I_{max} - 2I_a + I_{min})/4$$
$$B_3 = (I_{max} - 2I' + 2I'' - I_{min})/6$$
$$B_4 = (I_{max} - 4I' + 6I_a - 4I'' + I_{min})/12.$$

140. The quiescent point Q is determined by drawing the load line through the point $I_a = 0$, $V_a = 300$ V with a slope fixed by the resistance R_1 of the choke. Since R_1 is generally small the static load line is almost vertical. Since anode dissipation is 25 W, anode current permissible is $25/300 = 83$ mA. This corresponds to a grid

bias of about -20 V. Permissible grid swing is about 20 V peak; distortion occurs from the non-linear parts of the valve characteristics. Thus a minimum anode current of about 20 mA is set where
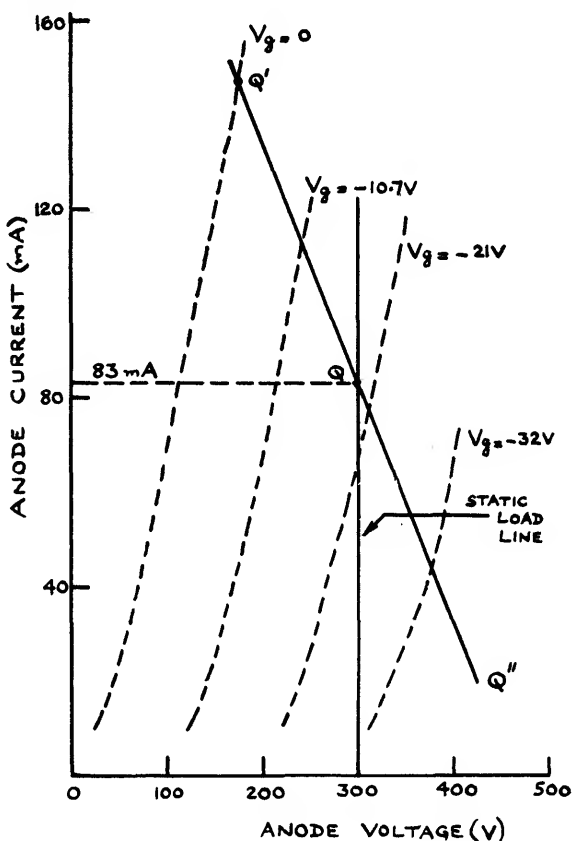
characteristics begin to curve. Therefore, dynamic load line is as shown ($Q'QQ''$). Voltage swing is $(425-175)$ V $= 250$ V. Corresponding current change is $(148-20)$ mA $= 128$ mA. Load resistance $= 250/(128 \times 10^{-3})\,\Omega = \underline{1\cdot95\ \text{k}\Omega}$.

Output power $= (V_{\text{max}} - V_{\text{min}})(I_{\text{max}} - I_{\text{min}})/8 = (250 \times 0\cdot128)/8$

$$= \underline{4\ \text{W}.}$$

Efficiency $= (4/25) \times 100\% = \underline{16\%}.$

141. (*a*) The analysis required can be found in many textbooks.*

(*b*) The method of determining the composite characteristic consists of inverting the characteristic of valve (2) with the quiescent point P_2 immediately under P_1 and then adding algebraically the corresponding ordinates of the two characteristics
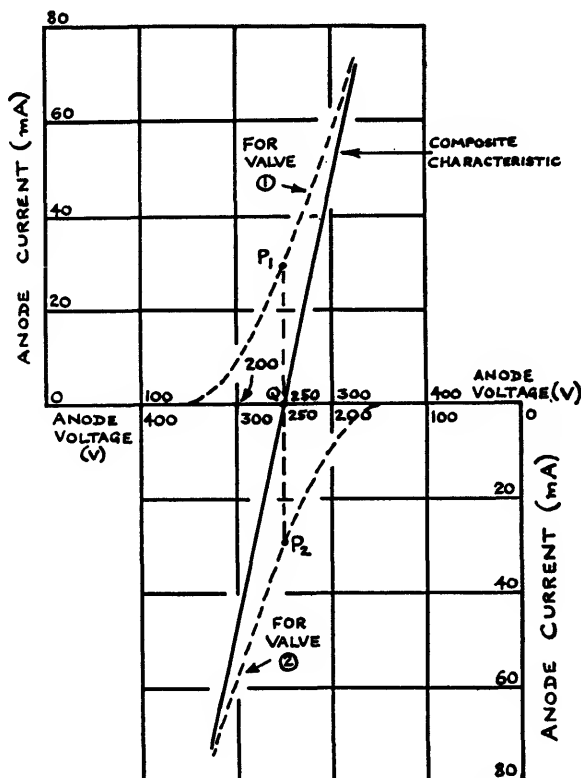
At the points P, slopes of characteristics

$$= 1/2,000 \text{ mho, so } r_a = 2,000 \, \Omega.$$

At Q, $r_a = 1,000 \, \Omega.$

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* See, for example, P. Parker, *Electronics*, Arnold, 1950, Section 73; or Cruft Electronics Staff, *Electronic Circuits and Tubes*, McGraw-Hill, 1947, Chapter 13.

142. The composite characteristics and the load line are as shown below. Having drawn the load line the power rectangle may
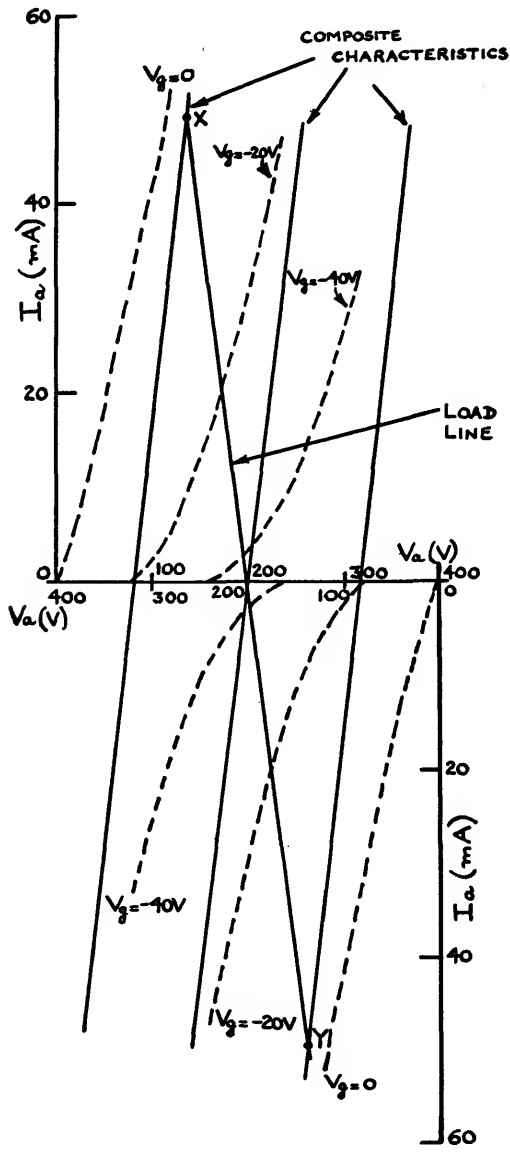
be constructed on diagonal XY and one-eighth of its area gives the amplifier power output.

For the rectangle the sides are approximately 98 mA and 125 V so power output

$$\simeq \tfrac{1}{8} \times 125 \times 98 \times 10^{-3} \text{ W} \simeq 1\cdot53 \text{ W.}$$

143. (a) The solution to this problem can be found in many standard textbooks.*

(b) Power output $= \tfrac{1}{2}V_l I_l$ where V_l and I_l are respectively the peak values of the voltage across, and the current through, one section of the output transformer primary. Now $I_l = I_{max}$ and $V_l = V - V_{min}$ where $V = 500$.

$\therefore$ power output $P_{a.c.} = \tfrac{1}{2}I_{max}(500 - V_{min})$.

But $V_{min} = 1,000 I_{max}$, so $P_{a.c.} = \tfrac{1}{2}I_{max}(500 - 1,000I_{max})$. This is a maximum when $I_{max} = \tfrac{1}{4}$ A when $P_{a.c.} = 31\cdot25$ W.

Power drawn from h.t. supply, $P_{d.c.} = 2I_a V$ where I_a is the mean anode current of either valve (valves assumed identical). The pulses of anode current may reasonably be taken as half sine waves so $I_a = I_{max}/\pi$ and therefore $P_{d.c.} = 2 V I_{max}/\pi$.

The efficiency $= P_{a.c.}/P_{d.c.} = \pi(1 - V_{min}/V)/4$.

In this case the efficiency $= \pi(1 - 250/500)/4 = 0\cdot393$

$= 39\cdot3\%$.

(c) The efficiency of a class-C amplifier can be shown to be†

$$(1 - V_{min}/V)(\theta - \sin 2\theta/2)/2 (\sin \theta - \theta \cos \theta)$$

where 2θ is the angle of flow, V_{min} is the minimum anode voltage and V is the supply voltage. In the ideal case, $V_{min} = 0$ and if 2θ is then $120°$ the efficiency is found to be $89\cdot6\%$.

144. Mean value of anode current $= \dfrac{1}{2} \times 2\cdot5 \times \dfrac{90}{360} = 0\cdot31$ A.

Power supplied by h.t. source is $0\cdot31 \times 2,500 = 775$ W.

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* For example, see P. Parker, *Electronics*, Arnold, 1950, p. 340.

† For the proof of this expression see, for example, P. Parker, *Electronics*, Arnold, 1950, pp. 343–4; or A. T. Starr, *Electronics*, Pitman, 2nd Edition, 1959, p. 272.

Output power $= 0{\cdot}8 \times 0{\cdot}8 \times 750 = 480$ W

$\therefore$ efficiency $= \dfrac{480}{775} \times 100\% = \underline{61{\cdot}9\%}.$

At the point where anode current commences to flow, the instantaneous anode voltage is:[*]

$$V_a = V_{\text{h.t.}} - V_{\text{a.c.}} \cos\theta \quad (2\theta \text{ is the angle of flow})$$
$$= 2,500 - 2,000 \cos 45°$$
$$= \underline{1,086 \text{ V.}}$$

145. Let Z be the impedance of the parallel resonant circuit shown.
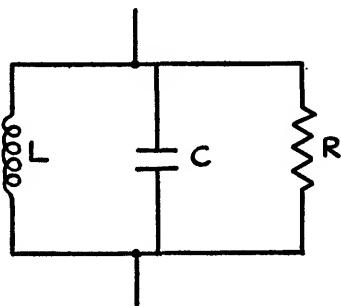
Then $\dfrac{1}{Z} = \dfrac{1}{j\omega L} + \dfrac{1}{R} + j\omega C.$

Let $\omega_0^2 LC = 1$ and $Q = R/\omega_0 L.$

$\therefore$ $\dfrac{1}{Z} = \dfrac{R(1 - \omega^2/\omega_0^2) + j\omega R/\omega_0 Q}{R j\omega R/\omega_0 Q}.$

$\therefore$ $Z = R/[1 + jQ(\omega/\omega_0 - \omega_0/\omega)].$

When ω is near to ω_0 let $\omega = \omega_0 + \Delta\omega.$

Then $Z = R/[1 + 2jQ\Delta\omega/\omega_0].$

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[*] See, for example, J. D. Ryder, *Electronic Fundamentals and Applications*, Pitman, 3rd Edition, 1964, Chapter 12.

For the anode load of the valve the voltage across L is

$$v_L = -\mu Z v_g/(Z + r_a).$$

∴   the voltage across L_2 is $\dfrac{v_L}{j\omega L} \cdot j\omega M = v_L M/L = v_0.$

∴ $v_0 = -\mu Z v_g M/L(Z + r_a)$

$$= -50 M v_g \Big/ \left[L \left\{ 1 + \frac{30{,}000}{15{,}000} \left(1 + 2j \cdot 45 \cdot \frac{\Delta\omega}{\omega_0} \right) \right\} \right].$$

Now $\dfrac{\Delta\omega}{\omega} = \dfrac{500}{20{,}000}$, $v_g = 1$ V, $M = 1$ mH,

$$L = R/\omega_0 Q = 15{,}000/(2\pi \times 20{,}000 \times 45).$$

∴ $\underline{|v_0| = 3 \cdot 485 \text{ V}.}$

146. The current-source equivalent circuit is chosen as shown at (*a*).

$$R = \omega_r L/Q = 2\pi \times 1{,}592 \times 10^3 \times 200 \times 10^{-6}/50 = 4\,\Omega.$$

This circuit can be replaced by that at (*b*), where

$$R' = \omega_r^2 L^2/R = Q\omega_r L = 10 \text{ k}\Omega.$$

∴ at resonance the load is $(2 + 100 + 2)$ micromhos.
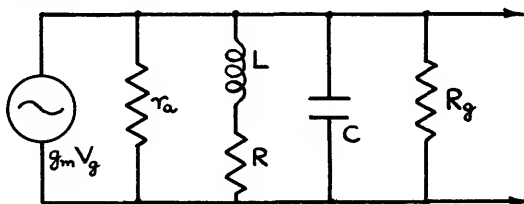
(a)
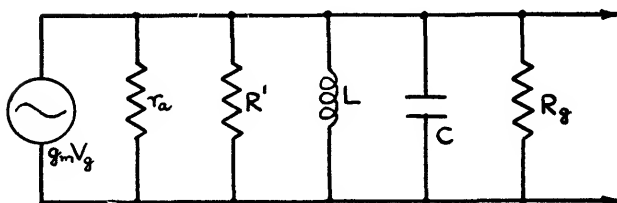
(b)

If the output voltage is V_o, $V_g g_m = V_o \times 104 \times 10^{-6}$
i.e. the gain

$$= V_o/V_g = g_m/104 \times 10^{-6} = 5 \times 10^{-3}/104 \times 10^{-6} = \underline{48}.$$

147. Referring to the diagram:

$$V_o = AV_g \qquad . \qquad . \qquad . \qquad . \qquad . \quad (1)$$

and
$$V_g = V_i + \beta V_o \qquad . \qquad . \qquad . \qquad . \quad (2)$$

$\therefore$
$$\frac{V_o}{V_i} = A_f = \frac{A}{1 - \beta A} \qquad . \qquad . \qquad . \quad (3)$$

In this case, gain with feedback $A_f = 20 \Big/ \Big(1 + \dfrac{1}{10} \cdot 20\Big) = \underline{6\cdot67}.$
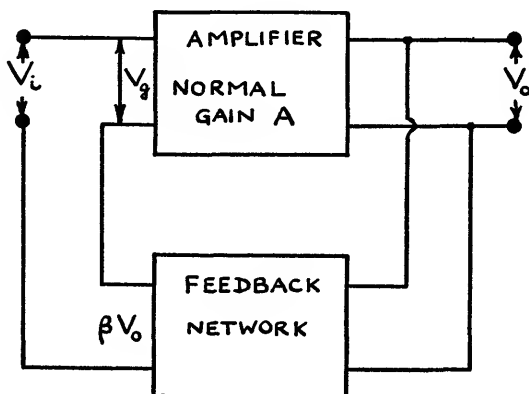
148. Without feedback gain $A = \dfrac{\mu R_l}{(r_a + R_l)}$

$$= \frac{1{,}000 \times 200 \times 10^3}{(200 + 200)10^3} = \underline{500}.$$

As in previous solution $A_f = \dfrac{500}{1 + 25} = \underline{19\cdot2}.$

$$A_f = \frac{A}{1 - \beta A} = \frac{\mu R_l/(r_a + R_l)}{1 - \beta \mu R_l/(r_a + R_l)}$$

$$= \frac{\{\mu/(1 - \mu\beta)\}R_l}{r_a/(1 - \mu\beta) + R_l} = \frac{\mu' R_l}{r_a' + R_l}$$

i.e. the effective amplification factor

$$\mu' = \mu/(1 - \mu\beta) = 1{,}000/51 = \underline{19\cdot6}$$

and the effective anode resistance

$$r_a' = r_a/(1 - \mu\beta) = 200 \times 10^3/51 = \underline{3\cdot92 \text{ k}\Omega}$$

The effective mutual conductance is $\mu'/r_a' = \underline{5 \text{ mA/V}}$.

149.   Original input voltage $= 60/120 = 0\cdot5$ V

      ,,   distortion ,, $= 10 \times 60/100 = 6$ V.

The distortion voltage has to be reduced 10 times, therefore:

$1 - \beta \times 120 = 10$, i.e. $\underline{\beta = -0\cdot075}$ where β is the feedback factor.

The gain will also be reduced by a factor of 10. $A_f = 120/10 = 12$. The added distortionless gain needed ahead of the feedback amplifier is $120/12 = 10$, and this amplifier must supply a signal voltage of $60/12 = 5$ V.

150. With feedback and normal supply voltage the gain

$$A_{f_1} = 24{,}000(1 + 24{,}000/1{,}000) = \underline{960}.$$

With feedback and a 25% reduction of supply voltage from its normal value the gain $A_{f_2} = 16{,}000(1 + 16{,}000/1{,}000) = \underline{941}.$

151. The gain of each stage $\simeq -g_m R_l \left\{ \dfrac{R_g}{R_g + 1/j\omega C} \right\}$

$$\simeq \frac{-j\omega C R_g g_m R_l}{1 + j\omega C R_g}$$

$\therefore$ loop gain of amplifier $= \beta \left\{ \dfrac{-j\omega C R_g g_m R_l}{1 + j\omega C R_g} \right\}^3$

$$= j\beta \left\{ \frac{\omega C R_g g_m R_l}{1 + j\omega C R_g} \right\}^3$$

According to Nyquist's criterion instability will occur if the loop gain > 1 when there is no phase-shift around the loop, i.e. when $\tan(\omega C R_g) = 30°$ or when $\omega C R_g = 1/\sqrt{3}$.

Under this condition the loop gain is $\beta(g_m R_l)^3/8$. Thus, for stability, $\beta(g_m R_l)^3/8 \not> 1$

i.e. $$\underline{\beta \not> 8/(g_m R_l)^3}$$

152. The cathode follower has been analysed by many authors.[*] The output impedance is nearly equal to $1/g_m$. In this case the output impedance $= 10^3/4 = \underline{250\ \Omega}$.

153. The solution to this problem can be found elsewhere.[†]

154. (*a*) For the solution to this problem see the book by Seely.[‡]

(*b*) For the solution to this and a similar problem see the book by Rideout.[§]

155. The input resistance of a common-base amplifier is given by[||]:

$$R_i = r_e + r_b - r_b(r_b + r_m)/(r_b + r_c + R_l)$$
$$= r_{11} - r_{12}r_{21}/(r_{22} + R_l),$$

where r_b, r_c, r_e, r_m, r_{11}, r_{12}, r_{21} and r_{22} are the usual transistor parameters and R_l is the load resistance.

In this case, $R_i = \{550 - 500 \times 1 \cdot 9 \times 10^6/(2 \times 10^6 + R_l)\}\ \Omega$. Thus, R_i varies from 75 Ω to 550 Ω as R_l changes from 0 to ∞.

The output resistance of the arrangement is given by[||]:

$$R_0 = r_c + r_b - r_b(r_m + r_b)/(r_b + r_e + R_g)$$
$$= r_{22} - r_{12}r_{21}/(R_g + r_{11}),$$

where R_g is the resistance of the source (zero in this case).

Thus,

$$R_0 = (2 \times 10^6 - 500 \times 1 \cdot 9 \times 10^6/550)\ \Omega = \underline{2 \cdot 72 \times 10^5\ \Omega}$$

The maximum possible voltage gain[||]

$$= r_{21}/r_{11} = 1 \cdot 9 \times 10^6/550 = 3{,}454.$$

* E.g. see S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, Chapter 5; or A. T. Starr, *Telecommunications*, Pitman, 1954, pp. 232 and 233.

† See S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, pp. 176–8.

‡ S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, pp. 164–6.

§ V. C. Rideout, *Active Networks*, Constable, 1954, pp. 175–8.

|| See, for example, S. Seely, *Electronic Engineering*, McGraw-Hill, 1956, Chapter 16; L. M. Krugman, *Fundamentals of Transistors*, Rider and Chapman & Hall, 2nd Edition, 1959; J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957, Chapter 12; R. B. Hurley, *Junction Transistor Electronics*, Wiley, 1958.

156. The solution to this problem can be found elsewhere.*

157. The solution to this problem can be found elsewhere.*

158.
$$r_e = r_{11} - r_{12} = 20 \ \Omega$$
$$r_b = r_{12} = 800 \ \Omega$$
$$r_c = r_{22} - r_{12} \simeq 2 \ \text{M}\Omega$$
$$r_m = r_{21} - r_{12} \simeq 1 \cdot 98 \ \text{M}\Omega$$

Voltage gain (A_v)

$$= \frac{R_l}{r_b + \{(r_b + r_e)(r_c + R_l)/(r_e - r_m)\}}$$

$$\simeq \frac{430}{800 - \{(820)(2 + 10^6 + 430)/(1 \cdot 98 \times 10^6 - 20)\}}$$

$$\simeq \underline{-15 \cdot 2}.$$

Current gain (A_i)

$$= \frac{1}{1 + (r_c + R_l)/(r_e - r_m)}$$

$$\simeq \frac{1}{1 - (2 \times 10^6 + 430)/(1 \cdot 98 \times 10^6 - 20)}$$

$$\simeq \underline{99}$$

Input resistance (R_i)

$$= r_b + \frac{r_e(r_c + R_l)}{(r_c + r_e - r_m + R_l)}$$

$$\simeq 800 + \frac{20(2 \times 10^6 + 430)}{(2 \times 10^6 + 20 - 1 \cdot 98 \times 10^6 + 430)}$$

$$\simeq \underline{2,755 \ \Omega}$$

159. The h parameters are defined by the equations:
$$v_e = h_{11}i_e + h_{12}v_c \quad . \qquad . \qquad . \qquad . \quad (1)$$
$$i_c = h_{21}i_e + h_{22}v_c \quad . \qquad . \qquad . \qquad . \quad (2)$$
[see equations (16) and (17) of the solution to Problem 87]

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* See, for example, S. Seely, *Electronic Engineering*, McGraw-Hill, 1956, Chapter 16; L. M. Krugman, *Fundamentals of Transistors*, Rider and Chapman & Hall, 2nd Edition, 1958; J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957, Chapter 12; R. B. Hurley, *Junction Transistor Electronics*, Wiley, 1958.

Also, $v_c = - i_c R_l$.   .   .   . (3)

From these equations:

Voltage gain $= v_c/v_e = \dfrac{-h_{21}}{h_{11}h_{22} - h_{12}h_{21} + h_{11}/R_l}$

Current gain $= i_c/i_e = h_{21}/(h_{22}R_l + 1)$

Input resistance $= v_e/i_e = \dfrac{(h_{11}h_{22} - h_{12}h_{21}) + h_{11}/R_l}{h_{22} + 1/R_l}$

160. It will be evident from the solution to Problem 159 for the common-base amplifier that the same expressions for voltage and current gains will hold for the common-emitter circuit if primed quantities are used for the h parameters.

$\therefore$ voltage gain

$$= \frac{- h_{21}{}'}{h_{11}{}'h_{22}{}' - h_{12}{}'h_{21}{}' + h_{11}{}'/R_l}$$

$$= \frac{- 47}{(800 \times 80 \times 10^{-6}) - (47 \times 5{\cdot}4 \times 10^{-4}) + 800/(20 \times 10^3)}$$

$$= \underline{- 598}$$

and current gain

$$= \frac{h_{21}{}'}{h_{22}{}'R_l + 1}$$

$$= \frac{47}{(80 \times 10^{-6} \times 20 \times 10^3) + 1}$$

$$= \underline{18}$$

161. The current gain of a common-emitter stage is given by:

$$A_i = \frac{(r_e - r_m)}{(r_e + r_c + R_l - r_m)} = \frac{1}{1 + (r_c + R_l)/(r_e - r_m)}$$

If $r_e \ll r_m$,

$$|A_i| \simeq r_m/(r_c - r_m + R_l) \simeq \frac{ar_c}{r_c(1 - a) + R_l}$$

Thus, for the circuit illustrated:

$$i_2 \simeq \frac{i_1 ar_c}{r_c(1 - a) + R_l}$$

where R_l in this expression is the total a.c. load of transistor 1.

If $R_2 \gg R_i$, the input resistance of the second transistor stage, then in the mid-frequency band, where the reactance of C may be neglected, $R_l \simeq R_1 R_i/(R_1 + R_i)$.

Now $i_3 \simeq i_2 R_1/(R_1 + R_i)$ so the overall current gain, at these intermediate frequencies Ai_i is:

$$Ai_i = i_3/i_1 \simeq \left\{ \frac{ar_c}{r_c(1-a) + R_l} \right\} \cdot \left\{ \frac{R_1}{R_1 + R_i} \right\}$$

At low frequencies the reactance of C is large so:

$$i_2 \simeq \frac{i_1 ar_c}{r_c(1-a) + R_1/(1 + j\omega CR_1)}$$

The overall current gain now, Ai_l, is therefore:

$$Ai_l = i_3/i_1 \simeq \left\{ \frac{ar_c}{r_c(1-a) + R_1/(1 + j\omega CR_1)} \right\} \cdot \left\{ \frac{R_1}{R_1 + 1/j\omega C} \right\}$$

which can be reduced to:

$$Ai_l \simeq \frac{a}{(1-a)\left[1 - \dfrac{j}{\omega c} \left\{ \dfrac{1}{R_1} + \dfrac{1}{r_c(1-a)} \right\} \right]}$$

As in the case of the corresponding expression for the valve amplifier this may be written as:

$$Ai_l \simeq \left(\frac{a}{1-a} \right) \left\{ \frac{1}{1 - j(f_1/f)} \right\}$$

where
$$f_1 = \frac{1}{2\pi C} \left\{ \frac{1}{R_1} + \frac{1}{r_c(1-a)} \right\}$$

The solution to the last part of the problem can be found elsewhere. For example, a good description of high-frequency effects in junction transistors has been given by Terman* and a calculation to determine the high-frequency response of an RC-coupled common-emitter amplifier by Ryder.†

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* F. E. Terman, *Electronic and Radio Engineering*, McGraw-Hill, 4th Edition, 1955.
† J. D. Ryder, *Electronic Fundamentals and Applications*, Pitman, 3rd Edition, 1964, Chapter 9.

162. The expression for the stability factor has been derived elsewhere.*

$$S = \frac{R_b + R_c}{R_c + (1 - \alpha)R_b} = \frac{110}{10 + 0 \cdot 02 \times 100} = \underline{9 \cdot 2}$$

163. The expression for the stability factor has been derived elsewhere.*

$$S = \frac{R_b + R_e}{R_e + R_b(1 - \alpha)} \quad \text{where } R_b = R_1 R_2/(R_1 + R_2)$$

Here, $R_b = (50 \times 20)/70 = 14 \cdot 3 \text{ k}\Omega$

so $S = \dfrac{14 \cdot 3 + 2 \cdot 5}{2 \cdot 5 + 14 \cdot 3(0 \cdot 02)} = \underline{6.}$

164. The condition for maintenance of oscillation in a dynatron oscillator is $R \leqslant L/Cr_a$,

∴ $R \leqslant 150 \times 10^{-6}/500 \times 10^{-12} \times 90{,}000 \leqslant \underline{3 \cdot 33 \ \Omega.}$

The corresponding frequency of oscillation

$$= \frac{1}{2\pi} \sqrt{\frac{1}{LC}\left(1 - \frac{R}{r_a}\right)}$$
$$= \underline{581 \text{ kc/s.}}$$

165. The condition for maintenance of oscillation in a tuned-anode oscillator is $M = (L + r_a RC)/\mu$, where M is the mutual inductance between the two coils.

With the higher anode voltage

$$M = (175 + 220 \times 10^{-6} \times 18 \times 9{,}000)/9 \ \mu\text{H} = 23 \cdot 4 \ \mu\text{H}.$$

The coefficient of coupling

$$= \frac{23 \cdot 4}{\sqrt{175 \times 60}} = \underline{0 \cdot 228.}$$

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* See, for example, H. Henderson, *Transistors*, B.B.C. Engineering Training Department, Supplement No. 12, 1958. Methods of obtaining bias and stabilizing the operating point are described in many books, e.g., J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957. J. D. Ryder, *Electronic Fundamentals and Applications*, Pitman, 3rd Edition, 1964. A. H. Seidman and S. L. Marshall, *Semiconductor Fundamentals*, Wiley, 1963. L. M. Krugman, *Fundamentals of Transistors*, Rider and Chapman and Hall, 2nd Revised Edition, 1958. Mullard Reference Manual on Transistor Circuits, 1st Edition, 1960.

With the lower anode voltage

$$M = (175 + 220 \times 10^{-6} \times 18 \times 11{,}000)/9 \ \mu\text{H} = 24 \cdot 3 \ \mu\text{H}.$$

The coefficient of coupling

$$= \frac{24 \cdot 3}{\sqrt{175 \times 60}} = \underline{0 \cdot 237.}$$

166. Maximum mutual inductance between grid and anode coils is $0 \cdot 4 \sqrt{200 \times 35} \ \mu\text{H} = 33 \cdot 5 \ \mu\text{H}.$

The condition for maintenance of oscillation is given in the previous solution.

$$\therefore \quad 33 \cdot 5 = (200 + 0 \cdot 0005 \times R \times 6{,}000)/8, \text{ i.e. } R = 22 \cdot 7 \, \Omega,$$

where R is the effective resistance of the tuned circuit.

The aerial may therefore contribute $(22 \cdot 7 - 8) \, \Omega$, i.e. $14 \cdot 7 \, \Omega$, to the tuned circuit.

Since the aerial is tuned the resistance reflected into the tuned circuit is $\omega^2 M^2 / R_1$, where $R_1 = 24 \, \Omega$ and M is the mutual inductance between the aerial and the tank circuit.

$$\therefore \qquad\qquad \omega M = \sqrt{24 \times 14 \cdot 7} = 18 \cdot 8.$$

But ω is approximately

$$\sqrt{\frac{1}{LC}} = \frac{10^6}{\sqrt{200 \times 0 \cdot 0005}} = 3 \cdot 167 \times 10^6.$$

$$\therefore \qquad\qquad M = 18 \cdot 8/3 \cdot 167 \ \mu\text{H} = \underline{5 \cdot 94 \ \mu\text{H}.}$$

This is the maximum permissible value of M if oscillations are to be maintained.

167. The frequency of oscillation of a tuned-anode oscillator is given by $\dfrac{1}{2\pi} \sqrt{\dfrac{r_a + R}{r_a LC}}$,

where R, L and C are the usual constants of the anode coil.

$$\therefore \qquad 25 = \frac{1}{2\pi} \sqrt{\frac{1{,}800 + 11}{1{,}800 \times 0 \cdot 6 \times C}} \quad \therefore \ C = 67 \cdot 9 \times 10^{-6} \ \text{F}.$$

The condition for maintenance of oscillation is given in the previous two solutions.

$$\therefore \qquad M = (0 \cdot 6 + 67 \cdot 9 \times 10^{-6} \times 11 \times 1{,}800)/5 = 0 \cdot 389 \ \text{H}.$$

M must have at least this value for oscillation.

The maximum mutual inductance available is

$$0.32 \times 0.6 = 0.192 \text{ H}$$

∴ the circuit will not oscillate.

From the maintenance condition the value of C corresponding to the mutual inductance 0.192 H is found to be $18.2\ \mu\text{F}$.

This capacitance gives a frequency of oscillation of 48.3 c/s.

168. The condition for maintenance of oscillation in a tuned-grid circuit is:

$$\frac{\mu M}{C}\left\{1 - \frac{M}{\mu L}\right\} = Rr_a$$

where M is the mutual inductance between the grid and anode coils.

∴ M must be as large as $\dfrac{\mu L}{2} - \sqrt{\left(\dfrac{\mu L}{2}\right)^2 - Rr_a LC}$

$$= \left[\frac{9 \times 180}{2} - \sqrt{\left(\frac{9 \times 180}{2}\right)^2 - 26 \times 11{,}000 \times 180 \times 0.0012}\right] \mu\text{H}$$

$$= 39\ \mu\text{H}.$$

But the maximum available M is $0.3\sqrt{180 \times 50} = 28.5\ \mu\text{H}$.

∴ the circuit will not oscillate.

169. As in the previous question the maintenance condition is

$$\frac{\mu M}{C}\left\{1 - \frac{M}{\mu L}\right\} = Rr_a.$$

Since $M < L$ and $\mu = 10$, then $\dfrac{\mu M}{C}$ is approximately equal to Rr_a

∴ M is approximately $\dfrac{r_a RC}{\mu} = \dfrac{10{,}000 \times 100 \times 0.01 \times 10^{-6}}{10}$ H

$$= 1 \text{ mH}.$$

This is the condition for sustained oscillation.

170. The theorem is well known and will not be proved here.

The equivalent circuit of the tuned-anode oscillator is shown.
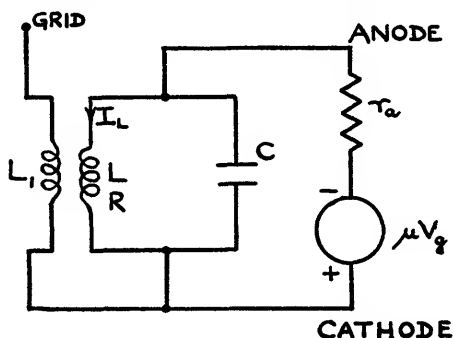
$$Z = \frac{(R + j\omega L)\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}}$$

$$I_L = V_a/(R + j\omega L).$$

$$\therefore \qquad V_g = -j\omega M . V_a/(R + j\omega L).$$

$$\therefore \qquad N = -j\omega M/(R + j\omega L).$$

Substitution of the expressions for Z and N in $Z + r_a/(1 + \mu N) = 0$, equating real and imaginary parts and rearranging gives:

(a) the frequency of oscillation $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{r_a + R}{r_a LC}}$,

and (b) the maintenance condition $M = (L + r_a RC)/\mu.$

171. The frequency of oscillation* $= 1/2\pi\sqrt{C(L_1 + L_2 + 2M)} = f.$ If $M = 0, f = 1/2\pi\sqrt{0 \cdot 1 \times 10^{-6} \times 40 \times 10^{-3}} = 2{,}517$ c/s.

In the second case, $2{,}000 = 1/2\pi\sqrt{0 \cdot 1 \times 10^{-6}(40 + 2M)10^{-3}}$, where M is in mH.

$$\therefore \qquad M = 11 \cdot 67 \text{ mH}.$$

The coefficient of coupling $= 11 \cdot 67/20 = 0 \cdot 584.$

172. For sustained oscillations* $\mu = (L_1 + M)/(L_2 + M).$

$$\therefore \qquad 20 = (45 + M)/(15 + M)$$

where M is in mH.

i.e. $\qquad M = 13 \cdot 42 \text{ mH}.$

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* See solution to Question 173.

173. The equivalent circuit is as shown.
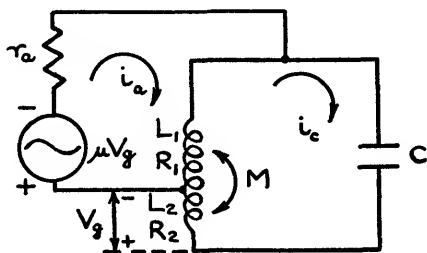
The voltage across the bottom coil is

$$V_g = (R_2 + j\omega L_2)i_c + j\omega M(i_c - i_a) \qquad . \qquad . \quad (1)$$

Applying Kirchhoff's second law to the i_a and i_c loops:

$$i_a(r_a + R_1 + j\omega L_1) = i_c\{R_1 + j\omega(L_1 + M)\} - \mu V_g \quad . \quad (2)$$

and

$$i_a(R_1 + j\omega L_1) = i_c(R_1 + j\omega L_1 + j\omega M + 1/j\omega C) + V_g \quad (3)$$

Substituting (1) in (2) and (3) and then dividing (2) and (3)

$$\frac{r_a + R_1 + j\omega L_1 - \mu j\omega M}{R_1 + j\omega L_1 + j\omega M}$$
$$= \frac{R_1 - \mu R_2 + j\omega(L_1 + M) - j\omega(L_2 + M)\mu}{R_1 + R_2 + j\omega L_1 + j\omega L_2 + 2j\omega M + 1/j\omega C} \qquad (4)$$

Simplifying, and equating the imaginary terms, gives:

$$\omega^2 = \frac{1 + R_1/r_a}{C\left[(L_1 + L_2 + 2M) + \dfrac{(R_1 L_2 + R_2 L_1)(1 + \mu)}{r_a}\right]} = (2\pi f)^2$$

where f is the frequency of oscillation.

Equating the real terms gives:

$$(r_a + R_1)(R_1 + R_2) - \omega^2(L_1 - \mu M)(L_1 + L_2 + 2M) + (L_1 - \mu M)/C$$
$$= R_1(R_1 - \mu R_2) - \omega^2(L_1 + M)(L_1 + M - \mu L_2 - \mu M).$$

174. The equivalent circuit is as shown.

$$V_g = i_c/j\omega C_2 \qquad . \qquad . \qquad . \qquad . \quad (1)$$

Applying Kirchhoff's second law to the i_a and i_c loops:

$$i_a/j\omega C_1 = i_c(R + j\omega L + 1/j\omega C_1 + 1/j\omega C_2) \qquad . \quad (2)$$

and $\quad i_a(r_a + 1/j\omega C_1) = i_c(1/j\omega C_1 - \mu/j\omega C_2)$ $\quad$. $\quad$. $\quad$. (3)
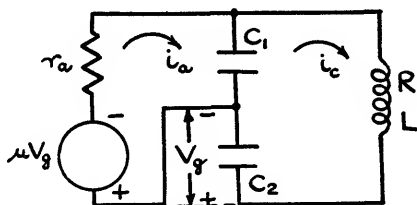
Dividing (2) and (3), simplifying, and equating imaginary and real terms, as in the previous solution gives:

$$\omega^2 = \frac{1}{L}\left\{\frac{1}{C_2} + \frac{1}{C_1}(1 + R/r_a)\right\} = (2\pi f)^2$$

where f is the frequency of oscillation.

$(1 + \mu)/\omega^2 C_1 C_2 = r_a R + L/C_1$ which is the condition for maintenance of oscillations.

175. The equivalent circuit is as shown.
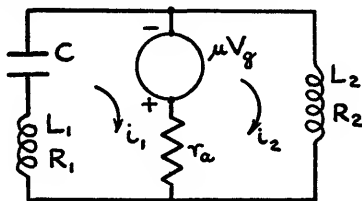
$$V_g = -(R_1 + j\omega L_1)i_1 \qquad . \qquad . \qquad . \quad (1)$$

Applying Kirchhoff's second law to the i_1 and i_2 loops:

$$i_1(r_a + R_1 + j\omega L_1 + 1/j\omega C) - \mu V_g = i_2 r_a \quad . \qquad . \quad (2)$$

and $\qquad i_2(r_a + R_2 + j\omega L_2) - r_a i_1 + \mu V_g = 0 \qquad . \qquad . \quad (3)$

From (1), (2) and (3), proceeding as in the previous two solutions:

$$\omega^2 = \frac{1 + R_2/r_a}{C[L_1 + L_2 + (R_1 L_2 + R_2 L_1)(1 + \mu)/r_a]} = (2\pi f)^2$$

where f is the frequency of oscillation.

$r_a(R_1 + R_2) + R_1 R_2(1 + \mu) + L_2/C - \omega^2 L_1 L_2(1 + \mu) = 0$ which is the condition for maintenance of oscillations.

176. The equivalent circuit is shown.

EQUIVALENT
CIRCUIT OF
CRYSTAL
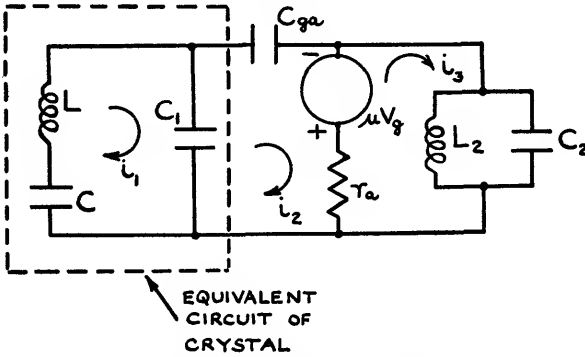
$$V_g = - i_1(j\omega L + 1/j\omega C) \qquad . \qquad . \qquad . \quad (1)$$

Applying Kirchhoff's second law to the three meshes in turn:

$$i_1(j\omega L + 1/j\omega C + 1/j\omega C_1) = i_2/j\omega C_1 \qquad . \qquad . \quad (2)$$

$$i_1(j\omega L\mu + \mu/j\omega C - 1/j\omega C_1) + i_2(1/j\omega C_1 + 1/j\omega C_{ga} + r_a) = i_3 r_a \quad (3)$$

and
$$i_1(j\omega L\mu + \mu/j\omega C)$$
$$+ i_2 r_a - i_3\{r_a + j\omega L_2/(1 - \omega^2 L_2 C_2)\} = 0 \quad (4)$$

Eliminating i_3 from (3) and (4), dividing by (2), cross-multiplying and equating the imaginary terms gives:

$$\omega^2 = \frac{1}{LC}\left[\frac{1 + (1 + C_1/C_{ga} + C/C_{ga})/\mu}{1 + (1 + C_1/C_{ga})/\mu}\right] = (2\pi f)^2$$

where f is the frequency of oscillation.

177. The feedback network is shown in the diagram.
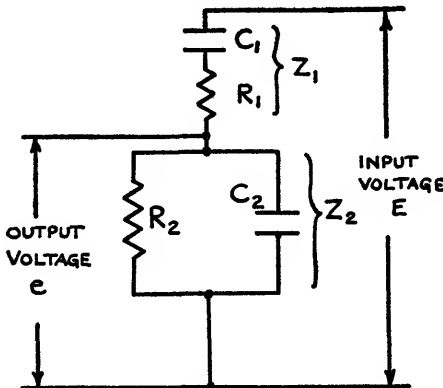
$$e = E \cdot Z_2/(Z_1 + Z_2)$$

$$Z_1 = R_1 + 1/j\omega C_1$$

$$Z_2 = R_2/(1 + jR_2\omega C_2)$$

$$\therefore \qquad e = \frac{R_2 E}{\left(R_2 + R_1 + \dfrac{R_2 C_2}{C_1}\right) + j\left(R_1 R_2 \omega C_2 - \dfrac{1}{\omega C_1}\right)}.$$

$\therefore$ e will be in phase with E at a frequency given by:

$$\omega^2 = \frac{1}{C_1 C_2 R_1 R_2} \text{ or } f = \frac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}}.$$

The system will then oscillate at this frequency if the associated amplifier gain is greater than $1 + R_1/R_2 + C_2/C_1$.
    If $C_1 = C_2 = 0 \cdot 001\ \mu\text{F}$ and $R_1 = R_2 = 120{,}000\ \Omega$,
$$f = 1{,}326 \text{ c/s}.$$

178. The phase-shift network is as shown.
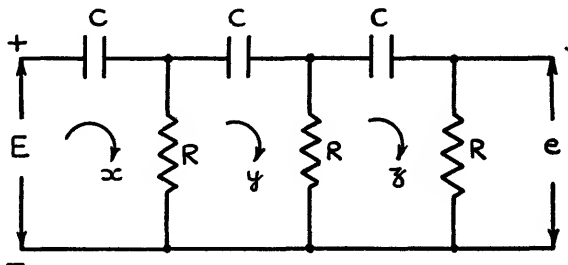
Let currents x, y and z circulate as shown:

Then
$$\left(R + \frac{1}{j\omega C}\right) x - Ry = E. \qquad . \qquad . \ (1)$$

$$\left(2R + \frac{1}{j\omega C}\right) y - Rx - Rz = 0. \qquad . \qquad . \ (2)$$

and
$$\left(2R + \frac{1}{j\omega C}\right) z - Ry = 0. \qquad . \qquad . \ (3)$$

From (3)
$$y = \left(2R + \frac{1}{j\omega C}\right) z/R. \qquad . \qquad . \ (4)$$

From (1) and (4) $x = \dfrac{E + \left(2R + \dfrac{1}{j\omega C}\right) z}{R + \dfrac{1}{j\omega C}}.$ . . . . (5)

Substituting (4) and (5) in (2) gives:

$$z\left(R^3 + \frac{6R^2}{j\omega C} - \frac{5R}{\omega^2 C^2} - \frac{1}{j\omega^3 C^3}\right) = ER^2. \quad . \quad . \ (6)$$

There is no j term when $\dfrac{6R^2}{\omega C} = \dfrac{1}{\omega^3 C^3}$, i.e. when $\omega^2 = 1/6R^2 C^2$,

i.e.   when $f = \dfrac{1}{2\pi RC\sqrt{6}}$.

At this frequency $z(-29R^3) = ER^2$ $\therefore$ $e = Rz = -E/29$, i.e. the attenuation ratio of the network is 29, and the total phase shift is 180° when $f = 1/2\pi RC\sqrt{6}$.

In this case, $f = 1/2\pi \times 10^5 \times 0.0005 \times 10^{-6}\sqrt{6} = 1{,}300$ c/s.

179. The phase-shift network is as shown.
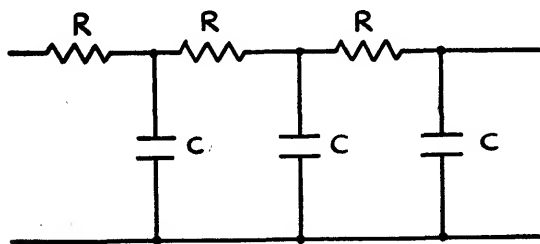
Proceeding with the analysis as in the previous solution it is found that the frequency f at which the network produces 180° phase shift is $\sqrt{6}/2\pi CR$.

In this case $f = \sqrt{6}/2\pi \times 10^5 \times 0.0005 \times 10^{-6} = 7{,}800$ c/s.

The attenuation ratio of the network is again 29.

180. Let currents x, y, z and p circulate as shown.
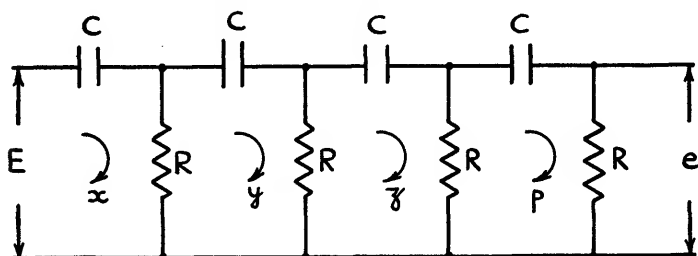
Then
$$\left(R + \frac{1}{j\omega C}\right) x - Ry = E. \qquad . \qquad . \qquad (1)$$

$$\left(2R + \frac{1}{j\omega C}\right) y - Rx - Rz = 0. \qquad . \qquad . \qquad (2)$$

$$\left(2R + \frac{1}{j\omega C}\right) z - Ry - Rp = 0. \qquad . \qquad . \qquad (3)$$

and
$$\left(2R + \frac{1}{j\omega C}\right) p - Rz = 0. \qquad . \qquad . \qquad (4)$$

From (4)
$$z = \left(2R + \frac{1}{j\omega C}\right) p/R \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (5)$$

From (3) and (5)
$$y = (3R^2 + 4R/j\omega C - 1/\omega^2 C^2)p/R^2 \qquad . \qquad . \qquad (6)$$

From (2), (5) and (6),
$$x = (4R^3 + 10R^2/j\omega C - 6R/\omega^2 C^2 - 1/j\omega^3 C^3)p/R^3 \qquad . \qquad (7)$$

From (1), (6) and (7),
$$p\left[R^4 + \frac{10R^3}{j\omega C} - \frac{15R^2}{\omega^2 C^2} - \frac{7R}{j\omega^3 C^3} + \frac{1}{\omega^4 C^4}\right] = ER^3 \qquad . \qquad (8)$$

There is no imaginary term when $\omega^2 = 0.7/R^2 C^2$.

At this frequency, $pR = e = -E/18.39$,

i.e. the attenuation ratio of the network is 18.39.

In this case
$$f = \sqrt{0.7}/2\pi RC = \sqrt{0.7}/2\pi \times 10^5 \times 0.0005 \times 10^{-6} = \underline{2{,}663 \text{ c/s.}}$$

181. When S is closed, the voltage across C rises exponentially as shown until it reaches V_s. C is then suddenly discharged until the voltage across it falls to V_e.
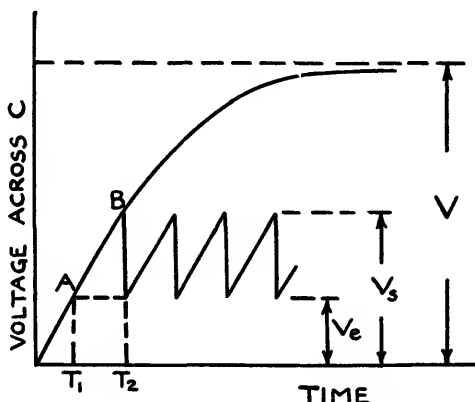
The voltage across $C(V_c)$ at any time t after closing S is given by

$$V_c = V(1 - e^{-t/CR})$$

∴ at points A and B $V_e = V(1 - e^{-T_1/CR})$

and $V_s = V(1 - e^{-T_2/CR})$

∴ period of oscillation

$$T = T_2 - T_1 = CR \log_e \{(V - V_e)/(V - V_s)\}.$$

182. Using the solution to the previous problem

$$\frac{1}{100} = 0.04 \times 10^{-6} \, R \log_e (80/40), \text{ so that } R = 360.7 \text{ k}\Omega.$$

When supply voltage drops to 198 V,

$$\frac{1}{f} = 0.04 \times 10^{-6} R \log_e (78/38), \text{ so that } f = 96.34 \text{ c/s,}$$

i.e. f drops by 3.66%.

183. Using the solution to Question 181 and the same symbols,

$$T = 0.01 \times 10^{-6} \times 500 \times 10^3 \log_e \{230/(250 - V_s)\}$$

Also $V_s - V_e = 100$ V, so that

$$V_s = 120 \text{ V and } T = 2.83 \times 10^{-3} \text{ sec.}$$

Since control ratio is 30, $V_g = (-120/30) \text{ V} = -4 \text{ V.}$

184. An expression for the period of oscillation of a multivibrator has been developed by Seely.* For a symmetrical multivibrator the expression can be reduced to:

$$T = 2CR_g \log_e \left[\frac{V_1 - V_s}{V_g} \right]$$

where here $R_g = 50 \times 10^3\,\Omega$, $C = 0.005\,\mu\text{F}$, $V_s = 250\,\text{V}$, $V_1 = 110\,\text{V}$ and $V_g = -20\,\text{V}$.

$\therefore$ $\qquad\qquad f = 1/T = \underline{1{,}027\ \text{c/s.}}$

185. The frequency of a 300-m signal is 1 Mc/s.

$\therefore$ $\qquad$ beat frequency $= (1 \cdot 3 - 1)\ \text{Mc/s} = \underline{300\ \text{kc/s.}}$

The frequency of a 400-m signal is 0·75 Mc/s.

$\therefore$ new oscillator frequency $= (750 + 300)\ \text{kc/s} = \underline{1{,}050\ \text{kc/s.}}$

186. (*a*) For feedback type of oscillator,

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{10{,}000}{50} \right)^2 = \underline{4 \times 10^4.}$$

(*b*) For beat-frequency oscillator,

$$\frac{C_{\max}}{C_{\min}} = \left(\frac{100 + 10}{100 + 0 \cdot 05} \right)^2 = \underline{1 \cdot 2.}$$

187. The solution to this problem can be found elsewhere.†

188. The method of solution is the same as that for Problem 187 so is not given here.

189. The thermal agitation noise voltage is given by

$$E^2 = 4kT \int_{f_1}^{f_2} R\,df$$

where E is the r.m.s. noise voltage in volts.

$\quad T$ „ „ absolute temperature.

$\quad R$ „ „ resistance in ohms.

$\quad f$ „ „ frequency in c/s.

$\quad f_1$ and f_2 are the limits of the frequency band.

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* See S. Seely, *Electron-tube Circuits*, McGraw-Hill, 2nd Edition, 1958, p. 429.
† See J. D. Ryder, *Engineering Electronics*, McGraw-Hill, 1957, Chapter 12.

When the integration is carried out over the band,

$$E = \sqrt{4kTR(f_2 - f_1)} \text{ volts.}$$

In this case

$$E = \sqrt{4 \times 1{\cdot}38 \times 10^{-23} \times 290 \times 1{,}000(10^7)} = \underline{12{\cdot}66\,\mu\text{V.}}$$

190. The effect of thermal-agitation noise may be expressed either as an e.m.f. in series with the resistor considered noiseless, or as a constant-current generator in parallel with the resistor considered noiseless, as shown.
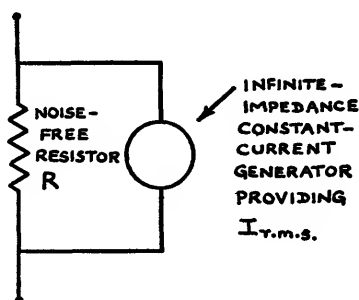
The output current of the generator is obtained by dividing the expression for the r.m.s. noise voltage, given in the previous solution, by R.

$$\therefore \qquad\qquad I_{\text{r.m.s.}} = \sqrt{\frac{4kT(f_2 - f_1)}{R}} \text{ amperes.}$$

In this case $I_{\text{r.m.s.}} = \underline{12{\cdot}66 \times 10^{-9} \text{ A.}}$

191. The r.m.s. value of the noise-current components in a bandwidth $(f_2 - f_1)$ c/s is given by

$$(i_{\text{r.m.s.}})^2 = 2\,eI(f_2 - f_1) \text{ amperes}^2$$

where I is the average current in amperes.

$$\therefore\ i_{\text{r.m.s.}} = \sqrt{2 \times 1{\cdot}602 \times 10^{-19} \times 10^{-3} \times 2 \times 10^4} = \underline{2{\cdot}53 \times 10^{-9} \text{ A.}}$$

192. The mean square of the fluctuation components of the current depends only on the magnitude of the emission current I_0 and the frequency bandwidth.

$$I_{\text{r.m.s.}} = \sqrt{2\, eI_0(f_2 - f_1)} \text{ amperes}$$
$$= \sqrt{2 \times 1\cdot59 \times 10^{-19} \times 10 \times 10^{-3} \times 20 \times 10^3}\, \text{A}$$
$$= 7\cdot98 \times 10^{-9}\, \text{A}.$$

193. As in the solution to Question 189 the thermal-agitation noise voltage is given by

$$E^2 = 4kT \int_{f_1}^{f_2} R\, df.$$

The impedance of the parallel combination is
$$(R_1 - jR_1^2\omega C)/(1 + R_1^2\omega^2 C^2)$$
$$\therefore \qquad R = R_1/(1 + \omega^2 C^2 R_1^2)$$
$$\therefore \qquad E^2 = 4kT \int_0^\infty R_1\, df/(1 + 4\pi^2 f^2 C^2 R_1^2) = kT/C$$
$$\therefore \qquad E = \sqrt{kT/C}.$$

194. The value of the noisy resistor is approximately $2\cdot5/g_m$* ohms $= 2\cdot5/2\cdot6 \times 10^{-3} = 961\,\Omega.$

195. The equivalent resistor is $\dfrac{2\cdot5}{g_m}\left(\dfrac{I_a}{I_a + I_s}\right)\left(1 + \dfrac{8I_s}{g_m}\right)$ *† ohms

$$= \frac{2\cdot5}{9 \times 10^{-3}}\left(\frac{10}{12\cdot5}\right)\left(1 + \frac{8 \times 2\cdot5 \times 10^{-3}}{9 \times 10^{-3}}\right) \text{ ohms} = 716\,\Omega.$$

196. The equivalent resistance‡ $= (20R_g^2 + 4 \times 10^4 I_a/g_m^3)I$ ohms where R_g is the shunt resistance of the grid circuit in ohms, and I is the control-grid current in amperes.

$\therefore$ the resistance

$$= \{20 \times 10^{10} + 4 \times 10^4 \times 10^{-3}/(5 \times 10^{-3})^3\}0\cdot01 \times 10^{-6}\,\Omega$$
$$= 2{,}003\cdot2\,\Omega.$$

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* See W. A. Harris, 'Space Charge Limited Current Fluctuations in Vacuum Tube Amplifiers and Input Systems,' *R.C.A. Review*, 5, p. 505, 1941 and 6, p. 114, 1941. K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1st Edition, 1948, Chapter 12. L. B. Arguimbau and R. B. Adler, *Vacuum Tube Circuits and Transistors*, Wiley, 1956, Chapter 15.

† D. O. North, 'Fluctuations in SpaceCharge Limited Currents in Multi-collectors,' *R.C.A. Review*, 5, p. 244, 1940.

‡ See K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1st Edition, 1948, Chapter 12.

197. Current through R due to V_1 with V_2 short-circuited

$$= \frac{R_2}{R + R_2} \left\{ \frac{V_1}{R_1 + RR_2/(R + R_2)} \right\}$$

$$= R_2 V_1/(RR_1 + R_2 R_1 + RR_2)$$

Similarly, current through R due to V_2 with V_1 short-circuited

$$= R_1 V_2/(RR_1 + R_2 R_1 + RR_2)$$

The Johnson formula for thermal noise generated in a resistor R_3 is:

$$\overline{V_n^2} = 4kTR_3 df$$

therefore total mean-square noise current through R per unit bandwidth

$$= \frac{4k(T_1 R_1 R_2^2 + T_2 R_2 R_1^2)}{(RR_1 + R_2 R_1 + RR_2)^2}$$

$$= \frac{4k R_1 R_2 (T_1 R_2 + T_2 R_1)}{(RR_1 + R_2 R_1 + RR_2)^2}$$

Noise power $P = \dfrac{4k R_1 R_2 (T_1 R_2 + T_2 R_1) R}{\{(R_1 + R_2) R + R_1 R_2\}^2}$

This is a maximum when $\underline{R = R_1 R_2/(R_1 + R_2)}$

$\therefore$
$$P_{\text{max.}} = \frac{4k R_1 R_2 (T_1 R_2 + T_2 R_1) R_1 R_2}{(R_1 + R_2)\{R_1 R_2 + R_1 R_2\}^2}$$

$$= \underline{k(T_1 R_2 + T_2 R_1)/(R_1 + R_2)}$$
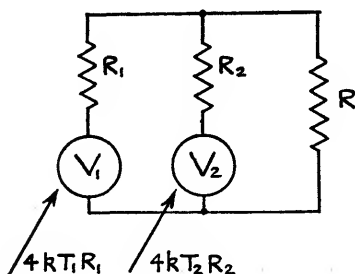
198. (*a*) For two resistors R_1 and R_2 in series, total resistance

$$= R_1 + R_2$$

∴ per unit bandwidth

$$4kT_1R_1 + 4kT_2R_2 = T_{\text{eff.}}\, 4k(R_1 + R_2)$$

where $T_{\text{eff.}}$ is the effective noise temperature.

$$\therefore \qquad\qquad T_{\text{eff.}} = \frac{T_1R_1 + T_2R_2}{(R_1 + R_2)}$$

Using the statement, $T_{\text{eff.}} = \dfrac{T_1R_1}{(R_1 + R_2)} + \dfrac{T_2R_2}{(R_1 + R_2)}$

$$= \frac{T_1R_1 + T_2R_2}{(R_1 + R_2)}$$

(*b*) When the resistors are in parallel

$$\frac{4kT_1R_1R_2{}^2}{(R_1 + R_2)^2} + \frac{4kT_2R_2R_1{}^2}{(R_1 + R_2)^2} = T_{\text{eff.}}\, \frac{R_1R_2}{(R_1 + R_2)}$$

$$\therefore \qquad\qquad T_{\text{eff.}} = \frac{T_1R_2 + T_2R_1}{(R_1 + R_2)}$$

Using the statement, $T_{\text{eff.}} = \dfrac{T_1\left(\dfrac{1}{R_1}\right)}{\dfrac{1}{R_2} + \dfrac{1}{R_1}} + \dfrac{T_2\left(\dfrac{1}{R_2}\right)}{\dfrac{1}{R_2} + \dfrac{1}{R_1}}$

$$\therefore \qquad\qquad T_{\text{eff.}} = \frac{T_1R_2 + T_2R_1}{(R_1 + R_2)}$$

Thus, the statement has been verified for the two cases.

Using the statement:

$$T_{\text{eff.}} = T_1\{1 - \exp(-2\alpha l)\} + T_2\exp(-2\alpha l) \quad \text{(see diagram)}$$

$1 - \exp(-2\alpha l)$

$\exp(-2\alpha l)$
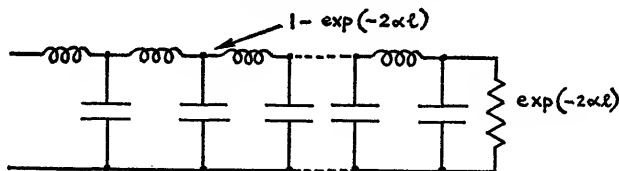
199. Open-circuit voltage due to R_1 with sources of e.m.f. in R_2 and R_3 branches short-circuited

$$= \frac{4kT_1R_1}{\left\{R_1 + \dfrac{R_2R_3}{(R_2 + R_3)}\right\}^2} \left\{\frac{R_2R_3}{(R_2 + R_3)}\right\}^2$$

$$= \frac{4kT_1R_1R_2{}^2R_3{}^2}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

By repeating this calculation for the other two resistors and adding the results the total voltage

$$= \frac{4kR_1R_2R_3(T_1R_2R_3 + T_3R_1R_2 + T_2R_3R_1)}{(R_1R_2 + R_1R_3 + R_2R_3)^2}$$

For single resistor at temperature T this voltage squared would be

$$\frac{4kTR_1R_2R_3}{(R_1R_2 + R_1R_3 + R_2R_3)}$$

because for the three resistors in parallel

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$\therefore \qquad T = \frac{T_1R_2R_3 + T_3R_1R_2 + T_2R_3R_1}{R_1R_2 + R_1R_3 + R_2R_3}$$

When the resistors are in series:

$$R = R_1 + R_2 + R_3$$

and $4kT_1R_1 + 4kT_2R_2 + 4kT_3R_3 = 4kT(R_1 + R_2 + R_3)$

$$\therefore \qquad T = \frac{T_1R_1 + T_2R_2 + T_3R_3}{(R_1 + R_2 + R_3)}$$

200. To receive maximum signal power from aerial the input impedance of the circuit is made equal to the radiation resistance of the aerial.

In bandwidth df power radiated from aerial

$$= d\overline{V_n{}^2}/4R_l$$

$\therefore$ thermal radiation power picked up

$$= 4kTR_l df/4R_l = kTdf \text{ watts}$$

If P is the noise power generated in the receiver and G is the power gain,

$$2(Gk\,300 + P) = Gk\,900 + P$$

$$\therefore \qquad\qquad P = 300\,Gk$$

Noise figure $= 10\log_{10} \times$

$$\left\{ \frac{\text{Output noise power from actual receiver at room temperature}}{\text{Output noise power from a perfect receiver that introduces no noise}} \right\}$$

$$\therefore \qquad \text{noise figure} = 10\log_{10}\left\{\frac{300\,Gk + P}{300\,Gk}\right\} = 10\log_{10} 2$$

$$\simeq 3\,\text{dB}.$$

201. Let calibration of signal generator read a power of P watts. Then voltage generated in signal generator $= V$ where $V^2/(4 \times 500) = P$ or $V^2 = 2 \times 10^3\,P$.

Signal on grid of triode $= (V \times 1{,}000)/1{,}500 = V/1{\cdot}5$.

Signal output from receiver $\propto (V/1{\cdot}5)^2$.

Noise output power produced by a 500-Ω resistor in parallel with a 1,000-Ω resistor $\propto \bar{V}_n^2$, i.e. $4kT\dfrac{1{,}000}{3}$. Δf since $1/500 + 1/1{,}000 = 3/1{,}000$.

$$4kT\frac{1{,}000}{3}\Delta f = 4 \times 1{\cdot}38 \times 10^{-23} \times \frac{1{,}000}{3} \times 300 \times 10^4$$

$$= 5{\cdot}52 \times 10^{-14}$$

$$\therefore \qquad (V/1{\cdot}5)^2 = 5{\cdot}52 \times 10^{-14}$$

$$\therefore \qquad V^2 = 5{\cdot}52 \times 10^{-14} \times 1{\cdot}5^2 = 2 \times 10^3\,P$$

$$\text{so } P = 6{\cdot}2 \times 10^{-17}\,W$$

202. The mean-square deflection $\bar{\theta}^2$ is given by:

$$\frac{1}{2}\,C\bar{\theta}^2 = \frac{kT}{2}$$

where C is the specific couple of the suspension

$$\therefore \qquad \bar{\theta}^2 = \frac{kT}{C} = \frac{1{\cdot}38 \times 10^{-23} \times 300}{10^{-10}} = 41{\cdot}4 \times 10^{-12}$$

i.e. $\qquad \bar{\theta} = \sqrt{41{\cdot}4 \times 10^{-12}} = 6{\cdot}44 \times 10^{-6}\,\text{radian.}$

Thus, r.m.s. deflection $= 2 \times$ (optical arm length) $\times \sqrt{\bar{\theta}^2}$
$$= 2 \times 1{,}000 \times 10^{-6} \times 6{\cdot}44 \text{ mm}$$
$$= 0{\cdot}0129 \text{ mm.}$$

Minimum detectable current $= \dfrac{0{\cdot}0129}{75} \times 1{,}000 \text{ m}\mu\text{A}$
$$= 0{\cdot}172 \text{ m}\mu\text{A.}$$

203. Torque $= 50i \times 1 \times 10^{-4} = 5i \times 10^{-3} = C\theta$ where i is the current and C the specific couple of the suspension.

$\therefore \ \theta = 5 \times 10^{-10} \times 10^{-3}/10^{-10} = 5 \times 10^{-3}$ radian

$\therefore$ deflection $= 2 \times 5 \times 10^{-3} \times 1{,}000$ mm
$$= 10 \text{ mm} = \underline{1 \text{ cm.}}$$

As in the previous Solution:

$$\frac{1}{2} C \bar{\theta}^2 = \frac{kT}{2}$$

$\therefore \qquad \bar{\theta} = \left[\dfrac{1{\cdot}38 \times 10^{-23} \times 300}{10^{-10}} \right]^{\frac{1}{2}} = 6{\cdot}44 \times 10^{-6}$ radian

Thus, $\quad 6{\cdot}44 \times 10^{-6} \times 10^{-10} = 5i \times 10^{-3}$

i.e. $\qquad\qquad\qquad i = \underline{1{\cdot}29 \times 10^{-13} \text{ A}}$

204. The carrier frequency

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times 0{\cdot}001 \times 10^{-6}}} = 712 \text{ kc/s.}$$

$\therefore$ sidebands are of frequencies 712 ± 10 kc/s,

i.e. frequency range occupied is 702 to 722 kc/s.

205. Let amplitude of carrier current $= I$, then sidebands each have amplitude $mI/2$.

Power in carrier $\propto I^2 = kI^2$ say.

,, ,, sidebands $= k\left(\dfrac{m^2 I^2}{4}\right) \times 2.$

$\therefore$ total power radiated $=$ (carrier power) $\left(1 + \dfrac{m^2}{2}\right),$

i.e. $8\cdot93^2 = 8^2 \left(1 + \dfrac{m^2}{2}\right).$

$\therefore$ $m = 0\cdot7$ and percentage modulation $= 70\%$.

Let new aerial current be I_1 when $m = 0\cdot8$.

Then $I_1^2 = 8^2 \left(1 + \dfrac{0\cdot8^2}{2}\right),$ i.e. $I_1 = 9\cdot19$ A.

206. As in the previous solution, total power radiated

$$= \text{(carrier power)} \left(1 + \frac{m^2}{2}\right).$$

$\therefore$ $10\cdot125 = 9 \left(1 + \dfrac{m^2}{2}\right),$ i.e. $m = 0\cdot5$.

Radiated power $= 9 \left(1 + \dfrac{(0\cdot5)^2}{2} + \dfrac{(0\cdot4)^2}{2}\right) = 10\cdot845$ kW.

207. Let the values of m for the several frequencies be m_1, m_2, m_3, etc. Then $m_1 + m_2 = m_3 +$, etc., must not exceed unity otherwise over-modulation will occur.

Total power of all sidebands

$$= \text{Carrier power} \left(\frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2} +, \text{etc.}\right).$$

If $m_1 + m_2 + m_3 + \ldots$, etc., does not exceed unity then $m_1^2 + m_2^2 + m_3^2 +$, etc., is less than unity.

$\therefore$ total power of all sidebands $< \frac{1}{2} \times$ carrier power.

208. Let $I_a = A + aV_g + bV_g{}^2$.

Let $\quad V_g = E_s \sin \omega_s t + E_c \sin \omega_c t$.

$\therefore \qquad I_a = A + a(E_s \sin \omega_s t + E_c \sin \omega_c t)$

$$+ b(E_s \sin \omega_s t + E_c \sin \omega_c t)^2$$

$$= A + \frac{b}{2}(E_c{}^2 + E_s{}^2) + aE_s \sin \omega_s t + aE_c \sin \omega_c t$$

$$- \frac{b}{2}E_c{}^2 \cos 2\omega_c t$$

$$- \frac{b}{2}E_s{}^2 \cos 2\omega_s t + bE_s E_c \cos (\omega_c - \omega_s)t$$

$$- bE_s E_c \cos (\omega_c + \omega_s)t.$$

Substituting the given values:

$$\underline{I_a = 10 \cdot 2725 + 3 \sin \omega_s t + 10 \sin \omega_c t}$$

$$\underline{- 0 \cdot 25 \cos 2\omega_c t - 0 \cdot 0225 \cos 2\omega_s t}$$

$$\underline{+ 0 \cdot 15 \cos (\omega_c - \omega_s)t - 0 \cdot 15 \cos (\omega_c + \omega_s)t \text{ mA.}}$$

where $\qquad\qquad \omega_s = 1{,}000t$ and $\omega_c = 4 \times 10^6 t$.

The carrier has an amplitude of 10 mA and the sidebands have amplitudes of 0·15 mA.

$\therefore \qquad\qquad\qquad m = (2 \times 0 \cdot 15)/10 = \underline{0 \cdot 03}.$

209. If f_m is the highest modulating frequency the highest sideband frequency is $f_c + f_m$, where f_c is the carrier frequency, and the lowest sideband frequency is $f_c - f_m$.

$\therefore$ bandwidth of transmission is $(f_c + f_m) - (f_c - f_m)$

$$= 2f_m = 2 \times 3 \cdot 4 \text{ kc/s} = \underline{6 \cdot 8 \text{ kc/s}}$$

The upper sideband will extend from (104 kc/s + 300 c/s) to (104 kc/s + 3·4 kc/s), i.e. from 104·3 kc/s to 107·4 kc/s.

The lower sideband will extend from (104 kc/s − 300 c/s) to (104 kc/s − 3·4 kc/s), i.e. from 103·7 kc/s to 100·6 kc/s.

These frequencies will be present in the transmitted wave in addition to the carrier frequency of 104 kc/s.

210. Let the carrier voltage be $E_c \sin \omega_c t$ and the audio-frequency voltage be $E_a \sin \omega_a t$.

Between the grid and cathode of valve (1) there is a voltage $(E_a \sin \omega_a t + E_c \sin \omega_c t)$.

Between the grid and cathode of valve (2) there is a voltage $(E_c \sin \omega_c t - E_a \sin \omega_a t)$.

$$\therefore I_{a_1} = I_a + a(E_a \sin \omega_a t + E_c \sin \omega_c t) + b_1(E_a \sin \omega_a t + E_c \sin \omega_c t)^2$$

and

$$I_{a_2} = I_a + a(E_c \sin \omega_c t - E_a \sin \omega_a t) + b_2(E_c \sin \omega_c t - E_a \sin \omega_a t)^2.$$

In the output $I_{a_1} - I_{a_2}$ results

i.e. $2aE_a \sin \omega_a t + \left(\dfrac{b_1 - b_2}{2} \right)(E_c^2 + E_a^2)$

$$- \left(\frac{b_1 - b_2}{2} \right)(E_c^2 \cos 2\omega_c t + E_a^2 \cos 2\omega_a t)$$

$$+ (b_1 + b_2)E_a E_c \{ \cos(\omega_c - \omega_a)t - \cos(\omega_c + \omega_a)t \}.$$

Therefore the carrier frequency is suppressed.

211. The circuit arrangement of the anode-modulated Class-C amplifier is shown below.

$$e_a = E_a + e_l = E_a + E_l \sin \omega_l t . \qquad . \qquad . \quad (1)$$

The amplitude of the r.f. current is directly proportional to the anode-supply voltage. This proportionality may be expressed as

$$I_t = k e_a \text{ where } k \text{ is a constant} . \qquad . \qquad . \quad (2)$$

From (1) and (2)

$$I_t = k E_a (1 + m \sin \omega_l t) \qquad . \qquad . \qquad . \quad (3)$$

where $\qquad\qquad m = E_l / E_a . \qquad . \qquad . \qquad . \qquad . \quad (4)$

The instantaneous r.f. current

$$i_t = I_t \sin \omega_c t = k E_a (1 + m \sin \omega_l t) \sin \omega_c t . \qquad . \quad (5)$$

which is of the usual form.

The r.f. voltage across the tank circuit is also proportional to the total anode-supply voltage, or

$$E_t = k' E_a (1 + m \sin \omega_l t) \text{ where } k' \text{ is a constant} \qquad . \quad (6)$$

Voltage v_a is the algebraic sum of E_a, $E_l \sin \omega_l t$ and e_t. The maximum value of e_t is $k'(E_a + E_l)$ as seen from equation (6). In practice k' is about 0·9. With $m = 1$ therefore the voltage v_a can reach the value $3·8E_a$.

With no modulation the power developed in the tank circuit is the carrier power

$$P_c = (E_t)^2 / 2R_l \text{ since } E_t \text{ is the peak value} \qquad . \qquad . \quad (7)$$

With modulation the power delivered is

$$P = P_c \left(1 + \frac{m^2}{2}\right) = (k' E_a)^2 \left(1 + \frac{m^2}{2}\right) \Big/ 2R_l . \qquad . \quad (8)$$

Assume that the anode-supply current $i_a = k'' e_a$ where k'' is a constant.

$$\therefore \qquad\qquad i_a = k''(E_a + E_l \sin \omega_l t) \qquad . \qquad . \qquad . \quad (9)$$

Average power from d.c. source is

$$E_a \overline{I_a} = k'' E_a{}^2 \text{ using (9)} \qquad . \qquad . \qquad . \quad (10)$$

Average power delivered by modulating transformer

$$= \tfrac{1}{2} E_l (k'' E_l) = \tfrac{1}{2} k'' m^2 E_a{}^2 \qquad . \qquad . \qquad . \quad (11)$$

Total average power supplied by anode-supply source is

$$P_a = k'' E_a{}^2 + k'' m^2 E_a{}^2 / 2 \qquad . \qquad . \qquad . \quad (12)$$

$$\text{Anode-circuit efficiency} = P/P_a = (k')^2 / 2k'' R_l \qquad . \quad (13)$$

In the problem under consideration:

(a) $m = E_t/E_a = 1,400/2,000 = 0.7$.

(b) Maximum value of $v_a = (E_a + E_t)(1 + k') = 3,400(1.9)$
$$= 6,460 \text{ V}.$$

(c) Power delivered by d.c. supply
$$= E_a \overline{I_a} = 2,000 \times 200/1,000 = \underline{400 \text{ W}}.$$

(d) Power delivered by modulation transformer $= E_t^2 k''/2$. From equation (10) $k'' E_a^2 = 400$ W, so $k'' = 400/(2,000)^2$.

∴ power delivered by transformer $= \underline{98 \text{ W}}$.

(e) R.f. output power without modulation $= 0.8 \times 400 = \underline{320 \text{ W}}$.

(f) R.f. output power with modulation $= 320(1 + m^2/2) = \underline{398 \text{ W}}$.

(g) The modulating voltage E_t causes the anode-supply current to have a component of amplitude $k'' E_t$ in phase with E_t. Thus the load on the modulation transformer is effectively a resistor whose resistance is $E_t/k'' E_t$, i.e. $E_a/k'' E_a = E_a/\overline{I_a}$. In this case the resistance $= 2,000/0.2 = 10,000 \, \Omega$.

212. The solution to this problem can be found in certain standard textbooks.*

213. The solution to the previous problem gives*
$$i = I[J_0(M) \sin ct + J_1(M)\{\sin (c + a)t - \sin (c - a)t\}$$
$$+ J_2(M)\{- \sin (c + 2a)t + \sin (c - 2a)t\}$$
$$+ J_3(M)\{\sin (c + 3a)t - \sin (c - 3a)t\}$$
$$+ J_4(M)\{- \sin (c + 4a)t + \sin (c - 4a)t\}$$
$$+ \ldots].$$

In this case $M = 50/5 = 10$ so that, disregarding signs

$J_0(M) \simeq 0.24, J_1(M) \simeq 0.05, J_2(M) \simeq 0.26, J_3(M) \simeq 0.05$, etc.,

as are readily found from graphs of these functions.*

∴ carrier amplitude $= 240 \, \mu\text{V/m}$ and sideband amplitudes are $50 \, \mu\text{V/m}, 260 \, \mu\text{V/m}, 50 \, \mu\text{V/m}$, etc.

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* See, for example, L. B. Arguimbau and R. B. Adler, *Vacuum Tube Circuits and Transistors*, Wiley, 1956, Chapter 12, Section 11.

214. The solution to this problem can be found in certain standard textbooks.*

215. The maximum permissible value for the time constant of a diode detector RC circuit is given by:†

$$RC \leqslant \sqrt{1 - m^2}/(2\pi f)m.$$

In this case, $220 \times 10^3 \times 100 \times 10^{-12} \leqslant \sqrt{1 - m^2}/(2\pi \times 6{,}000)$ m.

i.e. $\qquad\qquad\qquad m \leqslant 0{\cdot}77.$

216. The solution to this problem can be found elsewhere‡ but the I_a/V_g relationship for a valve having two control grids is stated without any discussion of why the relationship should hold. The reader can find such a discussion, however, in the book *Thermionic Valve Circuits*, by E. Williams.§

217. The oscillator frequency $= 700 + 465 = 1{,}165$ kc/s.

Let $I_a = a_0 + a_1 V_g + a_2 V_g^2 + a_3 V_g^3$ and assume $V_g = (E_s \cos \omega_s t - E_h \cos \omega_h t)$ where s refers to the signal and h to the oscillator. The expansion of the $a_3 V_g^3$ terms shows that there are eight frequencies in the output, namely f_s, $3f_s$, f_h, $3f_h$, $2f_s + f_h$, $2f_s - f_h$, $2f_h + f_s$, $2f_h - f_s$.

Undesired-signal frequencies of 351, 816, 1,867 and 2,797 kc/s produce 2 kc/s whistles because:

$$1{,}165 - 2 \times 351 = 463 \text{ kc/s}$$
$$2 \times 816 - 1{,}165 = 467 \text{ kc/s}$$
$$2 \times 1{,}165 - 1{,}867 = 463 \text{ kc/s}$$
$$2{,}797 - 2 \times 1{,}165 = 467 \text{ kc/s.}$$

218. (*a*) The oscillator circuit is as shown, where L is the tuning inductance, C_o the coil self capacitance and C_p the padding capacitance.

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* For example, see K. R. Sturley, *Radio Receiver Design*, Part 2, Chapman & Hall, 1954, p. 250.

† J. D. Ryder, *Electronic Fundamentals and Applications*, Pitman, 3rd Edition, 1964, pp. 493–4.

‡ K. R. Sturley, *Radio Receiver Design*, Part 1, Chapman & Hall, 3rd Edition, 1965, p. 430.

§ E. Williams, *Thermionic Valve Circuits*, Pitman, 4th Edition, 1961, pp. 270–2.
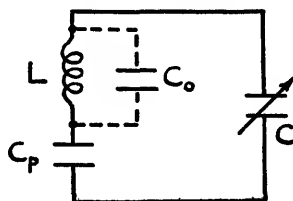
The oscillator frequency

$$f_h = 1/2\pi \sqrt{L\left(C_o + \frac{C_p C}{C_p + C}\right)} \quad . \quad . \quad . \quad (1)$$

This equation can be satisfied simultaneously for any two values f_{h_1} and f_{h_2} by a suitable choice of L and C_p. Suppose that for these two frequencies C has values C_1 and C_2.

Then
$$\frac{1}{(2\pi f_{h_1})^2} = L\{C_o + C_p C_1/(C_p + C_1)\} \quad . \quad . \quad (2)$$

and
$$\frac{1}{(2\pi f_{h_2})^2} = L\{C_o + C_p C_2/(C_p + C_2)\}. \quad . \quad . \quad (3)$$

From these two equations L and C_p can be found. Let f_{h_1} and f_{h_2} correspond to signal frequencies f_{s_1} and f_{s_2}. The most suitable values of f_{s_1} and f_{s_2} are those giving the least error over the frequency band and to find them the shape of the error/frequency curve must
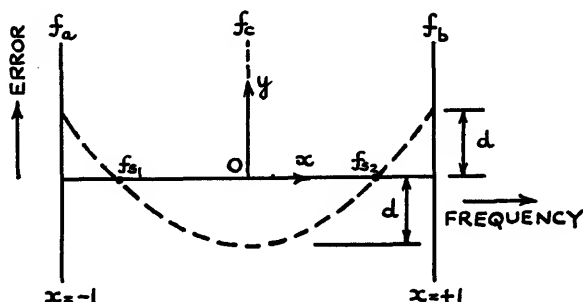
be known. Assume the error/frequency curve is parabolic and that there are equal errors at the ends and centre of the range as illustrated. Let frequency be represented by x and let $x = -1$ when $f = f_a$ the lowest frequency of the range and let $x = +1$ when $f = f_b$ the highest frequency of the range. The maximum error is d kc/s.

The general equation of the parabola is $y = ax^2 + bx + c$, but $dy/dx = 0$ when $x = 0$, $y = d$ when $x = \pm 1$ and $y = -d$ when $x = 0$.

$\therefore$ the equation is $y = d(2x^2 - 1)$.

Thus the frequencies for zero error are given by $x = \pm 1/\sqrt{2}$.

$\therefore$
$$f_{s_1} = f_c - 0 \cdot 707(f_c - f_a)$$
$$f_{s_2} = f_c + 0 \cdot 707(f_b - f_a).$$

In this case, $f_c = 1{,}025$ kc/s, $f_a = 550$ kc/s and $f_b = 1{,}500$ kc/s.

$\therefore$ $f_{s_1} = 689$ kc/s and $f_{s_2} = 1{,}361$ kc/s.

The capacitance required to tune the signal coil at these two frequencies can easily be calculated since the inductance is given as $156\,\mu$H. Not all this tuning capacitance is found in the tuning capacitor itself. It is reasonable to assume that about $40\,\mu\mu$F is due to stray capacitance (of range switch, wiring, self-capacitance of coil, trimmer). Hence the actual value of tuning capacitance is found by subtracting $40\,\mu\mu$F from the calculated figures. Similarly assume the oscillator tuning circuit has $20\,\mu\mu$F stray capacitance (there is no trimmer here).

The value of C at a signal frequency of 689 kc/s is therefore the calculated value $342\,\mu\mu$F $- 40\,\mu\mu$F $+ 20\,\mu\mu$F $= 322\,\mu\mu$F $= C_1$.

Similarly $C_2 = 67 \cdot 6\,\mu\mu$F.

Assume $C_0 = 10\,\mu\mu$F.

Also $f_{h_1} = (f_{s_1} + 465)$ kc/s and $f_{h_2} = (f_{s_2} + 465)$ kc/s.

From equations (2) and (3),

$$C_p = 288\,\mu\mu\text{F and } L = 117 \cdot 3\,\mu\text{H}.$$

(b) The oscillator circuit is as shown, where C_p is the trimmer capacitance. In the equations below C_t represents the total capacitance across L, including self and stray capacitances. The method of solution is similar to that of (a).

Zero error can now be obtained at three oscillator frequencies f_{h_1}, f_{h_2} and f_{h_3} corresponding to signal frequencies f_{s_1}, f_{s_2} and f_{s_3}. Suppose that for these frequencies C has values C_1, C_2 and C_3 respectively.
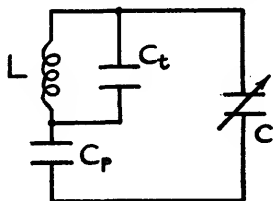
Then
$$\frac{1}{(2\pi f_{h_1})^2} = L\left[C_t + \frac{C_p C_1}{C_p + C_1}\right] \quad . \quad . \quad . \quad (1)$$

$$\frac{1}{(2\pi f_{h_2})^2} = L\left[C_t + \frac{C_p C_2}{C_p + C_2}\right] \quad . \quad . \quad . \quad (2)$$

and
$$\frac{1}{(2\pi f_{h_3})^2} = L\left[C_t + \frac{C_p C_3}{C_p + C_3}\right] \quad . \quad . \quad . \quad (3)$$

From these equations L, C_p and C_t can be found. Proceeding as before we can now assume the error/frequency curve is cubic of the form illustrated.
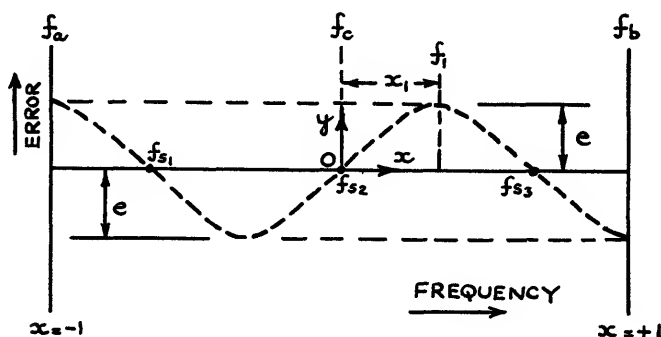
The general equation of the curve is

$$y = ax^3 + bx^2 + cx + d \quad . \quad . \quad . \quad (4)$$

When $x = 0$, $y = 0$ and when $x = \mp 1$, $y = \pm e$ so the equation reduces to

$$y = ax^3 + cx \quad . \quad . \quad . \quad . \quad (5)$$

and
$$(a + c) = -e \quad . \quad . \quad . \quad . \quad (6)$$

At frequency f_1 where $x = x_1$, $dy/dx = 0$.

$$\therefore \qquad x_1 = +\sqrt{-c/3a} \quad . \quad . \quad . \quad . \quad (7)$$

From (7), (5) and (6), since $y = e$ at $x = x_1$

$$c = -3a/4 \quad . \quad . \quad . \quad . \quad (8)$$

From (5) and (8), $\quad y = ax(x^2 - 3/4)$

$\therefore$ for zero error $\quad x = 0$ or $\pm \sqrt{3/4}$

i.e.
$$f_{s_2} = f_c$$
$$f_{s_1} = f_c - \sqrt{\tfrac{3}{4}}(f_c - f_a)$$
and
$$f_{s_3} = f_c + \sqrt{\tfrac{3}{4}}(f_b - f_c).$$

In this case, $f_a = 550$ kc/s, $f_b = 1,500$ kc/s and $f_c = 1,025$ kc/s, so $f_{s_1} = 614$ kc/s, $f_{s_2} = 1,025$ kc/s and $f_{s_3} = 1,436$ kc/s.

Making the same assumptions as in (a)

$\quad C_1 = 410 \cdot 5\,\mu\mu\text{F}, \; C_2 = 134 \cdot 58\,\mu\mu\text{F}$ and $C_3 = 58 \cdot 74\,\mu\mu\text{F}.$

Now $\quad f_{h_1} = (f_{s_1} + 465)$ kc/s, $f_{h_2} = (f_{s_2} + 465)$ kc/s

and $\quad\quad\quad\quad f_{h_3} = (f_{s_3} + 465)$ kc/s.

$\therefore$ from equations (1) to (3),

$\quad\quad C_p = 601\,\mu\mu\text{F}, \; C_t = 36 \cdot 5\,\mu\mu\text{F}$ and $L = 77 \cdot 4\,\mu\text{H}.$

219. Choose axes as shown.
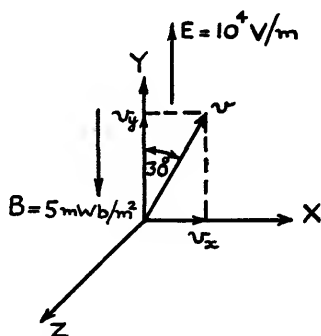
If the initial velocity of the electron was directed along the fields the magnetic field would exert no force on the electron. The electron would then move in a direction parallel to the fields with constant acceleration. If the initial velocity has a component perpendicular to the magnetic field, as in the present case, this component together with the magnetic field, will give rise to circular motion. Because of the field E the velocity along the fields changes with time so the resultant path of the electron is helical.

The velocity v can be resolved into two components v_x and v_y.

$$v_x = 1\cdot19 \times 10^7 \sin 30° = 5\cdot95 \times 10^6 \text{ m/sec}$$
$$v_y = 1\cdot19 \times 10^7 \cos 30° = 1\cdot03 \times 10^7 \text{ m/sec.}$$

The acceleration along the

$$- Y \text{ direction} = a = eE/m = 1\cdot759 \times 10^{15} \text{ m/sec}^2.$$

The projection of the electron path on the XZ plane is a circle of radius

$$r = mv_x/eB = 5\cdot95 \times 10^6/1\cdot759 \times 10^{11} \times 5 \times 10^{-3} = 0\cdot00677 \text{ m.}$$

The velocity along the fields is not constant but is given by

$$v_y' = 1\cdot03 \times 10^7 \text{ m/sec} - 1\cdot759 \times 10^{15}t$$

and y, the distance moved $= 1\cdot03 \times 10^7 t - \frac{1}{2} \times 1\cdot759 \times 10^{15}t^2$.

The electron begins to move in the $+ Y$ direction but because the acceleration is along the $- Y$ direction, it will gradually come to rest and will then reverse its motion in the Y direction. This reversal will occur after a time t' for which $v_y' = 0$,

i.e. $t' = 1\cdot03 \times 10^7/1\cdot759 \times 10^{15} = 5\cdot86 \times 10^{-9} \text{ sec.}$

The distance travelled in the $+ Y$ direction in this time t' is

$$y' = 1\cdot03 \times 10^7 \times 5\cdot86 \times 10^{-9}$$
$$- \tfrac{1}{2} \times 1\cdot759 \times 10^{15} \times (5\cdot86 \times 10^{-9})^2 = 0\cdot03 \text{ m.}$$

After the reversal of motion the electron continues moving in the $- Y$ direction and does not reverse again. There is, of course, no reversal of the direction in which the electron traverses the circular component of its path.

The angular velocity is constant

$$= \omega = \frac{Be}{m} = 1\cdot759 \times 10^{11} \times 5 \times 10^{-3} = 8\cdot8 \times 10^8 \text{ radians/sec.}$$

The periodic time $T = 2\pi/\omega = 7\cdot14 \times 10^{-9} \text{ sec.}$

220. The force on an electron due to E is directed along the $+ X$ axis. Any force due to the field B is always at right angles to B. Thus there is no component of force along the Y direction.

The two following equations therefore hold:

$$m \frac{dv_x}{dt} = eE - ev_z B \quad . \qquad . \qquad . \qquad . \quad (1)$$

$$m \frac{dv_z}{dt} = ev_x B \quad . \qquad . \qquad . \qquad . \qquad . \quad (2)$$

From (1) $\dfrac{d^2 v_x}{dt^2} = - \dfrac{eB}{m} \dfrac{dv_z}{dt} = - \omega \dfrac{dv_z}{dt} = - \omega^2 v_x$ from (2) . (3)

where $\omega = Be/m.$

The solution of equation (3) is of the form

$$v_x = P \cos \omega t + Q \sin \omega t \quad . \qquad . \qquad . \quad (4)$$

where P and Q are constants determined by the initial conditions that $v_x = v_z = 0$ when $t = 0.$

$$\therefore \qquad\qquad\qquad P = 0.$$

From (1) when $t = 0$ $\qquad \dfrac{dv_x}{dt} = \dfrac{eE}{m}$

and from (4) when $t = 0$ $\qquad \dfrac{dv_x}{dt} = Q\omega$

$$\therefore \qquad\qquad Q = \frac{eE}{m\omega} = \frac{E}{B} = u \text{ say.}$$

Thus, $\qquad\qquad v_x = u \sin \omega t \quad . \qquad . \qquad . \qquad . \qquad . \quad (5)$

Using this in equation (1),

$$v_z = u - \frac{1}{\omega} \frac{dv_x}{dt} = u - u \cos \omega t \quad . \qquad . \qquad . \quad (6)$$

From (5) $\quad x = \int v_x \, dt = \int u \sin \omega t \, dt = - \frac{u}{\omega} \cos \omega t + \text{constant.}$

Since $x = 0$ when $t = 0$, constant $= \dfrac{u}{\omega}.$

From (6)

$$z = \int v_z \, dt = \int (u - u \cos \omega t) \, dt = ut - \frac{u}{\omega} \sin \omega t + \text{constant.}$$

Since $z = 0$ when $t = 0$, constant $= 0.$

Thus $\qquad x = \dfrac{u}{\omega}(1 - \cos \omega t)$ and $z = ut - \dfrac{u}{\omega}\sin \omega t$.
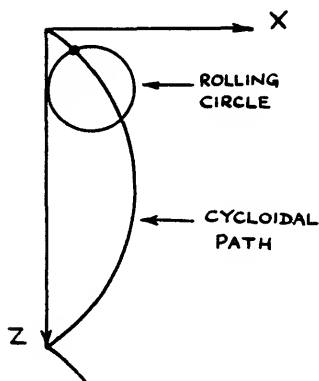
The electron path is a common cycloid, the path generated by a point on the circumference of a circle of radius u/ω which rolls along a straight line, the Z axis, as shown.

221. The axes are chosen as in Question 220.

As shown in the previous solution the electron path is cycloidal in the XZ plane. The electron travels with constant velocity along the Y axis.

Referring to the previous solution:

$$\omega = eB/m = 1{\cdot}759 \times 10^9 \text{ radians/sec}$$

$$u = E/B = 10^6 \text{ m/sec.}$$

The time t for which the electron is in the region between the plates $= 0{\cdot}02/10^6 = 2 \times 10^{-8}$ sec.

Angle θ turned through in this time t

$$= \omega t = (1{\cdot}759 \times 10^9)(2 \times 10^{-8}) = 35{\cdot}18 \text{ radians}$$

$$= (31{\cdot}42 + 3{\cdot}76) \text{ radians} = (10\pi + 3{\cdot}76) \text{ radians,}$$

i.e. the electron enters upon its sixth revolution before leaving the plates.

Referring to the previous solution and noting 3·76 radians = 215°:

$$x = \frac{u}{\omega}(1 - \cos \omega t) = 0.0569 \times 10^{-2}(1 - \cos 215°)$$

$$= 1.035 \times 10^{-3} \text{ m}$$

$$z = \left(ut - \frac{u}{\omega} \sin \omega t\right) = 0.0569 \times 10^{-2}(35.18 - \sin 215°)$$

$$= 2.034 \times 10^{-2} \text{ m.}$$

∴ distance from axis when electron leaves the region between the plates is $\sqrt{x^2 + z^2} = 2.04 \times 10^{-2}$ m.

222. The electric field E has two components $E_x = E \sin 20°$ and $E_y = - E \cos 20°$.

It follows that the equations of motion are those given as (1) and (2) in the solution to Question 220 where E is replaced by $- E \sin 20°$ but the force in the Y direction is no longer zero but is $e(E \cos 20°)$. The equations for x and z are then those given at the end of the solution to Question 220 with E replaced by $- E \sin 20°$ and the expression for y is $v_{0y}t + \frac{1}{2}\frac{e(E \cos 20)t^2}{m}$, where v_{0y} is the initial velocity in the Y direction.
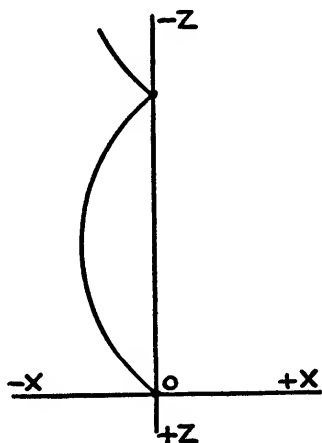
Thus, again, the projection of the path in the XZ plane is a common cycloid as shown. The equation of the cycloid is given by:

$$-x = \frac{u}{\omega}(1 - \cos \omega t)$$

and

$$-z = \frac{u}{\omega}(\omega t - \sin \omega t)$$

$$u = \frac{E \sin 20°}{B} = \frac{5 \times 10^3 \times \sin 20°}{10^{-3}} = \underline{1 \cdot 71 \times 10^6 \text{ m/sec}}$$

$$\omega = eB/m = \underline{1 \cdot 759 \times 10^8 \text{ radians/sec.}}$$

223. For equal sensitivities

$$R = 1/\omega C = 1/2\pi \times 50 \times 2 \times 10^{-6} \,\Omega = \underline{1{,}592 \,\Omega.}$$

For differing sensitivities

$$R = 1{,}592 \times 0 \cdot 55/0 \cdot 45 = \underline{1{,}946 \,\Omega.}$$

224. Voltage range $= (5 \times 30 - 20) = 130$ V.
Length of time base $= 130 \times 0 \cdot 8$ mm $= \underline{10 \cdot 4 \text{ cm.}}$
If T is the time of the sweep,

$$0 \cdot 01 \times 10^{-6} \times 130 = 1 \cdot 5 \times T \times 10^{-3}.$$

$$\therefore \qquad \text{Frequency } f = 1/T = \underline{1{,}154 \text{ c/s.}}$$

225. Charge on capacitor, $Q = CV = IT = I/f.$

$$\therefore \qquad \text{frequency } f = I/CV.$$

Here $V = (280 - 30) = 250$ and $I = 5 \times 10^{-3}$, so that
when $f = 20$ c/s, $C = 1 \,\mu\text{F}$ and when $f = 20$ kc/s, $C = 0 \cdot 001 \,\mu\text{F.}$

226. Let v be the voltage across C at any instant. Displacement current at any instant $i = dq/dt,$

$$\therefore \qquad\qquad i = C \, dv/dt \text{ since } q = Cv.$$

Also, $\qquad\qquad\quad i = (E - e)/R.$

$$\therefore \qquad\qquad v = E - CR \, dv/dt.$$
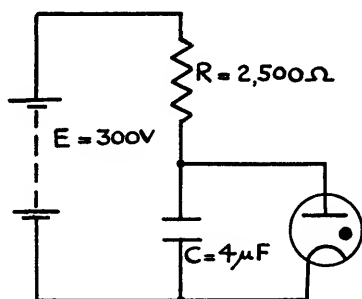
If $v = 0$ when $t = 0$ the solution of this equation is:

$$v = E(1 - e^{-t/CR}).$$

When $t = 20$ ms, $v = 300\{1 - e^{-(20 \times 10^{-3}/4 \times 2,500 \times 10^{-6}}\} = \underline{260 \text{ V}}.$

(Mean-square current)

$$= \frac{1}{20 \times 10^{-3}} \int_0^{20 \times 10^{-3}} i^2 \, dt = \frac{1}{20 \times 10^{-3}} \int_0^{20 \times 10^{-3}} \left\{ \left(\frac{300C}{CR} \right) e^{-t/CR} \right\}^2 dt$$

$$= \{3,600\} \text{ (mA)}^2,$$

i.e.   Root-mean-square current = 60 mA.

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227. The deflection at the screen* $= Vl(l/2 + L)/2Ed = \delta$, where
$l = 1\cdot5$ cm, $d = 0\cdot3$ cm, $L = 20$ cm, $E = 1,500$ volts and V is the
voltage applied to the deflector plates.

∴ $V/\delta = 2 \times 1,500 \times 0\cdot3/1\cdot5 \times 20\cdot75 = \underline{28\cdot9 \text{ V/cm}}.$

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228. The deflection at the screen† $= Hl\sqrt{e/2mE}[l/2 + L] = \delta$,
where $l = 0\cdot015$ m, $L = 0\cdot2$ m, $E = 1,500$ V and H is the magnetic
field produced by the poles.

∴ $H/\delta = 1/0\cdot015 \times 0\cdot2075\sqrt{1\cdot759 \times 10^{11}/3,000} = \underline{0\cdot042 \text{ Wb/m}^2\text{/m}}.$

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229. $\frac{1}{2} m\bar{v}^2 = \frac{3}{2} kT$

Thus, when $T = 273°$K,

$$\bar{v}^2 = \frac{3 \times 1\cdot38 \times 10^{-23} \times 273}{2 \times 14 \times 1\cdot67 \times 10^{-27}}$$

i.e. $\bar{v} = 492$ m/sec.

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* See F. A. Benson, *Electrical Engineering Problems with Solutions*, Spon, 1954,
pp. 219–20.
† *ibid.*, pp. 220–1.

When $T = 373°K$,

$$\bar{v}^2 = \frac{3 \times 1\cdot38 \times 10^{-23} \times 373}{2 \times 14 \times 1\cdot67 \times 10^{-27}}$$

i.e. $\bar{v} = 575$ m/sec.

230. The fractional number of particles having velocities in the range v to $v + dv$ is given by:

$$\frac{dN_v}{N} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv$$

The most probable velocity $v_p = \sqrt{\frac{2kT}{m}}$

When $v = v_p$:

$$\frac{dN_v}{N} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right) e^{-1} dv$$

$$= \frac{4}{\sqrt{2\pi}} \left(\frac{m}{kT}\right)^{\frac{1}{2}} e^{-1} dv$$

In this case $dv = \frac{2}{100} \sqrt{\frac{2kT}{m}}$

$\therefore$
$$\frac{dN_v}{N} = \frac{4}{\sqrt{2\pi}} \left(\frac{m}{kT}\right)^{\frac{1}{2}} e^{-1} \frac{2}{100} \sqrt{\frac{2kT}{m}}$$

$$= 0\cdot0166.$$

231. $\quad \dfrac{dN_{xyz}}{N} = \left(\dfrac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}(v_x^2 + v_y^2 + v_z^2)} dv_x\, dv_y\, dv_z$

so

$$dN_x = N \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv_x^2}{2kT}} dv_x \int_{-\infty}^{\infty} e^{-\frac{mv_y^2}{2kT}} dv_y \int_{-\infty}^{\infty} e^{-\frac{mv_z^2}{2kT}} dv_z$$

Now
$$\int_{-\infty}^{\infty} e^{-\beta s^2} ds = \sqrt{\pi/\beta}$$

$\therefore$
$$\int_{-\infty}^{\infty} e^{-\frac{mv_y^2}{2kT}} dv_y = \sqrt{2kT\pi/m}$$

and
$$\int_{-\infty}^{\infty} e^{-\frac{mv_z^2}{2kT}} \, dv_z = \sqrt{2kT\pi/m}$$

$\therefore$
$$dN_x = N \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv_x^2}{2kT}} \, dv_x \, (2kT\pi/m)$$

$$= N \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-\frac{mv_x^2}{2kT}} \, dv_x$$

Now
$$v_p = (2kT/m)^{\frac{1}{2}}$$

$\therefore$
$$dN_x = N \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-v_x^2/v_p^2} \, dv_x$$

$$= \frac{N}{\sqrt{\pi}} \left(\frac{1}{v_p}\right) e^{-v_x^2/v_p^2} \, dv_x$$

Let
$$\omega = v_x/v_p$$

Then
$$d\omega = dv_x/v_p$$

$\therefore$
$$dN_x = N e^{-\omega^2} \, d\omega / \sqrt{\pi}$$

232. First find the number of molecules with velocity v perpendicular to the wall of the box that strike the wall in unit time.

Let the wall be perpendicular to v_x. In unit time distance moved is v_x.

$\therefore$ number striking unit area of wall $= dN_x \, v_x$

so number with velocity greater than v is

$$\int_v^{\infty} dN_x \, v_x = \frac{N}{\sqrt{\pi}} \int_{v/v_p}^{\infty} e^{-\omega^2} d\omega \, v_p \, \omega$$

$$= \frac{Nv_p}{\sqrt{\pi}} \left[\frac{e^{-\omega^2}}{2}\right]_{v/v_p}^{\infty} = \frac{Nv_p}{2\sqrt{\pi}} e^{-(v/v_p)^2}$$

$$= N v_p \, e^{-\omega^2}/2\sqrt{\pi}$$

233. Number striking unit area of wall per second $= v_x \, dN_x$.
Partial pressure $= (2mv_x) \, v_x \, dN_x$

$\therefore$ total pressure $p = 2mN \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \int_0^{\infty} \exp\left(-\frac{mv_x^2}{2kT}\right) v_x^2 \, dv_x$

Put $mv_x^2/2kT = s^2$, i.e. $v_x = s\sqrt{2kT/m}$

$$\therefore \quad p = 2mN \left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} \frac{2kT}{m} \cdot \sqrt{\frac{2kT}{m}} \int_0^\infty \exp(-s^2)s^2 ds$$

$$= \frac{4NkT}{\sqrt{\pi}} \int_0^\infty \exp(-s^2) s^2 ds$$

Now

$$\int_{-\infty}^\infty e^{-\beta s^2} ds = \sqrt{\frac{\pi}{\beta}}$$

$$\therefore \quad \int_{-\infty}^\infty s^2 e^{-\beta s^2} ds = \frac{\sqrt{\pi}(\frac{1}{2})}{\beta\sqrt{\beta}}$$

When $\beta = 1$, $\quad \int_0^\infty s^2 e^{-s^2} ds = \sqrt{\pi}/4$

$$\therefore \quad \underline{p = NkT.}$$

234. The electrons which escape in unit time are those with a velocity v_x such that $\frac{1}{2}mv_x^2 > \phi$, i.e. $v_x^2 > 2\phi/m$

$$\therefore \quad \text{number} = \frac{Nv_p}{2\sqrt{\pi}} e^{-\omega^2} = \frac{Nv_p}{2\sqrt{\pi}} e^{-v_x^2/v_p^2}$$

$$= \frac{Nv_p}{2\sqrt{\pi}} e^{-2\phi/mv_p^2}$$

where $v_p^2 = 2kT/m$

$$\therefore \quad \text{number} = \frac{N}{2\sqrt{\pi}}\sqrt{\frac{2kT}{m}} e^{-2m\phi/m2kT}$$

$$= \frac{N}{2\sqrt{\pi}} \sqrt{\frac{2k}{m}} \sqrt{T} e^{-\phi/kT}$$

$$= A \sqrt{T} e^{-\phi/kT}$$

where

$$A = \frac{N}{2\sqrt{\pi}} \sqrt{\frac{2k}{m}}$$

235.
$$dN_E = 2\pi N \left(\frac{1}{\pi kT}\right)^{3/2} E^{1/2} \, e^{-E/kT} \, dE$$

$$\text{Mean energy} = 2\pi \left(\frac{1}{\pi kT}\right)^{3/2} \int_0^\infty E^{3/2} \, e^{-E/kT} \, dE$$

Put $E/kT = x^2$, then $dE/kT = 2x \, dx$ and $E^{3/2} = (kT)^{3/2} x^3$

$$\therefore \quad \text{mean energy} = 2\pi \left(\frac{1}{\pi kT}\right)^{3/2} (kT)^{3/2} \int_0^\infty x^3 \, e^{-x^2} 2x \, dx (kT)$$

$$= 4\pi \left(\frac{1}{\pi}\right)^{3/2} kT \int_0^\infty x^4 \, e^{-x^2} \, dx$$

But
$$\int_0^\infty x^4 \, e^{-x^2} \, dx = 3\sqrt{\pi}/8$$

$$\therefore \qquad \qquad \underline{\text{mean energy} = 3kT/2}$$

$E^{1/2} \, e^{-E/kT}$ is a maximum for most probable energy.

Write $y = E^{1/2} \, e^{-E/kT}$.

Then $\dfrac{dy}{dE} = E^{1/2} \, e^{-E/kT} \left(-\dfrac{1}{kT}\right) + \tfrac{1}{2} E^{-1/2} \, e^{-E/kT} = 0$ for maximum

$$\therefore \qquad \qquad \frac{E}{kT} = -\frac{1}{2}$$

$$\therefore \qquad \qquad \underline{\text{most probable energy} = kT/2}$$

$$3kT/2 = 1{\cdot}602 \times 10^{-19}$$

$$\therefore \qquad T = \frac{1{\cdot}602 \times 10^{-19} \times 2}{3 \times 1{\cdot}38 \times 10^{-23}} = \underline{7{,}740^\circ \text{K}}$$

236. (a) The kinetic energy $\tfrac{1}{2}mv^2 = Ve$.

$$\therefore \qquad \tfrac{1}{2} \times 9{\cdot}107 \times 10^{-31} \times v^2 = 11{\cdot}6 \times 1{\cdot}602 \times 10^{-19}$$

where v is in m/sec.

$$\therefore \qquad \qquad \underline{v = 2{\cdot}02 \times 10^6 \text{ m/sec.}}$$

(b) Excess energy
$$= \tfrac{1}{2} \times 9{\cdot}107 \times 10^{-31} \times (2{\cdot}02 \times 10^6)^2 - 10{\cdot}4 \times 1{\cdot}602 \times 10^{-19}$$
$$= 9{\cdot}107 \times 10^{-31} v^2.$$

$$\therefore \qquad \qquad \underline{v = 0{\cdot}459 \times 10^6 \text{ m/sec.}}$$

(c) Let the field strength be x volts/cm. Then the acceleration imparted to the electron is xe/m. If v is the electron velocity acquired in a distance s cm,

$$v^2 = 2xes/m$$

But

$$v^2 = 21 \cdot 5 \times 2 \times e/m.$$

$\therefore$

$$x = 21 \cdot 5/s = 21 \cdot 5/0 \cdot 079 = \underline{272 \text{ V/cm.}}$$

(d) If an electron falls from an energy level W_1 to another level W_2, then $W_1 - W_2 = hf$ where h is Planck's constant and f is the frequency of the emitted radiation. The wavelength of the emitted radiation is in Å given by $12,400/(W_1 - W_2)$.

In this case the two wavelengths λ_1 and λ_2 are given by:

$$\lambda_1 = 12,400/(7 \cdot 93 - 6 \cdot 71) \text{ and } \lambda_2 = 12,400/6 \cdot 71$$

i.e.

$$\underline{\lambda_1 = 10,160 \text{ Å and } \lambda_2 = 1,848 \text{ Å.}}$$

237. $\alpha/p = 15\,e^{-350p/E}$

For maximum multiplication $d\alpha/dp = 0$.
This requires that:

$$15\,p(-350/E)\,e^{-350p/E} + 15\,e^{-350p/E} = 0$$

or

$$350\,p/E = 1$$

In this case, $E = 100$ V/cm, so $p = (100/350)$ mm Hg

i.e.

$$\underline{p = 0 \cdot 29 \text{ mm Hg}}$$

Multiplication $= e^{\alpha d}$

where

$$\alpha = 15\,p\,e^{-350p/E} \text{ and } d = 1 \text{ cm.}$$

$\therefore$

$$\alpha = 15 \times 0 \cdot 29\,e^{-(350 \times 0 \cdot 29/100)} = \underline{1 \cdot 57}$$

and multiplication $= \underline{4 \cdot 81}$.

Average energy/ion pair $= E/\alpha = \underline{64 \text{ eV.}}$

238. Suppose n_0 electrons/sec are released from the cathode by external means. Consider a plane distant x from the cathode.

Number of electrons crossing this plane/sec $(n) = n_0 +$ number produced in x.

The n electrons produce in a further distance dx an additional number of electrons $dn = \alpha\,n\,dx$, where α is Townsend's first ionization coefficient. Hence,

$$\int_{n_0}^{n} \frac{dn}{n} = \alpha \int_0^x dx$$

i.e. $n = n_0 e^{\alpha x}.$

Thus, the value of the current i for a given point on the current/x characteristic $= i_0 e^{\alpha x}$, where i_0 is the current produced at the cathode by external means.

Experimentally it is found that a log plot of i against x does not give a straight line but increases faster than exponentially with x. This is attributed nowadays to secondary emission from the cathode. The theory above works well, however, for small currents.

Now let n_a = the number of electrons to the anode/sec;

n_+ = the number of electrons released from the cathode/ sec by positive-ion bombardment;

and γ = secondary emission coefficient at the cathode, i.e. the number of electrons from the cathode/ incident ion.

Thus, $n_a = (n_0 + n_+)e^{\alpha d}$

where d is the anode-cathode spacing.

Also, $n_+ = \gamma\{n_a - (n_0 + n_+)\}.$

$\therefore$ $n_a = n_0 e^{\alpha d}/\{1 - \gamma(e^{\alpha d} - 1)\},$

or $i_a = i_0 e^{\alpha d}/\{1 - \gamma(e^{\alpha d} - 1)\}.$

Here, from a plot of $\log_e I$ against x, for $0 < x < 0.8$ cm, the slope $\alpha = 3$/cm.

Also, $\log_e i_0 = 0$, so $i_0 = 1\ \mu\mu$A.

If $d = 1.6$ cm, from the above expression for i_a:

$$200 = e^{4\cdot8}/\{1 - \gamma(e^{4\cdot8} - 1)\}.$$

$$\gamma = 0.0033.$$

From the above expression for current i_a it is seen that $i_a \to \infty$ if $\gamma(e^{\alpha d} - 1) = 1$, or if $e^{\alpha d} = (\gamma + 1)/\gamma$. Usually $\gamma \ll 1$, so for breakdown $\gamma e^{\alpha d} \simeq 1$. Here $0.0033\ e^{3d} \simeq 1$, so $d \simeq 1.91$ cm.

239. At breakdown, $\gamma \, e^{\alpha d} = 1$

so $\qquad\qquad\qquad 0{\cdot}02 \, e^{0{\cdot}5\alpha} = 1$

i.e. $\qquad\qquad\qquad 0{\cdot}5 \, \alpha = \log_e 50$

$\therefore \qquad\qquad\qquad \alpha = 7{\cdot}82 \text{ cm}^{-1}$

At 200 V, with $d = 2{\cdot}5$ mm, α is the same since E is unaltered

$$\therefore \qquad \text{multiplication} = \frac{e^{7{\cdot}82 \times 0{\cdot}25}}{1 - 0{\cdot}02(e^{7{\cdot}82 \times 0{\cdot}25} - 1)}$$

$$= \underline{8{\cdot}06}.$$

240. The theory of probes and details of probe measurements can be found in many textbooks.*

The electron current I_e, corresponding to a probe current I_p, is given by:

$$I_e = I_p + I_i = I_p + 0{\cdot}08,$$

where I_i is the saturation value of the positive-ion current.

The slope of the $\log_e I_e/V_p$ graph is found to be $0{\cdot}65$/volt and this equals e/kT_e, where k is Boltzmann's constant and T_e is the electron temperature.

Thus, in this case,

$$T_e = 1{\cdot}602 \times 10^{-19}/0{\cdot}65 \times 1{\cdot}38 \times 10^{-23} = \underline{17{,}800°\text{K}}$$

The random electron current density

$$J_e = I_e/A = 36{\cdot}4/0{\cdot}033 = 1{,}100 \text{ mA/cm}^2 = 1{\cdot}1 \times 10^4 \text{ A/m}^2.$$

Therefore, $\qquad 1{\cdot}1 \times 10^4 = eN_e\sqrt{kT_e/2\pi m},$

where N_e is the electron concentration and m is the electron mass, i.e. $1{\cdot}1 \times 10^4$

$$= N_e \,.\, 1{\cdot}602 \times 10^{-19}\sqrt{1{\cdot}38 \times 10^{-23} \times 17{,}800/2\pi \times 9{\cdot}107 \times 10^{-31}}$$

$$\therefore \qquad\qquad \underline{N_e = 3{\cdot}3 \times 10^{17}/\text{m}^3 = 3{\cdot}3 \times 10^{11}/\text{c.c.}}$$

Plasma potential $= -11$ V, so $E = (11 - 5)/12 = \underline{0{\cdot}5 \text{ V/cm}}$, where E is the voltage gradient.

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* For example, see J. Millman and S. Seely, *Electronics*, McGraw-Hill, 2nd Edition, 1951, pp. 286–9; P. Parker, *Electronics*, Arnold, 1950, pp. 637–44; and F. G. Spreadbury, *Electronics*, Pitman, 1947, pp. 93–8.

The drift current density $= 1.1/6$ A/cm$^2 = 0.18$ A/cm^2.
Mobility of electrons $= 0.18/N_e eE$

$$= 0.18/3.3 \times 10^{11} \times 1.602 \times 10^{-19} \times 0.5$$
$$= \underline{6.8 \times 10^6 \text{ cm/sec/V/cm.}}$$

241. $0.25 = e^{-2.2/kT_e}$

so $\qquad\qquad 2.2/kT_e = \log_e 4$

from which $\underline{T_e = 18{,}200°K}$

$j_{es} = 18 \times 10^{-3}/0.025 = 0.72$ A/cm^2

so that $\qquad\qquad 10^4 \times 0.72 = e\, N_e \sqrt{\dfrac{kT_e}{2\pi m}}$

or $\qquad\qquad \underline{N_e = 2.15 \times 10^{17} \, m^{-3}}$

242. The current I at any point x can be represented by

$$I = I_0 e^{-Ax} = I_0(1 - Ax \ldots),$$

where I_0 is the current at $x = 0$ and $A = QN$. N is the number of particles/unit volume and Q is the total collision cross-section. Here, $Ax = 1/10$ and $x = 20$ cm, so $A = 1/200$. $N = 2.7 \times 10^{19}$ $(p/760)$, where p is the gas pressure in mm. of mercury.

$\therefore \quad 1/200 = 10^{-16} \times 2.7 \times 10^{19}(p/760)$ so $\underline{p = 1.405 \times 10^{-3}}$.

243. $\qquad\qquad f_0 = \sqrt{\dfrac{ne^2}{\pi m}}$ c/s.

At 8 mm, $f_0 = 3.75 \times 10^{10}$ c/s, so $n = 1.75 \times 10^{13}$ cm^{-3}
At 3 cm, $f_0 = 10^{10}$ c/s, so $n = 1.24 \times 10^{12}$ cm^{-3}.
If concentration is n_0 at $t = 0$, then at time t:

$$n = n_0/(1 + n_0 \alpha t)$$

$\therefore \qquad\qquad 1.75 \times 10^{13} = n_0/(1 + n_0 \alpha 5.7 \times 10^{-6})$

and $\qquad\qquad 1.24 \times 10^{12} = n_0/(1 + n_0 \alpha 81 \times 10^{-6})$

$\therefore \qquad\qquad \underline{\alpha \simeq 10^{-8}}$

244. (a) Light is considered as composed of particles of energy hf so that electron emission is possible only if the frequency of the impinging light is greater than the 'threshold' value $f_c = e\phi/h$; where ϕ is the work function in electron volts, e is the charge on the electron and h is Planck's constant.

Corresponding threshold wavelength $\lambda_c = \dfrac{ch}{e\phi} = \dfrac{12{,}400}{\phi}$ Å.

In this case ϕ must be less than $12{,}400/8{,}000 = 1\cdot55$ eV.
For caesium, $\lambda_c = (12{,}400/1\cdot8)$ Å $= 6{,}890$ Å.

(b) The electron-volt equivalent of the energy of the incident photons is $12{,}400/2{,}537$, i.e. $4\cdot89$ eV.

According to Einstein* the maximum energy of the emitted electrons is $4\cdot89 - 4\cdot3 = 0\cdot59$ eV.

The maximum velocity

$$v = \sqrt{2\frac{e}{m}E} = 5\cdot93 \times 10^5 \sqrt{E} \text{ m/sec}$$
$$= 5\cdot93 \times 10^5 \sqrt{0\cdot59} \text{ m/sec}$$
$$= 4\cdot56 \times 10^5 \text{ m/sec.}$$

245. If the charge/unit length of electrode is Q, then

$$E = Q/2\pi r\varepsilon_0 \quad \text{and} \quad V = \int_a^b E\, dr = Q \log_e (b/a)/2\pi\varepsilon_0,$$

where a and b are the anode and cathode radii respectively and r is any radius between a and b.

$$\therefore \qquad\qquad E = V/r \log_e (b/a)$$

Let the number of ion pairs be n at any point. Then,

$$dn/n = Ape^{-Bp \log_e(b/a)r/V}\, dr$$

Here $A = 13\cdot6$, $B = 17\cdot3 \times 13\cdot6 \simeq 235$, $b/a = 20$,

so $\log_e (b/a) \simeq 3$ and $p = 1$

$$\therefore \qquad dn/n = 13\cdot6\, e^{-235 \times 3r/V}\, dr,$$

and $\log_e (n/n_0) = 13\cdot6 \displaystyle\int_a^b e^{-705r/V}\, dr$

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* A. Einstein, 'Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt,' *Ann. Physik.*, 17, 32, 1905.

Write $705r/V = s$, then $705\,dr/V = ds$, so

$$\log_e (n/n_0) = 13 \cdot 6 \int_{705/20V}^{705/V} e^{-s}(V/705)\,ds$$
$$= (13 \cdot 6\, V/705)\{e^{-705/20V} - e^{-705/V}\}$$
$$\simeq (V/52)e^{-35 \cdot 3/V}.$$

246.   Sensitivity $= (24 \times 5^6)\,\mu\text{A/lumen}$
                        $= 375\,\text{mA/lumen}.$

Let maximum safe illumination $= x$ lumens.

$\therefore$ $x \times 375 = 3$

i.e. $x = 8$ millilumens.

247. The number of lumens L of light flux falling on an area A of a surface at a distance d from a point source of light of candle-power C is CA/d^2. Any units of length may be used provided they are the same for A and d^2.

In this case $L = 180 \times 19 \cdot 35/91 \cdot 4^2 = 0 \cdot 417$ lumen.

From the $R_l = 600\,\Omega$ characteristic $0 \cdot 417$ lumen gives a current of $120\,\mu\text{A}$.

248. It is easily shown* that the conductive component G of the input admittance is $\omega^2 L_c C_{cg} g_m$, where L_c is the cathode-lead inductance, C_{cg} is the input capacitance, ω is $(2\pi \times \text{frequency})$ and g_m is the mutual conductance.

Here $\omega = 2\pi \times 200 \times 10^6$, $L_c = 10^{-8}$ H, $C_{cg} = 10 \times 10^{-12}$ F, and $g_m = 2 \times 10^{-3}$ mho.

$\therefore$ $G = 316$ micromhos.

249. The solution to this problem can be found elsewhere.†

250. The solution to this problem can be found elsewhere.‡

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* See K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, p. 479; or A. T. Starr, *Telecommunications*, Pitman, 1954, p. 235.

† See, for example, P. Parker, *Electronics*, Arnold, 1950, pp. 50–3 and pp. 980–2; or R. Latham, A. H. King, L. Rushforth, *The Magnetron*, Chapman & Hall, 1952, Section 6.4.2.

‡ See, for example, P. Parker, *Electronics*, Arnold, 1950, Section 111.

251. The solution to this problem can be found elsewhere.*

252. The solution to this problem can be found elsewhere.†

253.

$$R + j\omega L = 10\cdot4 + j5{,}000 \times 3\cdot67 \times 10^{-3} = 10\cdot4 + j18\cdot35$$
$$= 21\cdot08\underline{/60^\circ\ 27'}\ \Omega.$$

$$G + j\omega C = 0\cdot8 \times 10^{-6} + j5{,}000 \times 0\cdot00835 \times 10^{-6}$$
$$= (0\cdot8 + j41\cdot75)10^{-6} = 41\cdot76\underline{/88^\circ\ 55'} \times 10^{-6}\ \text{mho.}$$

Characteristic impedance

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = 711\underline{/-14^\circ\ 14'} = (689 - j175)\ \Omega.$$

Propagation constant $P = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$, where α is the attenuation constant and β is the wavelength or phase constant.

$$\therefore \qquad P = 0\cdot0297\underline{/74^\circ\ 41'} = 0\cdot00785 + j0\cdot0287$$

$$\therefore \qquad \alpha = 0\cdot00785\ \text{neper/mile}$$

and $\qquad \beta = 0\cdot0287\ \text{radian/mile.}$

The wavelength

$$\lambda = 2\pi/\beta = \underline{219\ \text{miles.}}$$

Velocity of propagation

$$v = f\lambda = \omega/\beta = 5{,}000/0\cdot0287 = \underline{174{,}300\ \text{miles/sec.}}$$

254. Since the line is terminated by Z_0 the input impedance is also Z_0.

Input current when connected to generator (i_1)

$$= 2/(600 + 689 - j175)\ \text{A} = 0\cdot001539\underline{/7^\circ\ 44'}\ \text{A.}$$

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* See, for example, J. Millman and S. Seely, *Electronics*, McGraw-Hill, 1951, pp. 80–2. See also, P. Parker, *Electronics*, Arnold, 1950, Section 107.
† See W. W. Harman, *Fundamentals of Electronic Motion*, McGraw-Hill, 1953, pp. 207–15.

Current at receiving end (i_2)

$$= i_1 e^{-Pl} \text{ where } l \text{ is 300 miles} = \underline{0.0001458/-485° \ 16' \text{ A.}}$$

Voltage across load $= i_2 Z_0 = \underline{0.1036/-499° \ 30' \text{ V.}}$

255. In this case

$$R + j\omega L = 10.4 + j18.35 + (7.3 + j5{,}000 \times 0.246)/7.88$$
$$= 11.32 + j174.35 = \underline{174.35/86° \ 18' \ \Omega}$$
$$G + j\omega C = 41.76/88° \ 55' \times 10^{-6} \text{ mho.}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \underline{2{,}038/-1° \ 19' \ \Omega}$$
$$\underline{P = 0.0036 + j0.0850 = \alpha + j\beta}$$
$$\lambda = 2\pi/\beta = \underline{74 \text{ miles}}$$
$$v = \omega/\beta = \underline{58{,}800 \text{ miles/sec.}}$$

256. $Z_0 = 689 - j175 = \underline{711/-14° \ 14' \ \Omega}$

$$\alpha = 0.00785 \quad \therefore \ \alpha l = 0.785 \text{ and } e^{\alpha l} = 2.192$$
$$\beta = 0.0287 \quad \therefore \ \beta l = 2.87 \text{ radians} = 164° \ 20'.$$

Load impedance $= 500/45° = (353.5 + j353.5) \ \Omega = Z_r$.

Assume the initial voltage at the sending end is $1/0 \text{ V} = E_s$. Then the receiving-end voltage due to this is $E_r = (1/2.192)/-164° \ 20'$

$$= 0.456/-164° \ 20' = (-0.440 - j0.123) \text{ V.}$$

Reflected-wave voltage at receiving-end is

$$E_r' = E_r(Z_r - Z_0)/(Z_r + Z_0)$$
$$E_r' = 0.270/-51° \ 36' = (0.168 - j0.211) \text{ V.}$$

Voltage at receiving end due to E_r' is

$$E_r'' = \frac{0.270/-51° \ 36'}{2.192/164° \ 20'} = 0.123/-215° \ 56' = (-0.10 + j0.072) \text{ V.}$$

Let I_s be the component of the sending-end current due to E_s, then

$$I_s = 1/(711 \underline{/-14° \; 14'}) = 1·407 \times 10^{-3}\underline{/14° \; 14'}$$
$$= (1·364 + j0·347)10^{-3} \text{ A.}$$

If I_s'' is the component of the sending-end current due to E_r'', then

$$I_s'' = \frac{0·123\underline{/-215° \; 56'}}{711\underline{/-14° \; 14'}} \times 1\underline{/180°} = 0·173 \times 10^{-3}\underline{/-21° \; 42'}$$

$$= (0·161 - j0·064)10^{-3} \text{ A.}$$

Sending-end voltage is

$$E_s + E_r'' = 1 - 0·10 + j0·072 = (0·9 + j0·072)$$
$$= 0·9\underline{/4° \; 35'} \text{ V.}$$

Sending-end current is

$$I_s + I_s'' = (1·364 + j0·347)10^{-3} + (0·161 - j0·064)10^{-3}$$
$$= (1·525 + j0·283)10^{-3} = 1·55\underline{/10° \; 30'} \text{ A.}$$

Receiving-end voltage is

$$E_r + E_r' = (-0·440 - j0·123) + (0·168 - j0·211)$$
$$= (-0·272 - j0·334) = 0·431\underline{/230° \; 52'} \text{ V.}$$

Input impedance of line

$$Z_i = \frac{0·9\underline{/4° \; 35'}}{1·55\underline{/10° \; 30'}} \, \Omega = 580\underline{/-5° \; 55'} \, \Omega$$

$$= (576 - j60) \, \Omega.$$

Actual sending-end voltage $= \left| \dfrac{2 \times Z_i}{600 + Z_i} \right| = 0·986 \text{ V.}$

Since the original assumption that $E_s = 1\underline{/0}$ gave a value of sending-end voltage of 0·9 V,

$$|E_r + E_r'| = \frac{0·431 \times 0·986}{0·9} = 0·472 \text{ V} = \text{receiving-end voltage}$$

and $\quad |I_r| = \dfrac{|E_r + E_r'|}{|Z_r|} = \left(\dfrac{0·472}{500}\right) \text{ A} = 944 \text{ mA.}$

257.
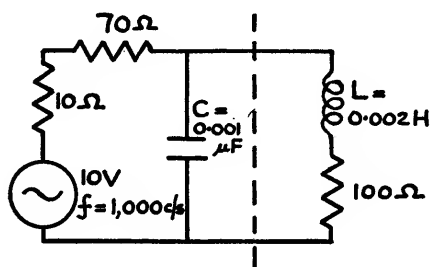
The actual circuit (above) can be replaced, using Thévenin's Theorem, by the one shown below.
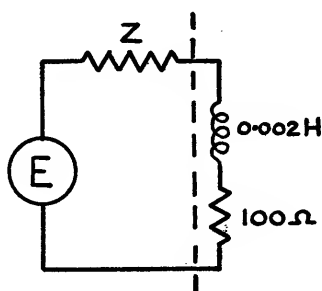
$$\therefore \qquad E = \left[\frac{10 \times \dfrac{1}{j\omega C}}{\left(10 + 70 + \dfrac{1}{j\omega C}\right)} \right] = \frac{-j1{,}592}{80 - j159 \cdot 2}$$

$$Z = \frac{1}{j\omega C} \times 80 \left/ \left(\frac{1}{j\omega C} + 80\right)\right. = -80(j159 \cdot 2)/(80 - j159 \cdot 2)$$

$$\therefore \qquad \text{load current} = E/(Z + 100 + j\omega L) = \underline{0 \cdot 71 \text{ mA.}}$$

258.
$$-\frac{\partial v}{\partial x} = Ri + L\frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = Gv$$

$$-\frac{\partial^2 v}{\partial x^2} = R\frac{\partial i}{\partial x} + L\frac{\partial^2 i}{\partial x\partial t}$$

$$-\frac{\partial^2 i}{\partial x\partial t} = G\frac{\partial v}{\partial t}$$

$$\therefore \qquad -\frac{\partial^2 v}{\partial x^2} = -RGv - LG\frac{\partial v}{\partial t}$$

Assume $V = V_0 \sin (\pi x/l)e^{-\gamma t}$

$$\frac{\partial^2 V}{\partial x^2} = -\frac{\pi^2}{l^2}\,V;\ \frac{\partial V}{\partial t} = -\gamma V$$

$$\therefore \qquad \pi^2/l^2 = -RG + \gamma LG$$

so $\qquad \underline{\gamma = R/L + \pi^2/LGl^2}$

259. The line must be half a wavelength long.

In free-space the wavelength corresponding to 20 Mc/s is 15 m.

$\therefore$ if ε is the permittivity of the dielectric $5 = 7{\cdot}5/\sqrt{\varepsilon}$,

i.e. $\qquad\qquad \underline{\varepsilon = 2{\cdot}25.}$

For a short-circuited line of length l the input impedance

$$Z_1 = Z_0 \tanh Pl.$$

For an open-circuited line of length l the input impedance

$$Z_2 = Z_0 \coth Pl$$

$$\therefore \qquad\qquad Z_1 Z_2 = Z_0{}^2.$$

In this case $\quad Z_0 = \sqrt{4{\cdot}61 \times 1{,}390} = \underline{80\ \Omega}.$

Also, $Z_1/Z_2 = \tanh^2 Pl,\quad \therefore\ \tanh Pl = \sqrt{\dfrac{4{\cdot}61}{1{,}390}} = 0{\cdot}05758.$

The attenuation constant $= \dfrac{0{\cdot}05758}{5}$ nepers/m $= \underline{0{\cdot}1\ \text{db/m}.}$

Velocity of propagation $=$ velocity of light$/\sqrt{\varepsilon} = \underline{2 \times 10^8\ \text{m/sec.}}$

260. If a line with a characteristic impedance Z_o is terminated by an impedance Z_L the voltage reflection coefficient $\rho e^{j\theta}$ is $(Z_L - Z_o)/(Z_L + Z_o)$.

Let the voltage of the wave travelling towards the load be $V \cos \omega(t + x/v)$ at a distance x from the load. Then the voltage at the load due to this wave is $V \cos \omega t$. Thus, at a distance x from the load, the reflected wave is $\rho V \cos \{\omega(t - x/v) + \theta\}$. The total voltage at distance $x = V_x = V \cos \omega\left(t + \dfrac{x}{v}\right) + \rho V \cos \{\omega(t - x/v) + \theta\}$.

The amplitude of $V_x = |V_x| = V\sqrt{1 + \rho^2 + 2\rho \cos(\theta - 2\omega x/v)}$.

Voltage standing-wave ratio (as a quantity > 1)

$$= r = \frac{|V_x|_{\max}}{|V_x|_{\min}} = \frac{1 + \rho}{1 - \rho}.$$

Also, position of first voltage maximum is given by $\theta - 2\omega x/v = 0$, i.e. when $\theta = 4\pi x/\lambda$.

In this case

$$r = 2 \qquad \therefore \ \rho = (r - 1)/(r + 1) = 1/3$$

$$x = \lambda/12 \qquad \therefore \ \theta = \pi/3.$$

$$(Z_L - Z_o)/(Z_L + Z_o) = \rho e^{j\theta} \quad \therefore \ Z_L/Z_o = \frac{1 + \rho e^{j\theta}}{1 - \rho e^{j\theta}}$$

$$= \frac{1 + \dfrac{e^{j\pi/3}}{3}}{1 - \dfrac{e^{j\pi/3}}{3}} = \frac{8 + j3\sqrt{3}}{7}.$$

$$\therefore \qquad Z_L = 70(8 + j3\sqrt{3})/7 = \underline{\underline{80 + j52 \ \Omega.}}$$

261. The solution to this problem can be found elsewhere.*

262. Normalized terminating impedance

$$= (37\cdot5 + j52\cdot5)/75 = 0\cdot5 + j0\cdot7.$$

This point (A) can be located on either the Cartesian or Smith Charts (not shown).

On the Cartesian Chart, at A the values of u and v are:

$$u_0 = 0\cdot325, \quad v_0 = 0\cdot11\lambda.$$

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* See W. Jackson, *High Frequency Transmission Lines*, Methuen, 3rd Edition, 1958, Chapter 6.

At the input B: $u_1 = 0.325$ since there is no loss and

$$v_1 = 0.11\lambda + 0.30\lambda = 0.41\lambda.$$

B corresponds to $z = 0.42 - j0.55$.

Hence, input impedance $= 75(0.42 - j0.55) = (31.5 - j41.2)\,\Omega$.

On the Smith Chart the value of u need not be found since the u circles are all centred at the origin of the chart. Hence, movement of point A on the circle centred at the origin through a distance 0.3λ towards the generator locates point B. Again B corresponds to $z = 0.42 - j0.55$ and so the input impedance is $(31.5 - j41.2)\,\Omega$.

The Cartesian diagram will now be used to find the input impedance when the loss in the line is 1.15 db.

$$1.15 \text{ db} = 1.15/8.686 = 0.132 \text{ neper.}$$

Hence, input impedance is located at a point B' for which $v_1' = v_0 + 0.30\lambda = 0.41\lambda$ and $u_1' = u_0 + 0.132 = 0.457$. Point B' gives $z = (0.56 - j0.48)$, i.e. input impedance $= (42 - j36)\,\Omega$.

In using the Smith Chart to solve this part of the problem the value of u_0 can be read off the pre-calibrated cursor to be 2.85 db. Then $u_1' = u_0 + 1.15 \text{ db} = 4.0$ db. Point B' is then located at the intersection of the 4 db circle and the 0.41λ line.*

The voltage standing-wave ratio existing in the line is equal to the intercept of the u circle on the resistive axis, i.e. 0.315.

263. The input impedance (Z_1) of a finite short-circuited line of length l is $Z_0 \tanh Pl$, where $P = \alpha + j\beta$ is the propagation constant.

If $l = n\lambda/4$ where n is an odd integer and λ is the wavelength, $Z_1 = Z_0/\tanh \alpha l$ which is approximately $Z_0/\alpha l$.

Now α is approximately $R/2Z_0$.

$$\therefore \qquad Z_1 = 8Z_0^2/Rn\lambda$$
$$R = 41.6\sqrt{f}[1/a + 1/b]10^{-7}\,\Omega/\text{m}$$
$$Z_0 = 138 \log_{10}(b/a)\,\Omega$$

$\therefore$ if $n = 1$, in this case, $Z_1 = 248,600\,\Omega$.

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* Further information on transmission-line charts can be found in: W. Jackson and L. G. H. Huxley, 'The Solution of Transmission Line Problems by Use of the Circle Diagram of Impedance,' *J.I.E.E.*, **91**, Part III, p. 105, 1944, and P. H. Smith, 'Transmission Line Calculator,' *Electronics*, **12**, p. 29, January, 1939.

264. The selectivity of a parallel tuned circuit may be expressed in terms of the 'Q' of the coil; and, by analogy, an expression for the 'Q' of a resonant line may be obtained.
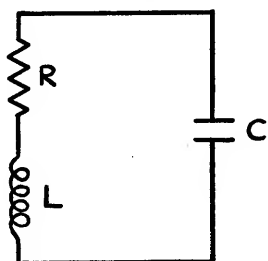
Consider the parallel tuned circuit shown. Its impedance

$$Z = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}. \qquad . \qquad . \qquad (1)$$

At resonance $\omega_R^2 LC = 1$ and $Q = \omega_R L/R$ is large compared with 1, hence

$$Z_R \text{ is approximately } L/CR \qquad . \qquad . \qquad (2)$$

Now the impedance at $\omega_R + \delta\omega$ near resonance is

$$Z = jL(\omega_R + \delta\omega)/[j(\omega_R + \delta\omega)CR - 2\omega_R\delta\omega LC] \qquad . \qquad (3)$$

$$\therefore \qquad\qquad Z = Z_R(1 - j2Q\delta\omega/\omega_R) \qquad . \qquad . \qquad (4)$$

The impedance of a short-circuited $\lambda/4$ line at resonance is $Z_R = Z_0/\alpha l$, as mentioned in the previous solution, and its impedance in general is $Z = Z_0 \tanh (\alpha + j\beta)l$, where $\beta = 2\pi f/c = \omega_R/c$ and c is the velocity of light.

When operation is at $\omega_R + \delta\omega$,

$$\beta = (\omega_R + \delta\omega)/c = 2\pi/\lambda + \delta\omega/c$$

$$\therefore \qquad\qquad \beta l = \pi/2 + l \cdot \delta\omega/c.$$

$$\therefore \qquad Z = Z_0 \tanh (\alpha l + j\delta\omega \cdot l/c + j\pi/2) \qquad . \qquad . \qquad (5)$$

Since αl and $\delta\omega \cdot l/c$ are small:

$$Z = Z_R[1 - j2(\omega_R/2c\alpha)\delta\omega/\omega_R] \qquad . \qquad . \qquad (6)$$

Now equations (4) and (6) are of the same form so the Q of the resonant line is

$$\omega_R/2c\alpha = \pi f/\alpha c = 2\pi f Z_0/Rc \qquad . \qquad . \qquad (7)$$

Substituting for Z_0 and R and putting in the values of the constants gives:

$$Q = 1{,}468.$$

265. For a short-circuited line $Z_1 = Z_0 \tanh(\alpha l + j\beta l)$, where α is the attenuation constant and β is the phase constant.

Expanding and manipulating:

$$Z_1 = Z_0\{\sinh 2\alpha l + j \sin 2\beta l\}/\{\cosh 2\alpha l + \cos 2\beta l\}$$

But, as αl is small, $\cosh 2\alpha l \simeq 1$ and $\sinh 2\alpha l \simeq 2\alpha l$

$\therefore$ $\qquad\qquad Z_1 = Z_0\{\alpha l/\cos^2 \beta l + j \tan \beta l\}.$

Now $\alpha \simeq R/2Z_0$ so,

$$Z_1 = Rc(l/\lambda)/2f \cos^2 2\pi(l/\lambda) + jZ_0 \tan(2\pi l/\lambda).$$

In the case of ordinary reactances, the change of reactance ΔX produced by a fractional change of frequency $\Delta f/f$ is $\Delta X = X \cdot \Delta f/f$.

In the case of the above line, $X = Z_0 \tan(2\pi l/\lambda) = Z_0 \tan(2\pi f l/c)$

$\therefore$ $\qquad\qquad dX = 2\pi(l/\lambda)Z_0(df/f)/\cos^2(2\pi l/\lambda).$

Multiplying both numerator and denominator by $\tan(2\pi l/\lambda)$ and reducing gives

$$dX = 4\pi(l/\lambda)(df/f)X/\sin(4\pi l/\lambda)$$

$$\frac{\text{Selectivity factor of line reactance}}{\text{Selectivity factor of lumped reactance}} = \frac{4\pi l/\lambda}{\sin(4\pi l/\lambda)}.$$

When $l/\lambda = 0{\cdot}2$ this ratio is $4{\cdot}28$.

266. The attenuation in a coaxial line is given by Jackson[*] as:

$$\alpha = 9{\cdot}95 \times 10^{-6}\sqrt{f}\sqrt{\varepsilon/\varepsilon_0}\{1/a\sqrt{\sigma_a} + 1/b\sqrt{\sigma_b}\}/\log_{10}(b/a)$$
$$+ 9{\cdot}10 \times 10^{-8}\sqrt{\varepsilon/\varepsilon_0}f \tan \delta \text{ db/m}$$

where f is the frequency in c/s, $\varepsilon/\varepsilon_0$ is the ratio of the permittivity of the line dielectric to that of air, a and b are the radii of the inner and outer conductors respectively in metres, σ_a and σ_b are the conductivities of the inner and outer conductors in mhos/metre cube and $\tan \delta$ is the power factor of the dielectric.

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[*] See W. Jackson, *High Frequency Transmission Lines*, Methuen, 3rd Edition, 1958, p. 50.

Here $\sigma_a = 5\cdot62 \times 10^7$ mhos/metre cube, $\sigma_b = 1\cdot54 \times 10^7$ mhos/metre cube, $a = 5\cdot65 \times 10^{-4}$ m, $b = 3\cdot97 \times 10^{-3}$ m, $\varepsilon/\varepsilon_0 = 1$, and tan $\delta = 0$.

$\therefore$ $$\alpha = 0\cdot331 \text{ db/m.}$$

The characteristic impedance of a coaxial line is also given by Jackson* as:

$$Z_0 = 138\sqrt{\varepsilon_0/\varepsilon} \log_{10} (b/a) \text{ ohms}$$
$$= 117\,\Omega.$$

267. Using the expression quoted in the previous solution and remembering that $\varepsilon/\varepsilon_0 = 2\cdot25$ and tan $\delta = 0\cdot0004$ it is found that,

$$\alpha = 0\cdot977 \text{ db/m and } Z_0 = 78\,\Omega.$$

268. The formulae for calculating the characteristic impedance Z_0 and the attenuation α are given by Blackband and Brown.†

$$\alpha = 8\cdot686 \tanh^{-1}\sqrt{g_{min}/g_{max}} \text{ db.}$$

From the circle (given with the problem),

$$g_{min} = 0\cdot489, g_{max} = 1\cdot202, \text{ so } \alpha = 6\cdot55 \text{ db.}$$

$Z_0 = Z_0'\sqrt{g_{min} \cdot g_{max}}$, where Z_0' is the characteristic impedance of the measuring line.

$\therefore$ $$Z_0 = 75/\sqrt{0\cdot489 \times 1\cdot202} = 97\cdot8\,\Omega.$$

269. The zero-susceptance points are seen from the circle (given with the problem) to be $g'_{min} = 0\cdot492$ and $g'_{max} = 1\cdot198$.
α and Z_0 are now calculated as in the previous solution.

$$\alpha = 6\cdot60 \text{ db.}$$
$$Z_0 = 97\cdot7\,\Omega.$$

270. A quarter-wavelength section of line of characteristic impedance $Z_0 = (\sqrt{150 \times 75})\,\Omega$, i.e. $106\,\Omega$ provides the desired transformation, eliminating a reflected wave on the $75\,\Omega$ line.

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* W. Jackson, *High Frequency Transmission Lines*, Methuen, 1945, p. 46.
† W. T. Blackband and D R. Brown, 'The Two-point Method of Measuring Characteristic Impedance and Attenuation of Cables at 3,000 Mc/s,' *J.I.E.E.*, **93**, Part IIIA, p. 1383, 1946.

271. The admittance of the load $= 1/(100 + j100)$ mho.
$$= (1 - j)/200 \text{ mho.}$$

Susceptance of load is thus $(-j/200)$ mho, so that of the stub must be $(+j/200)$ mho. The conductance of the load is then $(1/200)$ mho, i.e. the load resistance $= 200 \, \Omega$. To match this load to a line of characteristic impedance $500 /\underline{0°} \, \Omega$, the quarter-wave line must have an impedance of $\sqrt{500 \times 200} \, \Omega = 316 \, \Omega$.

The susceptance of a short-circuited line of characteristic impedance Z_0 and length l is

$$- j/Z_0 \tan \beta l = - j/Z_0 \tan (2\pi l/\lambda).$$

Thus, $\qquad\qquad - j/Z_0 \tan (2\pi l/\lambda) = j/200.$

$\therefore \qquad\qquad \tan (2\pi f l/c) = - 200/Z_0 = - 200/316 = 0\cdot6328$

$\therefore \qquad\qquad\qquad 2\pi f l/c = 2\cdot578 \text{ radians.}$

Now $f = 100 \times 10^6$ c/s, $c = 3 \times 10^8$ m/sec, so the minimum length of line l is $2\cdot578 \times 3 \times 10^8/2\pi \times 10^8 = \underline{1\cdot23 \text{ m.}}$

272. For the explanation asked for see the book by Kraus.*

273. The mesh equations are:

$$V_1 = 60I_1 - 50I_3 \quad . \qquad . \qquad . \qquad . \quad (1)$$
$$90I_3 = 50I_1 + 20I_4 \quad . \qquad . \qquad . \qquad . \quad (2)$$
$$80I_4 = 20I_3 + 10I_2 \quad . \qquad . \qquad . \qquad . \quad (3)$$
$$V_2 = 10(I_4 - I_2) \quad . \qquad . \qquad . \qquad . \quad (4)$$

If I_3 and I_4 are eliminated two equations remain which can be put in the form:

$$V_1 = AV_2 + BI_2 \quad \text{and} \quad I_1 = CV_2 + DI_2$$

where $A = 20\cdot8$, $B = 179 \, \Omega$, $C = 0\cdot68$ mho and $D = 5\cdot9$.

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* J. D. Kraus, *Electromagnetics*, McGraw-Hill, 1953, pp. 439–441.

274. Let the network be represented by the T-section shown. If the image impedance Z_i is connected across terminals 3 and 4 the impedance measured between 1 and 2 is also Z_i.
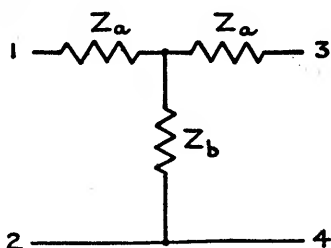
$$\therefore \qquad Z_i^2 = Z_a^2(1 + 2Z_b/Z_a) \qquad . \qquad . \qquad . \quad (1)$$

If the current flowing into 1, 2 is i_1 and the current flowing out of 3, 4 is i_2, the image transfer constant is given by

$$e^{+\theta} = i_1/i_2 \qquad . \qquad . \qquad . \qquad . \quad (2)$$

Voltage across terminals 3 and 4 $= i_2 Z_i$

Voltage across Z_b $\qquad\qquad = i_2(Z_i + Z_a)$

$$\therefore \qquad\qquad \text{Current in } Z_b = i_2(Z_i + Z_a)/Z_b = i_1 - i_2 \quad (3)$$

From (2) and (3),

$$\frac{Z_i}{Z_b} = e^\theta - 1 - \frac{Z_a}{Z_b} = \frac{Z_i}{Z_a} \cdot \frac{Z_a}{Z_b} \qquad . \qquad . \qquad . \quad (4)$$

$\therefore$ from (1) and (4),

$$Z_i^2 = Z_a^2\{1 + 2(Z_i/Z_a + 1)/(e^\theta - 1)\} \qquad . \qquad . \quad (5)$$
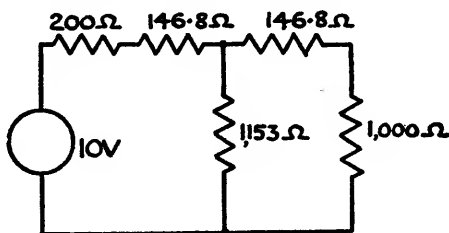
In this case $\theta = 0.5$ and $Z_i = 600\,\Omega$ $\quad\therefore\ \underline{Z_a = 146.8\,\Omega.}$

Also $\qquad Z_b = Z_a(Z_i/Z_a + 1)/(e^\theta - 1) = \underline{1,153\,\Omega.}$
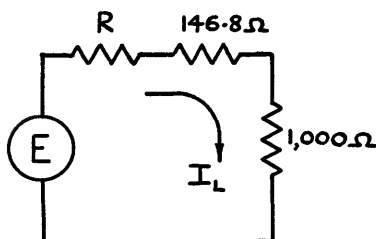
Using Thévenin's Theorem, the network can be replaced by the one shown where

$$E = 10 \times 1{,}153/(1{,}153 + 200 + 146{\cdot}8) = 7{\cdot}68 \text{ V}$$

and $$R = 1{,}153 \times 346{\cdot}8/(1{,}153 + 346{\cdot}8) = 266\,\Omega.$$

$$\therefore \quad I_L = 7{\cdot}68/(266 + 146{\cdot}8 + 1{,}000) = 5{\cdot}44 \text{ mA}.$$

275. The conditions for zero output are:[*]

$$\omega^2 = 1/R_3 C_1 C_2 (R_1 + R_2) \text{ and } \omega^2 = (C_1 + C_2)/(C_1 C_2 C_3 R_1 R_2).$$

In this case $f = \omega/2\pi = 1{,}240 \text{ c/s}$.

276. (*a*) With the output short-circuited,

$$Z_{sc} = R_1 + R_1 R_2/(R_1 + R_2)$$

„ „ „ open-circuited, $Z_{oc} = R_1 + R_2$

$$\therefore \qquad Z_i^2 = Z_{oc} Z_{sc} = R_1^2 + 2R_1 R_2$$

and $$\tanh^2 \theta = Z_{sc}/Z_{oc} = (R_1^2 + 2R_1 R_2)/(R_1 + R_2)^2.$$

It follows that $R_2 = Z_i/\sinh\theta$ and $R_1 = Z_i \tanh(\theta/2)$.

If N is the voltage ratio $= e^\theta$,

$$R_1 = Z_i(N - 1)/(N + 1) \text{ and } R_2 = 2NZ_i/(N^2 - 1).$$

When the loss is 10 db, $N = 3{\cdot}162$; also $Z_i = 600\,\Omega$

$$\therefore \qquad R_1 = 311{\cdot}8\,\Omega \text{ and } R_2 = 421{\cdot}6\,\Omega.$$

When the loss is 20 db, $N = 10$; also $Z_i = 600\,\Omega$

$$\therefore \qquad R_1 = 491\,\Omega \text{ and } R_2 = 121\,\Omega.$$

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[*] A. T. Starr, *Electronics*, Pitman, 2nd Edition, 1959, pp. 155–6.

(b) Let the elements of the attenuator have resistances $(R_1/2)$ and R_2 as illustrated.
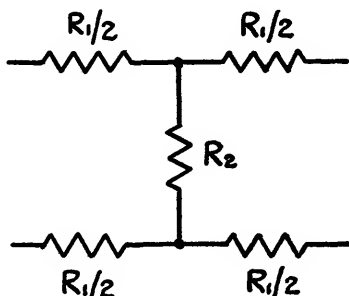
Then, $600 = \sqrt{R_1(R_1 + 2R_2)}$.   .   . (1)

and $\alpha = \cosh^{-1}(1 + R_1/R_2)$.   .   . (2)

From (1), $600 = 480(480 + R_2)$

$\therefore$ $\underline{R_2 = 135\ \Omega}$

From (2) $\alpha = \cosh^{-1}\left(1 + \dfrac{480}{135}\right)$ nepers

$= \underline{19 \cdot 1\ \text{dB}}$

277. With terminals 3 and 4 open-circuited:

$$Z_a + Z_c \doteq (250 + j100)\ \Omega \quad . \quad . \quad . \ (1)$$

With terminals 3 and 4 short-circuited:

$$Z_a + Z_b Z_c/(Z_b + Z_c) = (400 + j300)\ \Omega \quad . \quad . \ (2)$$

With terminals 1 and 2 open-circuited:

$$Z_b + Z_c = 200\ \Omega \quad . \quad . \quad . \quad . \ (3)$$

From (1), (2) and (3):

$$\underline{Z_a = (150 + j300)\ \Omega,}$$
$$\underline{Z_b = (100 + j200)\ \Omega,}$$
$$\underline{Z_c = (100 - j200)\ \Omega.}$$

278. The solution to this problem has been given elsewhere.*

279. The equations relating the input voltage V_1 and current I_1 with the output voltage V_2 and current I_2 are:

$$V_1 = AV_2 + BI_2 \quad . \qquad . \qquad . \qquad . \quad (1)$$

$$I_1 = CV_2 + DI_2 \quad . \qquad . \qquad . \qquad . \quad (2)$$

For circuit (a), $\qquad V_1 = V_2 + ZI_2 \quad . \qquad . \qquad . \qquad . \quad (3)$

and $\qquad\qquad\qquad\qquad I_1 = I_2 \quad . \qquad . \qquad . \qquad . \qquad . \quad (4)$

Comparing equations (1) and (3), also (2) and (4),

$$A = 1, \quad B = Z \text{ ohms}, \quad C = 0, \quad D = 1.$$

The transfer matrix $[A]$ is therefore $\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$.

For circuit (b), $\qquad\qquad V_1 = V_2 \quad . \qquad . \qquad . \qquad . \quad (5)$

and $\qquad\qquad\qquad I_1 = YV_2 + I_2 \quad . \qquad . \qquad . \quad (6)$

Comparing equations (1) and (5), also (2) and (6),

$$A = 1, \quad B = 0, \quad C = Y \text{ mhos}, \quad D = 1.$$

The transfer matrix $[A]$ is therefore $\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$.

The transfer matrix of the network and load in cascade is

$$[A] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1/Z_l & 1 \end{bmatrix} = \begin{bmatrix} A + B/Z_l & B \\ C + D/Z_l & D \end{bmatrix}.$$

Thus, the input impedance

$$Z_{11} = \frac{A + B/Z_l}{C + D/Z_l} = \frac{AZ_l + B}{CZ_l + D}$$

and the voltage gain

$$= \frac{1}{A + B/Z_l}$$

For the common-base transistor:

$$[Z] = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$$

so $\qquad\qquad\qquad [A] = \begin{bmatrix} \dfrac{r_{11}}{r_{21}} & \dfrac{|Z|}{r_{21}} \\ \dfrac{1}{r_{21}} & \dfrac{r_{22}}{r_{21}} \end{bmatrix}$

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* F. A. Benson and D. Harrison, *Electric Circuit Theory*, Arnold, 2nd Edition, 1963, p. 128.

where $$|Z| = r_{11} \cdot r_{22} - r_{12} \cdot r_{21}$$

$\therefore$ voltage gain $= \dfrac{1}{r_{11}/r_{21} + |Z|/r_{21}R_l}$

$$= \frac{r_{21}R_l}{r_{11}R_l + r_{11}r_{22} - r_{12}r_{21}}$$

The input resistance $= \dfrac{r_{11}R_l/r_{21} + |Z|/r_{21}}{R_l/r_{21} + r_{22}/r_{21}}$

$$= r_{11} - r_{12}r_{21}/(r_{22} + R_l)$$

280. Circuit (a) consists of a series impedance and a shunt admittance in cascade. Therefore, using the results of the previous solution, the transfer matrix $[A]$ is:

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} (1 + ZY) & Z \\ Y & 1 \end{bmatrix}$$

Circuit (b) consists of a shunt admittance and a series impedance in cascade. Thus,

$$[A] = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & (1 + ZY) \end{bmatrix}.$$

Circuit (c) can be considered as a series impedance, a shunt admittance and a series impedance in cascade. Therefore:

$$[A] = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1/Z_{12} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 + Z_1/Z_{12}) & (Z_1 + Z_2 + Z_1Z_2/Z_{12}) \\ 1/Z_{12} & (1 + Z_2/Z_{12}) \end{bmatrix}.$$

Similarly for circuit (d),

$$[A] = \begin{bmatrix} 1 & 0 \\ 1/Z_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_{12} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 + Z_{12}/Z_2) & Z_{12} \\ \{1/Z_1 + (1 + Z_{12}/Z_1)(1/Z_2)\} & (1 + Z_{12}/Z_1) \end{bmatrix}.$$

281. The transfer matrix for the first network excluding the load is:

$$[A] = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1 - 3\omega^2 LC + \omega^4 L^2 C^2) & (2j\omega L - j\omega^3 L^2 C) \\ (2j\omega C - j\omega^3 LC^2) & (1 - \omega^2 LC) \end{bmatrix}.$$

The transfer matrix of a network which has general parameters A, B, C and D, and a load Z_l in cascade is:

$$\begin{bmatrix} A + B/Z_l & B \\ C + D/Z_l & D \end{bmatrix}.$$

Thus, $V_2/V_1 = 1/(A + B/Z_l)$.

Here $A = 1 - 3\omega^2 LC + \omega^4 L^2 C^2$,

$B = 2j\omega L - j\omega^3 L^2 C$,

and $Z_l = \sqrt{L/C} + j\omega L/2$.

$\therefore$ V_2/V_1

$$= \frac{1}{1 - 3\omega^2 LC + \omega^4 L^2 C^2 + j\omega L(2 - \omega^2 LC)/(\sqrt{L/C} + j\omega L/2)}$$

The transfer matrix for the second network excluding the load is:

$$[A] = \begin{bmatrix} (1 - \omega^2 LC) & (2j\omega L - j\omega^3 L^2 C) \\ (j\omega C) & (1 - \omega^2 LC) \end{bmatrix}$$

$\therefore$ $$V_2/V_1 = \frac{R_l}{(1 - \omega^2 LC)R_l + (2j\omega L - j\omega^3 L^2 C)}$$

282. (a)

$$[A] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad [A] = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$
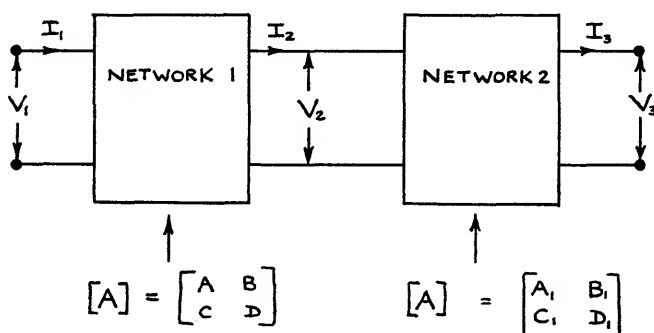
For network ①:

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

For network ②:

$$V_2 = A_1 V_3 + B_1 I_3$$
$$I_2 = C_1 V_3 + D_1 I_3$$

$$\therefore \qquad V_1 = (AA_1 + BC_1)V_3 + (AB_1 + BD_1)I_3$$
$$I_1 = (CA_1 + DC_1)V_3 + (CB_1 + DD_1)I_3$$

For the combined network, therefore,

$$[A] = \begin{bmatrix} (AA_1 + BC_1) & (AB_1 + BD_1) \\ (CA_1 + DC_1) & (CB_1 + DD_1) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

(b) When the two networks are connected in order 1, 2 the transfer matrix of the combination is the product of the original transfer matrices, i.e.

$$[A_{1,2}] = [A_1] \cdot [A_2] = \begin{bmatrix} 1\cdot50 & 11 \\ 0\cdot25 & 2\cdot5 \end{bmatrix} \cdot \begin{bmatrix} 1\cdot66 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 13\cdot5 & 39\cdot0 \\ 2\cdot92 & 8\cdot5 \end{bmatrix}.$$

The Y parameters of the networks are as follows:

	Network 1	Network 2
$Y_{11} = D/B$	2·5/11	3/4
$Y_{12} = -\|A\|/B$	−1/11	−1/4
$Y_{21} = 1/B$	1/11	1/4
$Y_{22} = -A/B$	−1·5/11	−1·66/4

The admittance matrix of the networks in parallel is

$$[Y_{1,2}] = [Y_1] + [Y_2]$$
$$= \begin{bmatrix} 2\cdot5/11 & -1/11 \\ 1/11 & -1\cdot5/11 \end{bmatrix} + \begin{bmatrix} 3/4 & -1/4 \\ 1/4 & -1\cdot66/4 \end{bmatrix}$$
$$= \begin{bmatrix} 43/44 & -15/44 \\ 15/44 & -73/132 \end{bmatrix}.$$

283. The mesh equations for the circuit are:

$$Z_2(I_1 - i) + Z_1(I_1 - i + I_2') = V_1 \qquad . \qquad . \quad (1)$$
$$Z_2(i - I_2') - Z_1(I_1 - i + I_2') = V_2 \qquad . \qquad . \quad (2)$$
$$Z_1 i + Z_2(i - I_2') = Z_1(I_1 - i + I_2') + Z_2(I_1 - i) \qquad . \quad (3)$$

From (3), $\qquad i = (I_1 + I_2')/2$ . . . . (4)

Substituting (4) in (1) and (2) there results:

$$Z_2(I_1 - I_2') + Z_1(I_1 + I_2') = 2V_1. \qquad . \quad . \text{ (5)}$$

$$Z_2(I_1 - I_2') - Z_1(I_1 + I_2') = 2V_2. \qquad . \quad . \text{ (6)}$$

Thus, the $[Z]$ matrix is seen to be

$$\begin{bmatrix} (Z_1 + Z_2)/2 & (Z_2 - Z_1)/2 \\ (Z_2 - Z_1)/2 & (Z_1 + Z_2)/2 \end{bmatrix}.$$

Equations (5) and (6) can be re-arranged to give the A, B, C and D parameters of the $[A]$ matrix. Alternatively,

$$A = Z_{11}/Z_{12} = Z_{22}/Z_{12} = D = (Z_1 + Z_2)/(Z_2 - Z_1),$$

$$B = (Z_{11}Z_{22} - Z_{12}{}^2)/Z_{12} = 2Z_1Z_2/(Z_2 - Z_1),$$

and $\qquad C = 1/Z_{12} = 2/(Z_2 - Z_1).$

$\therefore \qquad [A] = \begin{bmatrix} (Z_1 + Z_2)/(Z_2 - Z_1) & 2Z_1Z_2/(Z_2 - Z_1) \\ 2/(Z_2 - Z_1) & (Z_1 + Z_2)/(Z_2 - Z_1) \end{bmatrix}.$

284. *Circuit* (*a*)

Grid current of valve $i_g = I_1 = 0$.

Anode current of valve $i_a = -I_2 = g_m v_g + v_a/r_a$

$$= g_m V_1 + V_2/r_a.$$

In matrix form these equations become:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -g_m & -1/r_a \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}.$$

The admittance matrix is therefore: $\begin{bmatrix} 0 & 0 \\ -g_m & -1/r_a \end{bmatrix}.$

Circuit (*b*)

The arrangement can be regarded as a shunt admittance in cascade with the valve. The transfer matrix $[A_1]$ for the admittance is $\begin{bmatrix} 1 & 0 \\ Y_g & 1 \end{bmatrix}$. The admittance matrix for the valve is given above, from which the transfer matrix for the valve is:

$$[A_2] = \begin{bmatrix} -1/g_m r_a & -1/g_m \\ 0 & 0 \end{bmatrix}.$$

Thus, the transfer matrix for the combination is :

$$[A_{1,2}] = \begin{bmatrix} 1 & 0 \\ Y_g & 1 \end{bmatrix} \cdot \begin{bmatrix} -1/\mu & -1/g_m \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1/\mu & -1/g_m \\ -Y_g/\mu & -Y_g/g_m \end{bmatrix}.$$

$\therefore$ the admittance matrix for the combination is

$$\begin{bmatrix} Y_g & 0 \\ -g_m & -1/r_a \end{bmatrix}.$$

It is evident that the required matrix can also be found from the sum of the $[Y]$ matrices

$$\begin{bmatrix} Y_g & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 \\ -g_m & -1/r_a \end{bmatrix}$$

i.e.

$$\begin{bmatrix} Y_g & 0 \\ -g_m & -1/r_a \end{bmatrix}.$$

Circuit (c)

The interelectrode capacitances can be introduced by adding another $[Y]$ matrix to that just found above, the result is:

$$\begin{bmatrix} Y_g & 0 \\ -g_m & -1/r_a \end{bmatrix} + \begin{bmatrix} j\omega(C_{gc} + C_{ga}) & -j\omega C_{ga} \\ j\omega C_{ga} & -j\omega(C_{ac} + C_{ga}) \end{bmatrix}$$

$$= \begin{bmatrix} Y_g + j\omega(C_{ga} + C_{gc}) & -j\omega C_{ga} \\ (j\omega C_{ga} - g_m) & -\{1/r_a + j\omega(C_{ga} + C_{ac})\} \end{bmatrix}.$$

Circuit (d)

Grid current of valve $i_g = I_1 - Y_g V_1 = 0$.

Anode current of valve $i_a = -I_2 = g_m v_g + v_a/r_a$

$$= g_m(V_1 + Z_c I_2) + (V_2 + Z_c I_2)/r_a.$$

Thus, as for circuit (a), the admittance matrix can be found. The result is:

$$\begin{bmatrix} Y_g & 0 \\ -g_m/\{1 + Z_c(g_m + 1/r_a)\} & -(1/r_a)/\{1 + Z_c(g_m + 1/r_a)\} \end{bmatrix}.$$

285. The transfer matrices are:

For Z_1, $\begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}$, (see Solution 279)

For Y, $\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$, (see Solution 279)

For Z_2, $\begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}$, (see Solution 279)

For the transformer, $\begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$, (for a transformer $V_1 = nV_2$ and $I_1 = I_2/n$)

$\therefore$ Transfer matrix for whole arrangement is:

$$\begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \cdot \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} \cdot \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n(1 + Z_1 Y) & nZ_2(1 + Z_1 Y) + Z_1/n \\ n Y & nZ_2 Y + 1/n \end{bmatrix}$$

286. The solution to this problem can be found in certain standard textbooks.*

287. The solution to this problem can be found in certain standard textbooks.*

288. Critical wavelength $\lambda_c = 2 \times 7\cdot62 = 15\cdot24$ cm.

Guide wavelength λ_g is given by $1/\lambda_g{}^2 = 1/10^2 - 1/15\cdot24^2$, since a wavelength of 10 cm corresponds to the frequency of 3,000 Mc/s

$\therefore$ $\qquad\qquad\qquad \lambda_g = 13\cdot3$ cm.

289. The solution to this problem can be found in certain standard textbooks.†

290. It can be shown that the attenuation in nepers per unit length of a guide carrying an evanescent mode is given by:‡

$$\alpha = 2\pi\sqrt{1 - (\lambda_c/\lambda)^2}/\lambda_c.$$

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* E.g., see H. R. L. Lamont, *Wave Guides*, Methuen, 3rd Edition, 1950, Chapter 1.
† E.g., see H. R. L. Lamont, *Wave Guides*, Methuen, 3rd Edition, 1950, Chapter 2.
‡ See, for example, H. M. Barlow and A. L. Cullen, *Microwave Measurements*, Constable, 1950, p. 245; *or* L. G. H. Huxley, *A Survey of the Principles and Practice of Waveguides*, Cambridge U.P., 1947, p. 57.

Here $\lambda_c = 2\cdot61a$, where a is the cylinder radius* and $\lambda = 30$ cm.
Also $\alpha = 10$ db/cm $= 10 \times 0\cdot115$ nepers/cm.

$$\therefore \qquad\qquad\qquad \underline{a = 2\cdot06 \text{ cm.}}$$

291. A general formula for calculating the attenuation in a wave-guide caused by losses in the wall metal has been given by Kuhn.†
For the case of the H_{01} mode in a rectangular waveguide the attenuation α is given by the following expression:

$$\alpha = \left[\frac{c}{\sigma}\cdot\frac{\mu_1}{\mu}\left(\frac{\varepsilon}{\mu}\right)^{1/2}\right]^{1/2} \frac{1}{b^{3/2}}\cdot\frac{1}{\left[\frac{\lambda_e}{\lambda_{cr}}\left(1-\frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right]^{1/2}}\left(\frac{1}{2}\right)^{1/2}$$
$$\left[\frac{\lambda_e^2}{\lambda_{cr}^2}+\frac{b}{2a}\right] \qquad . \quad (1)$$

where a and b are the short and long internal dimensions of the guide respectively, λ_e is the wavelength in the unbounded dielectric, λ_{cr} is the critical wavelength of the guide, σ is the conductivity of the wall metal, μ_1 is the permeability of the wall metal, ε and μ are the dielectric constant and relative permeability of the dielectric respectively, and c is the velocity of electromagnetic waves. In the case of an air-filled copper guide with resistivity $1\cdot7 \times 10^{-6}$ ohm-cm the factor $\left[\frac{c}{\sigma}\cdot\frac{\mu_1}{\mu}\cdot\left(\frac{\varepsilon}{\mu}\right)^{1/2}\right]^{1/2} = 0\cdot2065$, if α is measured in db/m, the guide dimensions and wavelengths being in cm.
Expression (1) gives α in a form in which the ratio λ_e/λ_{cr} is the only parameter involving wavelength. An alternative expression is:

$$\alpha = \left[\frac{c}{\sigma}\cdot\frac{\mu_1}{\mu}\cdot\left(\frac{\varepsilon}{\mu}\right)^{1/2}\right]^{1/2}\frac{\lambda_g}{b(\lambda_e)^{3/2}}\left[\frac{\lambda_e^2}{\lambda_{cr}^2}+\frac{b}{2a}\right] \qquad . \quad . \quad (2)$$

where λ_g is the guide wavelength and

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_e^2} - \frac{1}{\lambda_{cr}^2} \qquad . \qquad . \qquad . \qquad . \quad (3)$$

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* See Problem No. 289.
† S. Kuhn, 'Calculation of Attenuation in Waveguides,' *J. Instn. Electrical Engrs.*, **93**, Part IIIA, pp. 663–78, 1946.

Consider the case where $\lambda_e = 3 \cdot 1$ cm,

$$b = 2 \cdot 54 \text{ cm}, \quad b/a = 2, \quad \lambda_{cr} = 2b = 5 \cdot 08 \text{ cm}$$

$$\therefore \qquad \frac{1}{\lambda_g{}^2} = \frac{1}{3 \cdot 1^2} - \frac{1}{5 \cdot 08^2}, \text{ i.e. } \lambda_g = 3 \cdot 92 \text{ cm.}$$

From (2),

$$\alpha = \frac{0 \cdot 2065 \times 3 \cdot 92}{2 \cdot 54} \frac{1}{(3 \cdot 1)^{3/2}} \times \left[\frac{(3 \cdot 1)^2}{(5 \cdot 08)^2} + 1 \right] \text{db/m} = \underline{0 \cdot 0801 \text{ db/m.}}$$

Similarly if $\lambda_e = 3 \cdot 2$ cm, α is found to be $\underline{0 \cdot 0814 \text{ db/m.}}$

292. Using the formula for attenuation given as (2) in the previous solution:

$b = 7 \cdot 62$ cm, $b/a = 3$, $\lambda_e = 10$ cm, $\lambda_{cr} = 2b = 15 \cdot 24$ cm and $\lambda_g = 13 \cdot 25$ cm.

$$\therefore \qquad\qquad \alpha = \underline{0 \cdot 022 \text{ db/m.}}$$

293. The losses due to the wall metal can be calculated from equation (2) of the solution to Question 291. It should be remembered, however, that the constant $0 \cdot 2065$ previously used for $\left[\frac{c}{\sigma} \cdot \frac{\mu_1}{\mu} \left(\frac{\varepsilon}{\mu} \right)^{1/2} \right]^{1/2}$ should now be multiplied by $(2 \cdot 55)^{1/4}$ since $\varepsilon = 2 \cdot 55$.

In this case $\lambda_e = 10/\sqrt{2 \cdot 55} = 6 \cdot 318$ cm, $\lambda_{cr} = 2b = 9 \cdot 6$ cm, so $\lambda_g = 8 \cdot 391$ cm. Further $b/a = 3$,

$$\therefore \qquad\qquad \alpha = 0 \cdot 05543 \text{ db/m.}$$

A formula for calculating the loss in the dielectric of a waveguide has been given by Kuhn.* The attenuation constant α_d in db/m is given by:

$$\alpha_d = \frac{2,726}{\lambda_e} [\tan \delta] \Big/ \left[1 - \left(\frac{\lambda_e}{\lambda_{cr}} \right)^2 \right]^{1/2}.$$

In this case, $\lambda_e = 6 \cdot 318$ cm, $\tan \delta = 0 \cdot 0006$ and $\lambda_{cr} = 9 \cdot 6$ cm.

$$\therefore \qquad\qquad \alpha_d = 0 \cdot 3438 \text{ db/m.}$$

The total value of the attenuation in this guide is

$$(0 \cdot 05543 + 0 \cdot 3438) \text{ db/m} = \underline{0 \cdot 399 \text{ db/m.}}$$

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* S. Kuhn, 'Calculation of Attenuation in Waveguides,' *J. Instn. Electrical Engrs.*, **93**, Part IIIA, pp. 663–78, 1946.

294. (a) Using the formula given as (2) in the solution to Question 291:

$b = 2\cdot54$ cm, $b/a = 2$, $\lambda_e = 3\cdot2$ cm, $\lambda_{cr} = 2b = 5\cdot08$ cm, and $\lambda_g = 4\cdot12$ cm.

$$\therefore \qquad \alpha = 0\cdot157 \text{ db/m}.$$

(b) It has been shown* that the formula used for the solution of part (a) of this problem can be modified to account for surface roughness. This is done by introducing three additional factors K_{T1}, K_{T2} and K_p. For the particular H_{01} mode the new expression is:

$$\alpha = \left[\frac{c}{\sigma} \cdot \frac{\mu_1}{\mu} \cdot \left(\frac{\varepsilon}{\mu}\right)^{1/2}\right]^{1/2} \frac{\lambda_g}{b(\lambda_e)^{3/2}} \left[\left(K_{T2} + \frac{b}{2a} K_{T1}\right) \frac{\lambda_e^2}{\lambda_{cr}^2} + K_p \frac{b}{2a}\left(1 - \frac{\lambda_e^2}{\lambda_{cr}^2}\right)\right].$$

Substituting the given figures α is found to be $0\cdot165$ db/m.

295. Using the formula given under part (b) of the previous solution and substituting the given figures μ_1 is found to be $3\cdot35$.

296. The guide wavelength $\lambda_g = 2(5\cdot731 - 3\cdot749) = 3\cdot964$ cm. The voltage standing-wave ratio is†‡

$$r = [K^2 - \cos^2(\pi w/\lambda_g)]^{\frac{1}{2}}/\sin(\pi w/\lambda_g) = 15\cdot25$$

[For high voltage standing-wave ratios $r \simeq (\sqrt{K^2 - 1})\lambda_g/\pi w$, i.e. $r \simeq 15\cdot17$, i.e. less than $0\cdot5\%$ error.]

The loss in the component

$$\alpha = 10 \log_{10}\{(r + 1)/(r - 1)\} \text{ db} = 0\cdot581 \text{ db}.$$

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* See J. Allison and F. A. Benson, 'Surface Roughness and Attenuation of Precision-drawn, Chemically-polished, Electropolished, Electroplated and Electroformed Waveguides,' *Proc. I.E.E.*, **102**, Part B, p. 251, 1955.

† See S. Roberts and A. von Hippell, 'A New Method of Measuring Dielectric Constant and Loss in the Range of Centimetric Waves,' *J. App. Phys.*, **17**, p. 610, 1946; *or* H. M. Barlow and A. L. Cullen, *Microwave Measurements*, Constable, 1950, Chapter 5; *or* F. A. Benson, 'Waveguide Attenuation and its Correlation with Surface Roughness,' *Proc. I.E.E.*, **100**, Part III, p. 85, 1953.

‡ Here the voltage standing-wave ratio is measured as a quantity greater than unity.

297. The cut-off frequency $f_c{}^* = 1/\pi\sqrt{LC} = 1{,}000$ c/s   . (1)
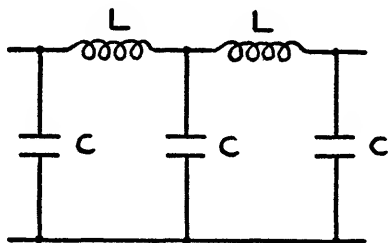
Now load resistance $= \sqrt{L/C} = 50\,\Omega$.   .   .   . (2)

From (1) and (2), $\underline{C = 6{\cdot}37\,\mu F}$ and $\underline{L = 15{\cdot}92\text{ mH}}$.

Attenuation constant α (per section)

$$= \cosh^{-1}(-1)(1 + Z_1/2Z_2)$$

where $Z_1 = j\omega L$, and $Z_2 = 1/j\omega C$,

$$= \cosh^{-1}(-1)(1 - 2f^2/f_c^2)$$

$$\therefore \qquad \alpha = \cosh^{-1}(-1)\left\{1 - 2\left(\frac{9}{4}f_c^2\right)\Big/ f_c^2\right\} = \underline{1{\cdot}928}.$$

298. (a) The frequencies f_1 and f_2 at the ends of the pass band are given by:*

$$f_1 = f_c\left[\sqrt{\frac{C_1}{C_2} + 1} - \sqrt{\frac{C_1}{C_2}}\right]$$

and

$$f_2 = f_c\left[\sqrt{\frac{C_1}{C_2} + 1} + \sqrt{\frac{C_1}{C_2}}\right]$$

where f_c is the resonant frequency of both arms $= 1{,}000$ c/s, C_1 is the capacitance in the series arm and C_2 is the capacitance in the shunt arm.

$$\therefore \qquad f_1 = 1{,}000[\sqrt{1{\cdot}01} - \sqrt{0{\cdot}01}] = 905 \text{ c/s}$$

and

$$f_2 = 1{,}000[\sqrt{1{\cdot}01} + \sqrt{0{\cdot}01}] = 1{,}105 \text{ c/s}$$

$$\therefore \qquad \underline{\text{bandwidth} = (f_2 - f_1) = 200 \text{ c/s}}.$$

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* See, for example, L. C. Jackson, *Wave Filters*, Methuen, 3rd Edition, 1960, Chapter 2.

(b) The T-section is illustrated in the figure.
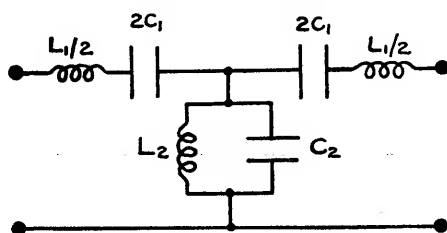
The angular frequencies ω_1 and ω_2, corresponding to frequencies f_1 and f_2 at the ends of the pass band are, from the previous solution:

$$\omega_1 = \omega_c \left[\sqrt{\frac{C_1}{C_2} + 1} - \sqrt{\frac{C_1}{C_2}} \right]$$

and

$$\omega_2 = \omega_c \left[\sqrt{\frac{C_1}{C_2} + 1} + \sqrt{\frac{C_1}{C_2}} \right]$$

where $\omega_c{}^2 = 1/L_1 C_1 = 1/L_2 C_2$

The iterative impedance*

$$Z_0 = R \sqrt{ 1 - \frac{\left(\dfrac{\omega}{\omega_c} - \dfrac{\omega_c}{\omega} \right)^2}{\left(\dfrac{\omega_1}{\omega_c} - \dfrac{\omega_c}{\omega_1} \right)^2} }$$

where $R = \sqrt{L_1/C_2} = \sqrt{L_2/C_1}$

From these equations and assuming $Z_0 = R$:

$$L_1 = 2R/(\omega_2 - \omega_1) = 2 \times 600/3000 \times 2\pi = 63 \cdot 6 \times 10^{-3}\,\text{H}$$

∴   each series inductance $= \underline{31 \cdot 8\ \text{mH}}$

$$C_2 = L_1/R^2 = \{(63 \cdot 6 \times 10^{-3} \times 10^6)/600^2\}\mu\text{F} = \underline{0 \cdot 177\ \mu\text{F}}$$

$$L_2 = (\omega_2 - \omega_1)R/2\omega_2\omega_1$$

$$= (3000 \times 2\pi \times 600 \times 10^6/2 \times 2\pi \times 120 \times$$
$$2\pi \times 123 \times 10^6)\mu\text{H}$$

$$= \underline{9 \cdot 7\ \mu\text{H}}$$

$$C_1 = \underline{L_2/R^2} = (9 \cdot 7 \times 10^{-6} \times 10^{12}/600^2)\ \mu\mu\text{F} = 26 \cdot 95\ \mu\mu\text{F}$$

∴   each series capacitance $= \underline{53 \cdot 9\ \mu\mu\text{F}}$.

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* See, for example, L. C. Jackson, *Wave Filters*, Methuen, 3rd Edition, 1960 Chapter 2.

299.   The cut-off frequency $f_c* = 1/4\pi\sqrt{LC} = 2,500$ c/s   .   (1)

Now   load resistance $= \sqrt{L/C} = 600\,\Omega$.   . (2)

$\therefore$ $L = 19{\cdot}1$ mH and $C = 0{\cdot}053\,\mu$F.
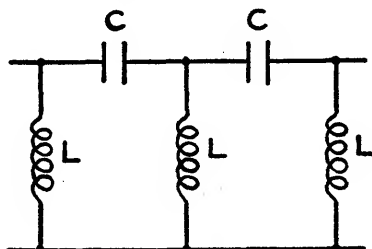
300. The solution to this problem can be found elsewhere.†

301. The solution to this problem can be found elsewhere.‡

302. The solution to this problem can be found elsewhere.§

303. The solution to this problem can be found elsewhere.∥

304. Cut-off frequency¶ $f_c = 1/\pi\sqrt{LC} = 796$ c/s.
Terminating impedance¶ $= \sqrt{L/C} = 600\,\Omega$.

It follows that $L = 240$ mH and $C = 0{\cdot}666\,\mu$F. A T-section would therefore be constructed from two 120-mH $(L/2)$ series inductors and a $0{\cdot}666$-μF capacitor in the shunt arm.

For a π-section there would be a 240-mH series inductor with two $0{\cdot}333\,\mu$F $(C/2)$ capacitors as shunt elements.

305. The series-derived T-section is shown in diagram (a) and the shunt-derived π-section in diagram (b).

Here $Z_1 = \omega L$ and $Z_2 = 1/\omega C$

$\therefore$ for (a) each series inductance

$$= mL/2 = 0{\cdot}6 \times 120 \text{ mH} = \underline{72 \text{ mH.}}$$

* See, for example, L. C. Jackson, *Wave Filters*, Methuen, 3rd Edition, 1960, Chapter 2.
† *ibid.*, pp. 12 and 13.
‡ *ibid.*, pp. 27 to 30. See, also, the solution to Problem 305.
§ *ibid.*, p. 32.
∥ *ibid.*, p. 43.
¶ *ibid.*, pp. 15–17.

(a)   (b)
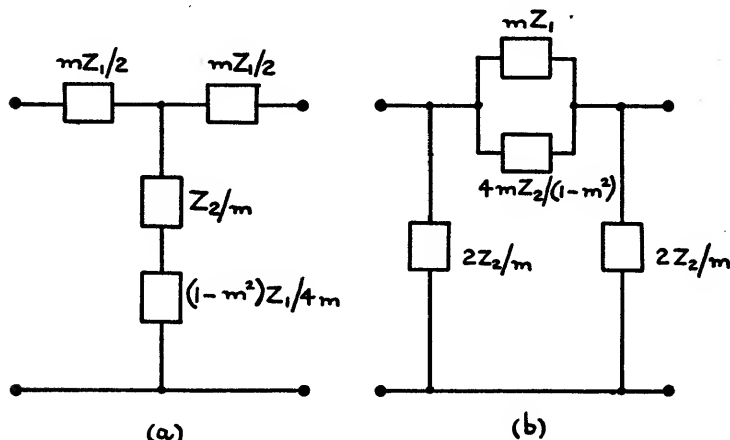
Also, inductance in shunt arm

$$= (1 - m^2)240/2\cdot4 \text{ mH} = \underline{64 \text{ mH}}.$$

Capacitance in shunt arm

$$= 0\cdot666 \times 0\cdot6 \,\mu\text{F} = \underline{0\cdot4 \,\mu\text{F}}.$$

Similarly, for π-section, inductance in series arm

$$= 0\cdot6 \times 240 \text{ mH} = \underline{144 \text{ mH}}.$$

Also, capacitance in series arm

$$= 0\cdot666 \times (1 - m^2)/4m \,\mu\text{F} = \underline{0\cdot178 \,\mu\text{F}}.$$

Capacitance of each shunt arm

$$= 0\cdot333 \times 0\cdot6 \,\mu\text{F} = \underline{0\cdot2 \,\mu\text{F}}.$$

306. The radiation resistance of a small loop aerial* is

$$R = 31,171 \, (A/\lambda^2)^2 \text{ ohms}$$

where A is the area of the loop (m²) and λ is the wavelength (m).

Here $A = \pi(\lambda/4\pi)^2/4$ and $\underline{R = 0\cdot77\,\Omega}$.

307. The radiation resistance R of a short dipole is† $80\pi^2(l/\lambda)^2$ ohms
where l/λ is the line length in wavelengths.

In this case,

$$R = 80\pi^2 \times (1/12)^2 \,\Omega = \underline{5\cdot5\,\Omega}.$$

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* See J. D. Kraus, *Electromagnetics*, McGraw-Hill, 1953, pp. 486–9; *or* S. Ramo
and J. R. Whinnery, *Fields and Waves in Modern Radio*, Wiley, 2nd Edition, 1953,
pp. 189–190 and 457–8.
† See J. D. Kraus, *Electromagnetics*, McGraw-Hill, 1953, pp. 500–1.

The directivity is defined as the ratio of the maximum radiation intensity to the average radiation intensity. For a short dipole this is* 1·5.

308. It is often useful to consider that a receiving aerial possesses an aperture, or equivalent area, over which it extracts energy from a radio wave.

The aperture A is given by† $D\lambda^2/4\pi$ where D is the directivity and λ is the wavelength.

In this case $A = 90 \times 4/4\pi \text{ m}^2 = 28\cdot6 \text{ m}^2.$

309. The field strength due to a distant transmitting station is, neglecting absorption, given by‡:

$E = 377(hI/\lambda d)$ volts/m

where $h = $ effective height of transmitting aerial in metres,

   $d = $ distance in metres,

   $\lambda = $ wavelength in metres

and $I = $ aerial current in amperes.

The power radiated from an aerial‡ $W = 1\cdot58h^2I^2/\lambda^2$ kW.

∴ $E = 300\sqrt{W}/d$ volts/m.

Here $W = 100, d = 100 \times 10^3$ m, so $E = 0\cdot03$ V/m.

310. Using the expression for W in the previous solution and noting that $h = 100$ m, $I = 450$ A and $\lambda = 7\cdot5 \times 10^3$ m, W is found to be 56·9 kW.

Radiation resistance‡ $= 1{,}580 \times h^2/\lambda^2$ ohms
$$= 15\cdot8/7\cdot5^2 \,\Omega$$
Efficiency $= 15\cdot8/(7\cdot5^2 \times 1\cdot12) = 0\cdot251 = 25\cdot1\%.$

311. In an aerial array with finite spacing, the total field in a direction at an angle θ with the normal to the array is‡

$$E = E_1 \sin(N\alpha/2)/\sin(\alpha/2),$$

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* See J. D. Kraus, *Electromagnetics*, McGraw-Hill, 1953, pp. 502–4.

† *ibid.*, pp. 504–7.

‡ See, for example, *Admiralty Handbook of Wireless Telegraphy*, Vol. II, 1938, Section R.

where E_1 is the field due to one aerial,

N „   „ number of vertical aerials,

and α „   „ phase difference between the radiations of consecutive aerials in the given direction.

Also $\alpha = (2\pi a/\lambda) \sin \theta \pm \phi$,

where a is the aerial spacing,

λ „   „ wavelength,

and ϕ „   „ phase difference between the currents in adjacent aerials.

E is zero when $N\alpha/2 = \pi, 2\pi$, etc.

If the aerial currents are in phase, as in the present problem, $\phi = 0$ and the first zero occurs when $\sin \theta = \lambda/Na$. In this case,

$$\sin \theta = \lambda/(10\lambda/2) = 1/5.$$

The angular width of the broadside beam $= 2\theta = 23° \, 4'$.

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312. The voltage received in a frame aerial in the plane of propagation of the wave is* $2\pi EAN/\lambda$ volts, where A is the frame area in sq. m., N is the number of turns, E is the field strength in V/m and λ is the wavelength in m.

Here $E = 0·01$, $A = 1$, $N = 12$ and $\lambda = 300$, so the voltage received $= 25·14 \times 10^{-4}$ V.

---

313. The Schering bridge circuit is shown in the diagram.

At balance:

$$\frac{1/j\omega C_1}{\rho_2 + 1/j\omega C_2} = \frac{1/(1/R_4 + j\omega C_4)}{R_3} \qquad . \qquad . \qquad . \quad (1)$$

Equating real and imaginary parts of this equation gives:

$$C_2 = R_4 C_1/R_3 \qquad . \qquad . \qquad . \qquad . \quad (2)$$

and $$\rho_2 = C_4 R_3/C_1 \qquad . \qquad . \qquad . \qquad . \quad (3)$$

The loss angle δ_2 is given by

$$\tan \delta_2 = \omega \rho_2 C_2 \qquad . \qquad . \qquad . \qquad . \quad (4)$$

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* See, for example, *Admiralty Handbook of Wireless Telegraphy*, **Vol. II, 1938,** Section T.
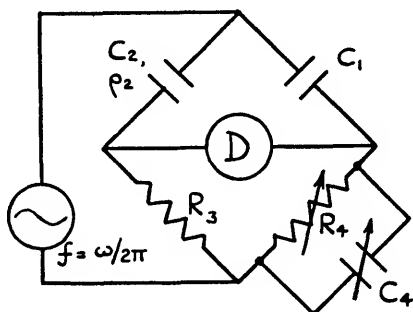
In this case, $C_2 = \varepsilon\varepsilon_0 A/t$ farads, where ε is the dielectric constant, $\varepsilon_0 = 8 \cdot 855 \times 10^{-12}$, A is the area of the plates in sq. metres and t is the distance between the plates in metres.

$\therefore$ $C_2 = 0 \cdot 000213 \, \mu\text{F}.$

From (2) therefore

$$R_4 = 4{,}260 \, \Omega.$$

From (4) $\tan 9' = (2\pi \times 50)\rho_2 \times 0 \cdot 000213 \times 10^{-6}$

i.e. $\rho_2 = 39{,}200 \, \Omega.$

$\therefore$ from (3) $C_4 = 0 \cdot 00196 \, \mu\text{F}.$

314. At balance the following equations are obtained:

$$i_1(R + j\omega L) = i_2 \, Q + i_3 \, r \qquad . \qquad . \qquad . \quad (1)$$

$$i_1 P = i_3/j\omega C \qquad . \qquad . \qquad . \qquad . \quad (2)$$

and $i_3 \, r + i_3/j\omega C = (i_2 - i_3)S \qquad . \qquad . \qquad . \quad (3)$

From (1), (2) and (3), eliminating i_1, i_2 and i_3:

$$\{(R + j\omega L)/j\omega CP - r\}S = Q(r + S + 1/j\omega C) \qquad . \quad (4)$$

Equating real and imaginary parts of (4):

$$R = PQ/S \text{ and } L = CP(rQ + SQ + rS)/S.$$

In this case, $R = 500 \times 1{,}000/1{,}000 = \underline{500 \, \Omega}$

and $L = 2 \times 10^{-6} \times 500(200 + 1{,}000 + 200)10^3/1{,}000 = \underline{1 \cdot 4 \text{ H}}.$

315. At balance the following equation holds:

$$(R_1 + j\omega L_1)/(R_2 + j\omega L_2) = (R_4 - j/\omega C_2)/(R_3 - j/\omega C_1) \quad . \quad (1)$$

Equating the real and imaginary parts of (1):

$$R_1 R_3 + L_1/C_1 = R_2 R_4 + L_2/C_2 \qquad . \qquad . \quad (2)$$

and $\quad - R_1 C_2 + \omega^2 L_1 C_1 R_3 C_2 = - R_2 C_1 + \omega^2 L_2 C_2 C_1 R_4 \quad . \quad (3)$

From (3) it follows that $L_1 C_1 R_3 C_2$ must equal $L_2 C_2 C_1 R_4$.

i.e. $\qquad\qquad\qquad\qquad L_1 R_3 = L_2 R_4 \quad . \qquad . \qquad . \qquad . \quad (4)$

Also from (3) $R_1 C_2$ must equal $R_2 C_1 \qquad . \qquad . \qquad . \qquad . \quad (5)$

From (2) it follows that if $R_1 R_3 = R_2 R_4$, L_1/C_1 must equal L_2/C_2.

From (4) and (5) $L_1/C_1 = \dfrac{L_2}{C_2} \cdot \dfrac{R_2 R_4}{R_1 R_3} = L_2/C_2$ when $R_1 R_3 = R_2 R_4$.

Similarly, if $L_1 = C_1 R_2 R_4$ it follows from (2) that $R_1 R_3$ must equal

L_2/C_2. From (4) and (5) $R_1 R_3 = \dfrac{R_2 C_1}{C_2} \cdot \dfrac{L_2 R_4}{C_1} = \dfrac{L_2}{C_2}$ when $L_1 = C_1 R_2 R_4$

$\therefore$ either $R_1 R_3 = R_2 R_4$ or $L_1 = C_1 R_2 R_4$.

316. The network is shown in the diagram.
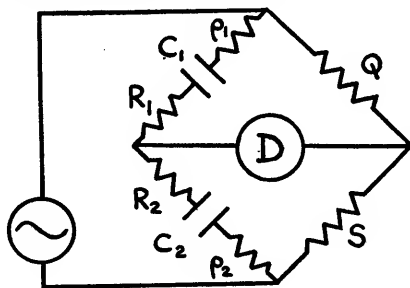
At balance:

$$Q/S = (R_1 + \rho_1 + 1/j\omega C_1)/(R_2 + \rho_2 + 1/j\omega C_2) \qquad . \quad (1)$$

Equating real and imaginary parts of (1):

$$Q/S = (R_1 + \rho_1)/(R_2 + \rho_2) = C_2/C_1 \qquad . \qquad . \quad (2)$$

Here $R_1 = 11\cdot4\,\Omega$, $\rho_2 = 0$, $R_2 = 10\,\Omega$, $C_2 = 0\cdot023\,\mu\text{F}$

$\therefore$ from (2) $\rho_1 = 1\cdot1\,\Omega$ and $C_1 = 0\cdot0184\,\mu\text{F}$.

317. Let the currents and voltages be as shown on the diagram.
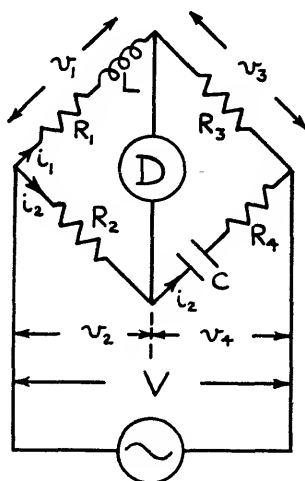
At balance:

$$(R_1 + j\omega L)i_1 = R_2 i_2 \qquad . \qquad . \qquad . \qquad . \quad (1)$$

$$R_3 i_1 = (R_4 + 1/j\omega C)i_2 \quad . \qquad . \qquad . \quad (2)$$
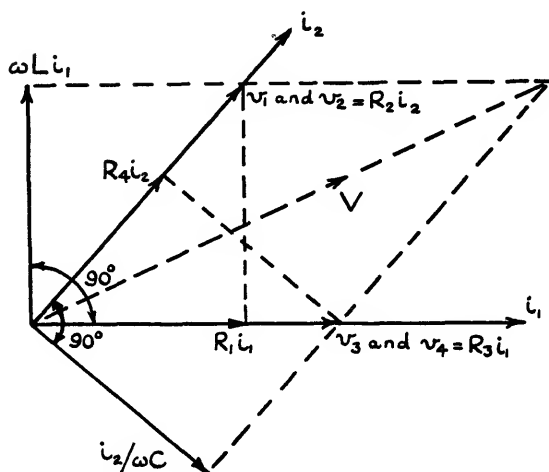
Dividing (1) and (2) and equating real and imaginary parts gives two equations in terms of L and R_1 from which:

$$L = R_2R_3C/(1 + \omega^2R_4^2C^2)$$

and

$$R_1 = R_2R_3R_4\omega^2C^2/(1 + \omega^2R_4^2C^2).$$

The vector diagram for the network is as shown.

318. At balance let the current through L_1, R_1 and R_3 be i_1 and the current through L_2, R_2 and R_4 be i_2.

Then:

$$i_2(R_2 + j\omega L_2) + j\omega M(i_1 + i_2) = (R_1 + j\omega L_1)i_1 \qquad . \quad (1)$$

and

$$i_2R_4 = i_1R_3 \quad . \qquad . \qquad . \quad (2)$$

Dividing (1) and (2) and equating real and imaginary parts

$$\underline{R_2R_3 = R_1R_4 \text{ and } R_3(L_2 + M) = R_4(L_1 - M).}$$

319. With S open the balance condition is:

$$C_1 = C_2C_3/(C_2 + C_3) \qquad . \qquad . \qquad . \qquad . \quad (1)$$

With S closed the balance condition is:

$$C_1 = C_2'(C_3 + C_x)/(C_2' + C_3 + C_x) . \qquad . \quad (2)$$

From (1) and (2),

$$\underline{C_x = C_3^2(C_2 - C_2')/(C_2'C_2 + C_2'C_3 - C_2C_3).}$$

With S open and with C_1 adjusted then at balance

$$C_1 = 1{,}000 \times 50/1{,}050 = 47 \cdot 6\ \mu\mu\text{F}.$$

With S closed, $47 \cdot 6 = C_2' \times 51/(C_2' + 51) \quad \therefore \quad C_2' = 714\ \mu\mu\text{F}.$

Thus $\qquad C_2 - C_2' = (1{,}000 - 714)\ \mu\mu\text{F} = 286\ \mu\mu\text{F}.$

There are two readings, one at the maximum setting of C_2 and the other at $C_2 = 286\ \mu\mu\text{F}$, so the reading error is $\pm\ 10 \times 100/286\%$ $\underline{= \pm\ 3 \cdot 5\%.}$

320. The equivalent circuit of the arrangement is shown.
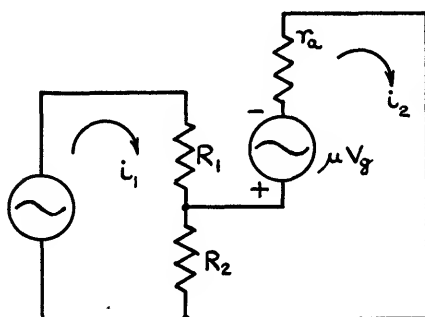
Using Kirchhoff's law for the i_2 mesh

$$\mu V_g + i_2(r_a + R_2) - i_1 R_2 = 0.$$

Also $\quad V_g = i_1 R_1$

$$\therefore \ \mu i_1 R_1 + i_2(r_a + R_2) - i_1 R_2 = 0.$$

At balance $i_2 = 0$,

$$\therefore \qquad \mu = R_2/R_1.$$

In this case μ is approx. 20.

321. Denote the anode resistance by r_a.

At balance, $r_a = R_2 R_3/R_1 = R_2 = \underline{8,000\ \Omega}$, since $R_1 = R_3$.

322. The solution to this problem can be found elsewhere.* If

$$\mu R_1/R_2 \gg 1, \text{ then } g_m = R_2/R_3 R_1.$$

323. $$I_a = (a_0 + a_1 V_{g_1})(b_0 + b_1 V_{g_2}).$$

Let $\quad V_{g_1} = V_s \cos \theta - E_{b_1}$ and $V_{g_2} = V_h \cos \theta - E_{b_2}$.

[Suffix s refers to signal and h to oscillator.]

For the in-phase condition of V_s and V_h:

$$I_a = \{a_0 + a_1(V_s \cos \theta - E_{b_1})\}\{b_0 + b_1(V_h \cos \theta - E_{b_2})\}.$$

The d.c. component of this is

$$I_{a_1} = [a_0 b_0 - a_1 b_0 E_{b_1} - a_0 b_1 E_{b_2} + (a_1 b_1 V_s V_h/2)].$$

For the out-of-phase condition $V_{g_1} = - V_s \cos \theta - E_{b_1}$ and the d.c. component of I_a is

$$I_{a_2} = [a_0 b_0 - a_1 b_0 E_{b_1} - a_0 b_1 E_{b_2} - (a_1 b_1 V_s V_h/2)].$$

$$\therefore \qquad I_{a_1} - I_{a_2} = a_1 b_1 V_s V_h.$$

But $$g_c = a_1 b_1 V_h/2$$

$$\therefore \qquad \underline{g_c = (I_{a_1} - I_{a_2})/2V_s.}$$

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* See S. Seely, *Electron Tube Circuits*, McGraw-Hill, 2nd Edition, 1958, pp. 71 and 72.

324. Let $V_x = V_1 \sin(\omega t + \theta_1)$ and $V_y = V_2 \sin(\omega t + \theta_2)$.

$$\therefore \qquad V_x = V_1 (\sin \omega t \cos \theta_1 + \cos \omega t \sin \theta_1) \qquad . \qquad . \qquad (1)$$

and $\qquad V_y = V_2 (\sin \omega t \cos \theta_2 + \cos \omega t \sin \theta_2) \qquad . \qquad . \qquad (2)$

From (1) and (2) eliminating ωt gives:

$$\frac{V_x^2}{V_1^2} + \frac{V_y^2}{V_2^2} - \frac{2 V_x V_y}{V_1 V_2} \cos(\theta_1 - \theta_2) = \sin^2(\theta_1 - \theta_2).$$

This is the equation of an ellipse whose major and minor axes coincide with the x and y axes respectively when $(\theta_1 - \theta_2) = \pi/2$. In general, the trace gives an ellipse the orientation of which depends on the phase difference between the two voltage waves.

325. The solution to this problem can be found elsewhere.*

326. The solution to this problem can be found elsewhere.*

327. The solution to this problem can be found elsewhere.*

328. The solution to this problem can be found elsewhere.†

329.
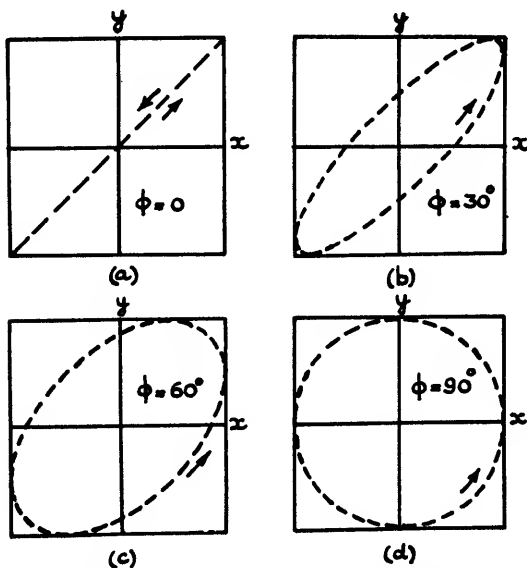
* F. A. Benson and A. O. Carter, 'A Critical Survey of Some Phase-angle Measurements Using a Cathode-ray Tube,' *Electronic Engineering*, 22, pp. 238–42, 1950.
† F. A. Benson and A. O. Carter, 'A New Method of Measuring Phase Angle,' *Journal of Scientific Instruments*, 26, pp. 285–88, 1949.

330.

$n = 2$

$n = 3$

331.

(a) $\phi = 0$

(b) $\phi = 30°$

(c) $\phi = 60°$

(d) $\phi = 90°$

(e) $\phi = 120°$

(f) $\phi = 180°$
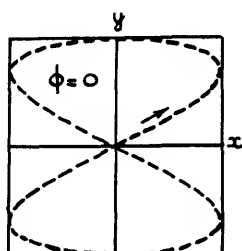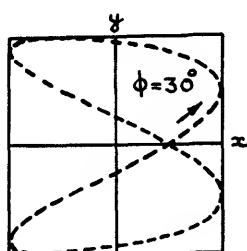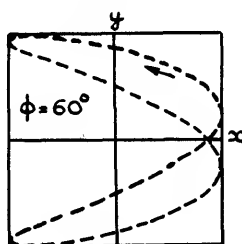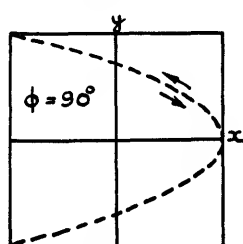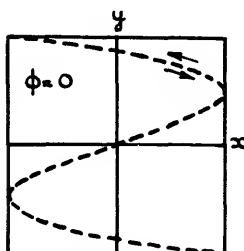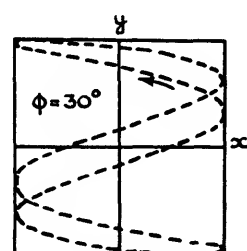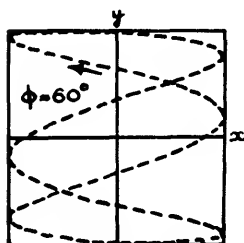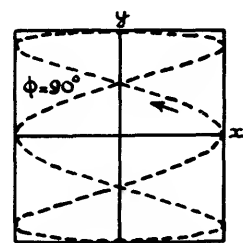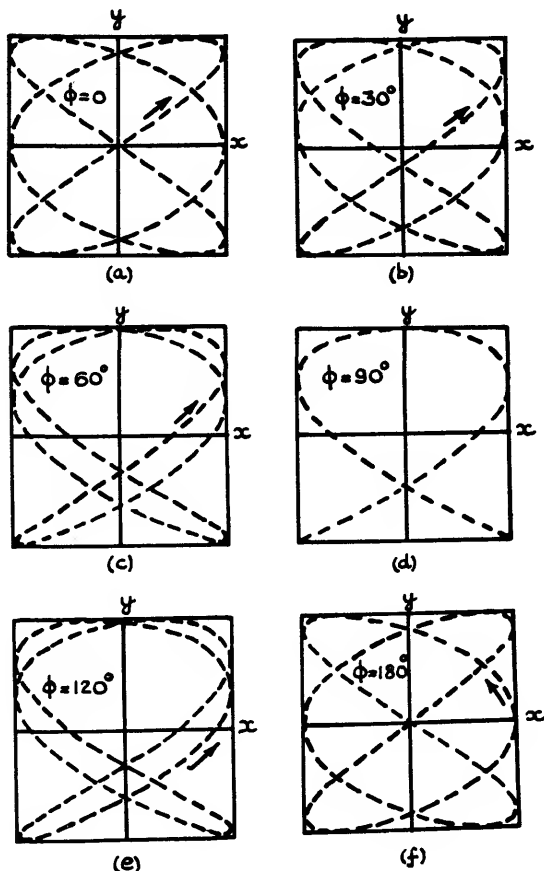
332. One method of finding the frequency ratio is as follows: draw a line parallel to the x-axis, which does not pass through any intersections of different parts of the curve, and count the number of points where the line intersects the curve; repeat this procedure with a line drawn parallel to the y-axis. It should be noted that the lines must not be part of the boundary rectangle. The ratio of the two numbers so found will be the required frequency ratio. The number of intersections on the line parallel to the x-axis is proportional to the frequency of the y-variation, and vice versa.*

* For further information see G. Parr and O. H. Davie, *The Cathode-Ray Tube and its Applications*, Chapman & Hall, 3rd Edition, 1959, Chapter 6. See also, F. E. Terman, *Radio Engineer's Handbook*, McGraw-Hill, 1943, p. 955.

333. The skin depth $\delta = 1/\sqrt{\pi\mu f\sigma}$

where σ is the conductivity, μ the permeability and f the frequency.[*]

For copper,

$$\delta = 1/\sqrt{\pi \times 4\pi \times 10^{-7} \times f \times 5\cdot88 \times 10^7}\ \text{m}$$

When $f = 300$ Mc/s, $\delta = 3\cdot79 \times 10^{-4}$ cm.

When $f = 10{,}000$ Mc/s, $\delta = 6\cdot56 \times 10^{-5}$ cm.

334. Referring to the previous solution, the conductivity of $5\cdot88 \times 10^7$ mhos/m corresponds to a resistivity of $1\cdot7 \times 10^{-6}$ ohm-cm.

$\therefore$ skin depth in nickel at 10,000 Mc/s,

$$= 6\cdot56 \times 10^{-5}\sqrt{9\cdot39/(1\cdot7 \times 3)} = 8\cdot9 \times 10^{-5}\ \text{cm}.$$

335. The inductance of a straight piece of wire at very high frequencies is:[†]

$$L = 0\cdot002l(2\cdot303 \log_{10} 4l/d - 1 + d/2l)\ \mu\text{H},$$

where l is the length of the wire in centimetres and d is the wire diameter in centimetres.

In this case, $l = 2\cdot54$ cm and $d = 0\cdot254$ cm so $L = 0\cdot014\ \mu\text{H}$.

At 500 Mc/s the reactance

$$= 2\pi \times 500 \times 10^6 \times 0\cdot014 \times 10^{-6}\ \Omega = 44\ \Omega.$$

336. The intrinsic impedance of the plate $Z_0 = \sqrt{Z_1 Z_2}$,

where $Z_1 =$ intrinsic impedance of air

and $Z_2 =$ intrinsic impedance of dielectric medium.

Now $Z_1 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$ and $Z_2 = 377/\sqrt{4}\ \Omega = 188\ \Omega$

$\therefore \quad Z_0 = \sqrt{377 \times 188}\ \Omega = 266\ \Omega.$

The relative permittivity required $= (377/266)^2 = 2.$

* See, for example, J. C. Slater, *Microwave Transmission*, McGraw-Hill, 1942, p. 114; *or* R. A. Bailey, 'A Resonant-cavity Torque-operated Wattmeter for Microwave Power,' *Proc. I.E.E.*, Part C, **103**, p. 60, 1956.

† See K. R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, 1948, p. 477.

337. Number of picture elements in one picture
$$= 405 \times 405 \times 5/4.$$

Total number of elements to be transmitted/sec
$$= 405 \times 405 \times 5 \times 25/4 = 5{,}125{,}781$$

$\therefore$ frequency $= 5{,}125{,}781/2$ c/s and bandwidth for double-side-band transmission is 5·13 Mc/s.

338. The output voltage is as shown in the diagram.
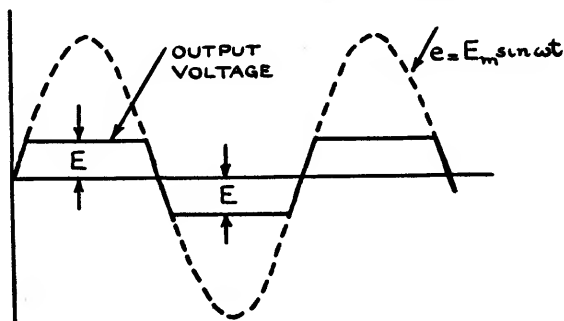
Rate of rise of waveform $= \dfrac{de}{dt} = \omega E_m \cos \omega t \simeq \omega E_m.$

Time taken to rise from $- E$ to $+ E$ is $\delta T \simeq 2E/\omega E_m,$

i.e. $\qquad \delta T = 2 \times 2/(2\pi \times 10^5 \times 200)$ sec $= 0{\cdot}032 \, \mu s.$

339. The charges e_1, e_2 and e_3 on the conductors and the potentials V_1, V_2 and V_3 are related by the following equations:
$$e_1 = q_{11}V_1 + q_{21}V_2 + q_{31}V_3$$
$$e_2 = q_{12}V_1 + q_{22}V_2 + q_{32}V_3$$
$$e_3 = q_{13}V_1 + q_{23}V_2 + q_{33}V_3$$

Let e_1 and $e_2 = 0$ and suppose conductor 3 is raised to unit potential by e_3. Since 1 and 2 are uncharged their potentials are also unity.

Therefore, the above equations become:
$$0 = q_{11} + q_{21} + q_{31}$$
$$0 = q_{12} + q_{22} + q_{32}$$
$$e_3 = q_{13} + q_{23} + q_{33}$$
$\therefore \qquad\qquad e_3 = q_{33} - (q_{11} + 2q_{12} + q_{22})$

and capacitance of 3 $= e_3/1 = q_{33} - (q_{11} + 2q_{12} + q_{22}).$

340. Let the initial charge on the movable sphere 4 be e and let the charges received by spheres 1, 2 and 3, after contact with 4, be e_1, e_2 and e_3 respectively.
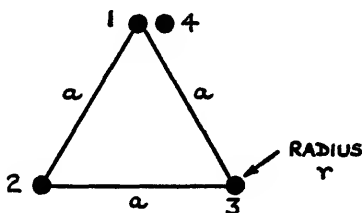
When 1 and 4 are in contact let the common potential be V_1.

Then

$$V_1 = p_{11}e_1 + p_{14}(e - e_1) \qquad . \qquad . \qquad . \quad (1)$$

and

$$V_1 = p_{14}e_1 + p_{44}(e - e_1) \qquad . \qquad . \qquad . \quad (2)$$

where the p's are coefficients of potential.

When 2 and 4 are in contact let the common potential be V_2. The charge on 4 is now $e - e_1 - e_2$. By symmetry the coefficients of potential of 2 and 4 are the same as the coefficients of potential of 1 and 4 when 1 and 4 were in contact, although the potentials of 2 and 4 will contain a term due to the charge e_1 on 1 which can be taken as e_1/a since $a \gg r$.

$$\therefore \qquad V_2 = p_{11}e_2 + p_{14}(e - e_1 - e_2) + e_1/a \qquad . \qquad . \quad (3)$$

and

$$V_2 = p_{14}e_2 + p_{44}(e - e_1 - e_2) + e_1/a \qquad . \qquad . \quad (4)$$

Similarly if V_3 is the common potential when 3 and 4 are in contact,

$$V_3 = p_{11}e_3 + p_{14}(e - e_1 - e_2 - e_3) + e_1/a + e_2/a \qquad . \quad (5)$$

and

$$V_3 = p_{14}e_3 + p_{44}(e - e_1 - e_2 - e_3) + e_1/a + e_2/a \qquad . \quad (6)$$

Subtracting (2) from (1),

$$0 = (p_{11} - p_{14})e_1 + (p_{14} - p_{44})(e - e_1) \qquad . \qquad . \qquad . \quad (7)$$

Subtracting (4) from (3),

$$0 = (p_{11} - p_{14})e_2 + (p_{14} - p_{44})(e - e_1 - e_2) \qquad . \qquad . \quad (8)$$

Subtracting (6) from (5),

$$0 = (p_{11} - p_{14})e_3 + (p_{14} - p_{44})(e - e_1 - e_2 - e_3) \qquad . \quad (9)$$

$$\therefore \quad \frac{e_1}{e - e_1} = \frac{e_2}{e - e_1 - e_2} = \frac{e_3}{e - e_1 - e_2 - e_3}, \text{ so that } \underline{e_2{}^2 = e_1 e_3.}$$

341. Let the charges on A, B and C be e_1, e_2 and e_3 respectively. Then the charges and potentials (V's) are related by the following equations:

$$e_1 = q_{11}V_1 + q_{21}V_2 + q_{31}V_3 \quad . \quad . \quad . \quad (1)$$

$$e_2 = q_{12}V_1 + q_{22}V_2 + q_{32}V_3 \quad . \quad . \quad . \quad (2)$$

$$e_3 = q_{13}V_1 + q_{23}V_2 + q_{33}V_3 \quad . \quad . \quad . \quad (3)$$

where q_{11}, q_{22} and q_{33} are coefficients of capacitance and the other q's are coefficients of induction. Consider the special case where B is at zero potential and A is uncharged. Then the potential inside B is zero.

$\therefore$ from (1), $q_{31}V_3 = 0$ and so $q_{31} = 0$ since the value of V_3 is unrestricted.

i.e. C can have any potential without affecting A.

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342. Consider an element of wire dx at A distant x from B. The
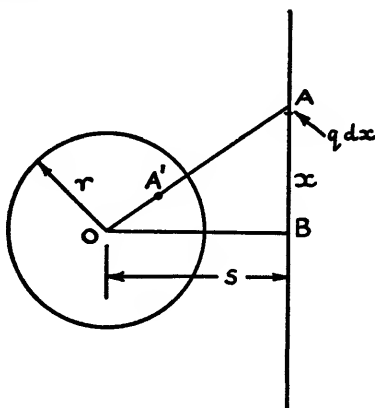
charge on the element is $q\,dx$. There is an image at the inverse point A' with a charge $- q\,dx \cdot r/OA = - qr\,dx/\sqrt{x^2 + s^2}$.

$\therefore$ total induced charge

$$= \int_{-L/2}^{+L/2} - qr\,dx/\sqrt{x^2 + s^2} = - 2qr \sinh^{-1}(L/2s).$$

343. The conditions of the problem are satisfied if an image charge $- ea/f$ is placed at A', where $OA \cdot OA' = a^2$, and a charge $(Q + ea/f)$ at O.

20
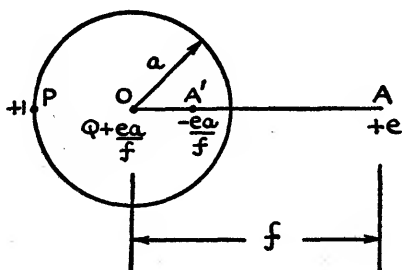
The electric force on a unit positive charge at P is

$$(Q + ea/f)/a^2 + e/(f + a)^2 - ea/f(a + a^2/f)^2.$$

This is zero, and hence the surface density, when

$$Q = - a^2 e(3f + a)/f(f + a)^2.$$

344. The charges are $+ e$ at A, an image $- ea/f$ at A', where $OA \cdot OA' = a^2$, and ea/f at O.
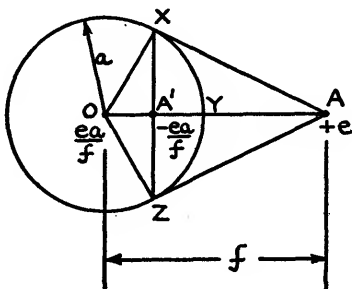
The solid angles which the spherical cap XYZ subtends at A, A' and O respectively are:

$$2\pi(1 - \cos OAX), 2\pi \text{ and } 2\pi(1 - \cos XOA)$$

i.e. $\qquad 2\pi(1 - \sqrt{1 - a^2/f^2}), 2\pi \text{ and } 2\pi(1 - a/f)$

$\therefore$ the charge induced on this portion of the sphere is

$$- e(1 - \sqrt{1 - a^2/f^2})/2 - ea/2f + ea(1 - a/f)/2f$$
$$= - e\{1 + a^2/f^2 - \sqrt{1 - a^2/f^2}\}/2.$$

345. Let the original charge on I be e.

Then $V = \dfrac{e}{a} - \dfrac{e}{b}$ or $e = abV/(b - a)$.

Let P_1 be the inverse of P in conductor I
   „ R_1 „ „   „   „ P_1 „   „ II
   „ P_2 „ „   „   „ R_1 „   „ I
   „ R_2 „ „   „   „ P_2 „   „ II
etc.
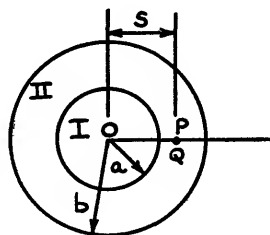
Then $OP_1 = a^2/s$ and the image charge at $P_1 = -\,Qa/s$,

$OR_1 = b^2/OP_1$ and the image charge at $R_1 = -\,(-\,Qa/s)b/OP_1$
$= Qb/a$,

$OP_2 = a^2/OR_1$ and the image charge at $P_2 = (-\,Qb/a)a/OR_1$
$= -\,Qa^2/bs$,

$OR_2 = b^2/OP_2$ and the image charge at $Q_2 = -\,(-Qa^2/bs)b/OP_2$
$= Qb^2/a^2$,

etc.

Similarly let R_1' be the inverse of P in conductor II

and   „ P_1' „ „   „   „ R_1' „   „   I

etc.

Then the image charge at $R_1' = -\,Qb/s$,

   „   „   „   „   „ $P_1' = \quad Qa/b$

   „   „   „   „   „ $R_2' = -\,Qb^2/as$

etc.

$\therefore$ the total charge on conductor I is:

$e + (-\,Qa/s) - Qa^2/bs + Qa/b$, etc.

$$= abV/(b - a) - \frac{Qa}{s}\left(1 + \frac{a}{b} + \frac{a^2}{b^2} + \ldots\right)$$

$$+\, Q\frac{a}{b}\left(1 + \frac{a}{b} + \frac{a^2}{b^2} + \ldots\right)$$

$$= \frac{ab}{(b - a)}\left(-\frac{Q}{s} + \frac{Q}{b} + V\right).$$

The total charge on conductor II is minus the sum of the charges it contains, i.e.

$$- Q - \frac{ab}{(b-a)} \left(-\frac{Q}{s} + \frac{Q}{b} + V \right) = \frac{ab}{(b-a)} \left(-\frac{Q}{a} + \frac{Q}{s} - V \right).$$

Ratio of charges on I and II is therefore

$$(Q/s - Q/b - V)/(Q/a - Q/s + V).$$

346. If V is independent of z:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} = 0$$

Let $V = f(r) \cos 4\theta$

$\therefore$
$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) \cos 4\theta - \frac{16f}{r^2} \cos 4\theta = 0$$

or
$$r \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) = 16f$$

Suppose $f = r^n$

$$\frac{df}{dr} = nr^{n-1} \quad \text{and} \quad r \frac{df}{dr} = nr^n$$

$\therefore$
$$r \frac{d}{dr} \left(r \frac{df}{dr} \right) = n^2 r^n = n^2 f$$

$\therefore$
$$n^2 = 16 \quad \text{or} \quad n = \pm 4$$

i.e.
$$f(r) = r^4 \quad \text{or} \quad r^{-4}$$

The solution is r^{-4} since $V \to 0$ as $r \to \infty$

$$E_r = -\frac{\partial V}{\partial r}$$

$$V = Ar^{-4} \cos 4\theta$$

so
$$E_r = -4Ar^{-5} \cos 4\theta$$

$$\frac{E_{(2r_0)}}{E_{(r_0)}} = \frac{(2r_0)^{-5}}{(r_0)^{-5}} = 2^{-5}$$

i.e.
$$E_{(2r_0)} = E_0/32.$$

347. For the first layer: capacitance $= A\varepsilon_0 K/d$

For the second layer: capacitance $= A\varepsilon_0 K/d$

conductance $= A\sigma/d.$

$$\text{Total impedance} = \frac{d}{j\omega A\varepsilon_0 K} + \frac{\dfrac{d}{A\sigma} \cdot \dfrac{d}{j\omega A\varepsilon_0 K}}{\dfrac{d}{A\sigma} + \dfrac{d}{j\omega A\varepsilon_0 K}}$$

$$= \frac{d}{j\omega A\varepsilon_0 K} \left\{ 1 + \frac{1}{1 + \sigma/j\omega\varepsilon_0 K} \right\}$$

For the composite dielectric impedance $= 2d/j\omega A\varepsilon_0 K'$

$$\therefore \qquad \frac{2}{K'} = \frac{1}{K} \left\{ 1 + \frac{1}{1 + \sigma/j\omega\varepsilon_0 K} \right\}$$

or $\qquad \underline{K' = 2K(K - j\sigma/\omega\varepsilon_0)/(2K - j\sigma/\omega\varepsilon_0).}$

348. The Clausius–Mossotti relationship* is:

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{N\alpha}{3\varepsilon_0}$$

To calculate the polarizability α of a sphere:

$$\text{Dipole moment} = 4\pi\varepsilon_0 \, r^3 \, E$$

where E is the electric field and r the sphere radius.

$$\therefore \qquad \alpha = 4\pi\varepsilon_0 r^3$$

$$= 4\pi \times \frac{10^{-9}}{36\pi} \times (10^{-3})^3$$

$$= 10^{-18}/9$$

$$\therefore \qquad N = \left(\frac{1}{4/1000} \right)^3 = 10^9/64.$$

Thus, $\qquad \dfrac{\varepsilon - 1}{\varepsilon + 2} = \dfrac{10^9}{64} \cdot \dfrac{10^{-18}}{9} \cdot \dfrac{36\pi}{3 \times 10^{-9}}$

$$\therefore \qquad \underline{\varepsilon = 1\cdot 21.}$$

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* See, for example, B. I. Bleaney and B. Bleaney, *Electricity and Magnetism*, Oxford, 1957, p. 493.

349. The Clausius–Mossotti relation* is:

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{N\alpha}{3\varepsilon_0}$$

$\therefore$
$$\frac{N\alpha}{3\varepsilon_0} = \frac{1 \cdot 25}{4 \cdot 25}$$

For the expanded polythene,

$$\frac{N\alpha}{3\varepsilon_0} = \left(\frac{1 \cdot 25}{4 \cdot 25}\right) \frac{5}{100} = 0 \cdot 0148$$

$\therefore$
$$\frac{\varepsilon - 1}{\varepsilon + 2} = 0 \cdot 0148$$

i.e.
$$\varepsilon = 1 \cdot 058.$$

* See, for example, B. I. Bleaney and B. Bleaney, *Electricity and Magnetism*, Oxford, 1957, p. 493.

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